artificial and natural classifications is then one of degree, and it would be well to discard the terms altogether.

6. **General or Scientific Classifications.**—The aim of all scientific classification is to group objects in thought according to their actual affinities. These affinities, as has been said, are many, and differ according to the aspect of truth under consideration. The grouping of plants according to their poisonous qualities would be different from the botanical grouping. But plants only come into the former classification incidentally—they are not the reason and justification of its existence. In the botanical classification they hold the supreme position. Yet our knowledge of the world needs to embrace the former relations as well as the latter; and it would be absurd to say that from that widest of all points of view the one is more essential than the other.

Yet, as we have seen, certain classifications agree most closely with the most marked and obvious groupings of things. The botanical is thus the most natural classification for plants, and indeed the only one which regards them simply as plants. But it is as plants that they have their place in existence; as drugs or as food their existence is regarded not in itself, but simply in certain relations to man or animal. So the botanical classification of plants may be regarded as more fundamental than the medicinal or the agricultural, and as dealing more directly with the nature of the things themselves.

Still, this must not be pushed too far. For the complete nature of a poisonous plant is only known when its power of harmful reaction on animal life is known. So long as a classification aims at setting out truly relations and affinities actually existing in things it is scientific in its reference and in its nature. Whatever the purpose in view—as indicated by the highest genus chosen—the questions to be asked at each step are: In this relation what is the true nature of this object? what are its affinities to other objects? how, then, for this purpose should it be classed so that the arrangement of classes shall correspond with the degree of closeness of these affinities?

AN

INTERMEDIATE LOGIC

of the order in which the two aspects of reality are presented to us, and this difference can have no effect either upon reality itself or upon our final conception of it, when we know it in both its aspects.

5. Nature of Inference.—It is now time to examine the nature of inference which is the most important characteristic of method. It is evident from the general account of it just given that induction and deduction, and also analysis and synthesis to which they to some extent correspond, imply inference. But it may be urged that testimony and observation, which give the particulars from which induction starts, are not on quite the same footing in this respect. Testimony, it will be granted, is one remove or more from the facts which it affirms, and the process by which we get back to them is clearly inferential, but surely in observation the contact with fact is direct, and things *are* as a man senses them. Yet mistakes in observation clearly show that things often are not what they are observed to be, and it will be evident later that error creeps in for the very reason that inference was present and was incorrectly made.[1] We may assert, then, that method is characterised in all its stages by inference.

The challenge of the truth of a judgment may arise from the practical needs of life or from the ideal of demonstration which science sets before itself. In any case it can only be met by developing the judgment either in the direction of the reasons which support it or in that of the consequences which flow from it. At every step both reasons and consequences may be true or false, and the evidence for them must be examined until data are reached which are undisputed. The chain of reasoning will be complete when the judgment is seen to follow necessarily through all the intermediate steps from those facts or principles which are beyond question. The proposition is then said to be proved, and the process of inference is complete. Now it is of no importance from the logical

[1] See Ch. 26.

AN INTERMEDIATE LOGIC

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PREFACE.

This book is based upon the *Manual of Logic* of which the last edition was first published fifteen years ago. Since that time the need has arisen for a simpler and briefer treatment adapted to the intermediate university examinations to which Logic has been transferred from the final degree examinations.

Moreover, controversy on the more modern parts of logical doctrine has had its legitimate outcome in a general agreement among thinkers on the subject. We are glad, in consequence, to be able to adopt a less controversial tone than was deemed advisable in the presentation of induction in the former work.

Especially in the material logic of method and of induction, then, a new version was desirable, and the opportunity was seized for a thorough revision of the treatment of the long accepted doctrines of formal logic, and for some important re-arrangement of topics. In particular we believe that much is gained by placing the treatment of 'Method' in such a position that the old doctrine of the syllogism is seen still to be an essential part of logical theory, and not merely an archaeological monument of the perverted ingenuity of our forefathers. We have also deemed it convenient to our readers to treat of the various forms of fallacy in immediate connexion with those parts of logical doctrine against which they offend. If this has the further

effect of brightening the way along which the student must of necessity find much of difficulty, it will be all to the good; and nothing is usually so entertaining as the mistakes of others.

We have added some questions on each part of logical doctrine, and we would earnestly urge every student who desires to become master of the subject to struggle with those exercises till he has overcome them.

J. W.
A. J. M.

THE UNIVERSITY, LEEDS,
June 1911.

NOTE TO REVISED EDITION.

THIS edition has been enlarged by the addition of a new chapter (XVI.) dealing with applications of general method to the great branches of knowledge, and of a section on the logical aspect of Probability. Other changes are confined to correction of errata.

J. W.
A. J. M.

Sept. 1922.

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CHAPTER I.

THE NATURE OF LOGIC.

1. **Knowledge.**—Men have always thought as naturally as they have eaten, and they have found the matter on which their thoughts work, as well as that which satisfies their hunger, among the things around them. But the advance of what is known as civilisation is attended by an advance in man's demands in each case. The civilised and cultured man rejects with equal disdain the rude dishes which gratify the palate of the primitive savage and the explanations of the phenomena of the world which satisfy his mind.

Such an advance would be quite inconceivable if each generation had to start anew at the beginning of things. It is only because men of our day can profit by the experiences of those who have preceded them that advance either in knowledge or in practical efficiency is possible. This implies that the knowledge gained by one can be communicated to others, so that human advance is in all its aspects a kind of co-operative movement. In other words, without the means of communication—in its broadest sense, language—man must have failed to improve the character of his life as surely as the lower animals have done.

There is probably no need to demonstrate at length that man's advance has not been an uninterrupted triumphal march. Every sane person is conscious of the truth of the old Latin proverb "*Humanum est errare.*" We all fall

into errors—both practical and theoretical. Savage man, whose interests are very directly practical, avoids death and disaster less surely than we do with all the resources of medical science—that is, knowledge—at our disposal. And savage man can do much less than we can to lead the forces of nature into bondage to his desires The great ocean-liner is an immense advance on the dug-out canoe, yet the ocean-liner at times meets with disaster. The express train rushing along the rails at sixty miles an hour is striking evidence of the extent to which man's practical knowledge has advanced: that train hurling itself in collision upon another, which is—but ought not to be—in the way, is an illustration of his liability to error. In how many human disasters is it not possible to find the explanation in "Some one has blundered"?

Practical failures bring conviction home to all of us that we are not infallible. In matters of merely theoretical knowledge we are much more apt to regard our results, not only as the last word which has been said on the subject, but as the last which can be said. Yet here, too, a modest demeanour would be more becoming. Nor is such modesty at all inconsistent with justifiable pride in what has already been accomplished. That certainly is *much*. We do know *much* about the operations of many of the forces of nature, and we prove it by the skill and success with which we utilise them. But much is still unexplained, and the further we get from the forces we can use directly the more uncertain are our steps. The unexpected discovery of radium illustrates how scientific knowledge thought to be the most perfect may be shown to be inadequate and incomplete.

Nor again have we any reason to suppose that the thought of this age is more final than that of earlier times. The path along which thought has advanced is strewn with the wreckage of discarded explanations, which in their day were regarded as securely established, that is, as known. Their fate has always been due to the same cause—increase of knowledge. Whenever a fact, or class of facts, new to us is proved to exist, our thought of the nature of the world must find an appropriate place for it.

Thus in our day the theories of physics must undergo any modification that is needed to give such a place to radium as will permit its qualities and powers to fit into the general scheme of things physical. So in the sixteenth and seventeenth centuries men had gradually to readjust their thoughts of the relations of the heavenly bodies so as to include the continually increasing knowledge of the facts which the telescope was making possible. In this case the readjustment of the theory was fundamental—the earth was deposed from the position men's thoughts had for centuries assigned it, and was made but a minor satellite in a system whose centre was the sun.

If we look at this striking historical instance we see that it was not the perceived facts which were proved to be other than men had thought: the sun, moon, and stars were—and are—still seen to rise in the east and to set in the west. No, it was only man's theory about their appearance—his explanation of how they could be explained—which was overthrown, because it could not find room for the facts first known when the telescope brought into the range of perception facts which had hitherto been beyond it. In this case only two theories were possible; if one were rejected the other must be accepted. And the heliocentric theory has been found to accommodate within its scope all subsequent discoveries of pertinent facts. So we regard it as certain—that is, as established truth.

It appears, then, that man seeks explanations, but that his explanations are subsequently discarded if they are found not to account for growing knowledge. This is to say that he regards consistency in the whole sphere of his knowledge as the one essential condition of sane thought. If the facts are so and so, then the explanation must fit them. For the facts constrain us. They are parts of the real world of things of which each one of us is also a part—and they are the parts which are most directly open to our experience.

The explanations, however, are constructions, made by our own minds, of possible relations between things; and they may, or they may not, reproduce in thought the relations which really exist. The savage believes the

tempest to be raised by a spiritual enemy. We, equally with him, accept the tempest as real, but we reject his explanation of its origin as not in correspondence with the truth. And this rejection is based on fuller knowledge of natural occurrences than the savage has attained. To us many things are 'natural' because we can explain them by relations of thing to thing, which to the savage are 'supernatural' because inexplicable by any known relations of natural things.

Human thought, then, accepts the facts of experience, and tries to explain them. The explanation always takes the form of fitting them into such a system that each one of them appears as the necessary result of the assumed relations of the others. Into such a system no contradiction can be admitted; that immediately destroys it. The whole advance of science—*i.e.* of exact knowledge, no matter what the topic—is the building up in thought of such systems. Each science has its own system. Chemistry regards material things from one point of view, physics from another. Yet they study the same objects. So that thought must not only demand that chemistry be consistent with itself and physics with itself, but that chemistry and physics be consistent with each other; and so throughout.

Our separate sciences, then, are all concerned with the same great system of things which we call the world or the universe. But each deals with only one aspect of it—that is, with one class of the forces which make it what it is. Because of the frailty of our intellects it must be so, for no mind is powerful enough to embrace more than one very small corner of existence. Each science works in its own artificially segregated domain, but in reality these domains interpenetrate. So if the sciences are together to explain that reality they must not only be actually in harmony but must be seen to be so in their own relations to each other.

It is plain, however, that we are far indeed from this complete construction in thought of what the universe is in reality. Each science is, as it were, working largely in the dark as to the bearing of its results on the general

scheme of things. Surely we have here an unanswerable proof that our knowledge is very imperfect, and that our explanations, therefore, may need more or less profound modification as that knowledge increases.

So far we have spoken only of the knowledge of mankind as a whole, and we have seen that in its gradual growth it has had to reject some of its beliefs and to adapt others by profound modifications to meet the new conditions of thought due to increase of knowledge—often of the most unexpected kind. But if all this is true of the combined search of men after truth, how much more is it true of the efforts of individual enquirers? One need not go outside one's memory of one's own life to find plenty of examples in which one's own thought has been proved by subsequent experience to be wrong. Have we not harboured suspicions afterwards seen to be unjustified? Have we not believed explanations which we now reject? Have we not discovered errors of thought in others? Have they not believed that they have found similar errors in ourselves?

There is no need to labour the point. Every one who thinks at all knows that at times he thinks wrongly—that is, the system he constructs in his thoughts does not correspond to the relations which hold in reality. His mistakes may or may not be found out either by himself or by others. In matters of small moment many such errors never are discovered, for the time once past no more thought is given to the matter. If, however, further knowledge in any form is brought to bear, then any inconsistency may become manifest, and the fact of error is laid bare even though the exposure does not carry with it in all cases the power of rectification.

2. **Origin and Function of Logic.**—The liability to error in thought naturally raises the question whether rules can be laid down by adherence to which such error can be avoided. Can we set forth a scheme by following which thought can advance safely and surely? The idea is a tempting one, and for centuries men believed it to be realised. Unhappily in those very centuries men's know-

ledge of the world in which they lived made little, if any, advance.

The reason is not far to seek. Such a universally applicable instrument of thought assumes of necessity that the validity of thought can be estimated apart from the matter thought about. In a very narrow sense it can. Within the four corners of any given argument it *can* be decided whether the thought is consistent with itself—whether the conclusion is rightly drawn from the given premises. But it can go no further. And if we wish to know the world, or any part of the world, we need further to ask whether the premises are true, and if so whether they are sufficient, and whether the conclusion agrees with fact. In short, we want, not a little artificial system of thought, shut up in itself and claiming validity merely because in these narrowest of bounds it does not contradict itself, but a system which is in relation to all other systems both of thought and of existence and the validity of which must be judged as a whole by its relations to those systems, not by its own little insignificant self. 'If S is M, and M is P, then S is P' is no doubt true no matter what S, M, and P may be held to represent, so long as each remains uniform in its reference. But the essential questions remain—whether S *is* M and M *is* P; whether these two statements are *all* the knowledge that we have pertinent to the case in hand; and whether 'S is P' is *true in fact*, in just that form.

This traditional *Logic*, then, which claimed to lay down the laws of *pure* thought—*i.e.* of thought considered by itself and regardless of the matter on which it was exercised—was powerless before the vast complexity of reality. It could deduce consequences from given data, and so long as men were agreed on the ultimate assumptions which could be made, the application of this logic in argumentative discussion went merrily on. Eventually such agreement became largely traditional—men tacitly agreed to accept as true all that certain authors of antiquity—especially Aristotle—had written, and to reject as false all that was not to be found in their writings. The result was much acuteness of thought and consequent

clearness of mental vision as to what is really involved in general statements—a point in which the modern mind is apt to be far inferior. But from the very nature of this conception of logic and of thought men's minds were thrown back on themselves, and occupied themselves with their own thoughts regardless of their origin or of their agreement with the real world. As a natural result the knowledge attained of mental processes was much more valuable than that which was gained of the external world.

Yet this logic was not valueless, and it is subject of regret that the conviction of its inadequacy has led to its general neglect Men continually make errors in reasoning of the most elementary formal character, and their readers or hearers fail to detect them. Distinctions are ignored or dismissed as "verbal quibbling" which yet have a very real pertinence to the matter in hand. The general neglect of a study of the formal implications and relations of propositions has been accompanied by an appalling amount of inaccurate and careless reasoning, and by a slipshod habit of mind which is much to be regretted.

With the growth of interest in the world of things and forces, the inadequacy of the formal logic of pure thought led to its being more and more despised. Bacon set himself to substitute for it a new "organon" or instrument of thought, though his effort was not markedly successful. Indeed, the hour was not yet come. The mediaeval notion that logic is the art of thinking, and that its province is to dictate to men *how* they should think, had first to be given up, for it rested on a false conception of the relation of theory to practice.

The truth is that in every department of activity practice comes first. We learn to do things before we attempt to discover the general principles that doing has involved. So men fell themselves, and saw other things fall, long before they hit upon the theory of gravitation as the principle unifying all those diverse facts. Just so men learned the new processes of thought practically in the very attempt to penetrate the hidden mysteries of things. They had to invent modes of ascertaining facts more exactly than by casual observation, and to utilise those

facts when found. Thoughts are at once processes and results, and thoughts were what men were seeking. Their interest was in reaching knowledge—that is, systems of facts and explanations by relation of fact to fact that would stand the onslaught of new discoveries. Many ways they tried, and some they abandoned as unfruitful. Others they found effective, and gradually set themselves to perfect.

There, then, was the material for the new logician—the systematiser of *real* thought, not merely of 'pure' or empty thought. It was not a little because the old logicians stood aloof and declared that logic had nothing to say to the march of real knowledge—was indifferent to truth, as Mansel maintained—that men of sense said "All the worse for logic." The time when logic could dictate to thought was long gone by. So when the upholders of 'logic' took up this preposterous attitude they only succeeded in drawing contempt upon their science.

That time is happily passing away. The scientific worker sees the value of a systematic exposition of the principles which can be found inherent in successful attempts to win explanation, though he is still often shy of naming such an exposition 'logic.' On the other hand, the present-day logician is a much more modest person than was his early Victorian predecessor. He is content to follow the advance of knowledge, and no longer claims to dictate to it the way it should go. From successful workers in science he learns, in the words of the late Lord Acton, "how to test proof, how to secure fullness and soundness of induction, how to restrain and employ with safety hypothesis and analogy."

We may say, then, that there is a general agreement that by analysis and comparison of actual pieces of successful thought there may be formed a body of common principles, though, of course, those principles are found under various forms according to the kind of matter thought about. The kind of evidence which will establish a truth in history is different from that which will establish one in physics; yet both must in some way be adequate, and from examination and comparison of the relation of evidence to conclusion the general conditions

of adequacy may be ascertained. The systematisation of such principles of validity in thought yields the science of **Logic,** which may, indeed, be defined as the **science of the principles of valid thought.**

3. Relation of Logic to other Knowledge.—Shall we say, then, that the province of logic is a wide or a limited one? It is wide surely in the sense that it is interested in all thought and so embraces all the sciences. But it is limited in its power to express adequately the grounds of men's conclusions. It can say that if such and such evidence be true and be all that is pertinent, then such and such a conclusion is justified or unjustified. But whether the particular evidence *is* true and *is* adequate logic cannot pronounce. That is the domain of the particular branch of knowledge to which the case most definitely belongs.

Is, then, logic of only theoretical interest? Probably it is true to say that that is its chief attraction to most people who study it, just as in geometry the demonstration of necessary relations of space is essentially the scientific interest. But as the theory of geometry is derived from examination of actual space relations, it may be consciously applied to new cases, and may be used critically to judge the value of new space-constructions. And the same is true of logic. For as logic results from analysis of actual thought—and not otherwise—it is evident that men both can and do think without any conscious use of logic, and certainly without any preliminary study of it. Some think well, some ill. Or perhaps it would be truer to say, all think sometimes well, sometimes ill, though the proportions of the 'sometimes' are different with different people. Logic is derived from thought about thought, just as botany is derived from thought about plants. So logic could no more originate thought than botany could originate plants. In each case that is "putting the cart before the horse." And in the same sense in which botany may be said to be inherent in plants logic may be said to be inherent in thought.

We grant then that the study of logic is not a pre-

liminary to thought. Further, when men criticise each other's arguments they do not usually *consciously* apply logical principles—that is, they do not separately enunciate such principles though they implicitly use them. Yet further, we will grant that even the professed logician does not call to mind the principles of his science when he sets himself to think about some other topic.

All this is not to deny a practical value to the study of logic. But no service is ever done to a good cause by claiming too much for it. We have seen how logic has suffered from its friends in the past, and we would save it from such damaging support in the future. So we assert frankly and unreservedly that the practical value of logic to the student thereof is indirect, and only indirect.

The clear apprehension of the conditions of validity of thought helps to form a critical habit of mind and to develop a fine scent for fallacies. Modern life with its incessant platform chatter supplies endless occasions for the application of this power; if these opportunities are seized it grows in strength and in efficiency, and it may be added that the more unconsciously it works the more effective it is. It follows that the student of logic will do well to work special exercises—special because made simpler than the common experiences of life.

This habit of mind will guard us against the abuse of logic itself. People often, we know, scornfully dismiss many arguments as "mere logic." No doubt in many cases that is only following the old advice to shelter a weak case behind abuse of the adversary's attorney. But not always. It is often felt that the argument, though unexceptional in itself, does not meet the requirements of the case; in other words that to some extent it is of the nature of that narrow formal logic of 'pure' thought of which we have spoken. The really logical mind apprehends when this is the case—that is, when *all* the evidence is not brought forward—even where it cannot supply the deficiency. And this is a great point. In matters that concern actual affairs of human life it is seldom that all the evidence *can* be explicitly set forth. Why do we distrust so-and-so? We can probably give *some* definite reasons, but we are con-

scious that from our relations with him there has been derived a great mass of unfavourable impressions which have never become sufficiently definite to be formulated in words. And it is only with thoughts that can be expressed in words that logic can deal. This is evident when it is remembered that logic must be general, and can therefore only be based on recorded thoughts.

A little consideration will convince the reader that though words do express thoughts they do so more or less inadequately. In fact many things we cannot express, and when we do resort to expressions in speech, tone of voice, gesture, circumstances in which we speak, all add a colour of meaning which the mere naked words cannot convey. It follows that logical thought is most successful in dealing with matters into which the element of personal feeling does not appreciably enter. So it is that the conclusions of the physicist and the chemist meet with more universal acceptance than do those of the historian and the economist.

Perhaps it is true to say that the chief obvious practical use of logic is the negative one of detecting error. Though negative this is of the first importance, for to clear away error is the essential first step in the advance towards truth.

With a perfect and universal knowledge, logic, in a sense, would indeed be the universal science. With knowledge in its present fragmentary condition, it must partly accept its data from outside itself—as, indeed, all the sciences have to do. But with all branches of knowledge logic has a general connexion. With those which investigate the processes of men's minds and the modes of man's expression in speech—*i.e.* with psychology and grammar—the relation is more intimate. Yet in neither case does logic merge in the other, or the other merge in it. Psychology deals with the actual ways in which men think, whether ill or well. Logic is ideal in that it concerns itself directly only with valid thought, and with invalid thought only indirectly and with the object of making clear how and why it is invalid. A yet more important distinction is that psychology deals with thought as part of the life process of individuals: logic is not

concerned with individuals at all; it looks at thought in quite a detached and impersonal way. Nor can logic consider—as psychology is bound to do—how the thoughts of actual men and women are determined by other considerations than those of reason. There is no more call to confuse logic and psychology than to confuse physics and chemistry.

Similar remarks apply to the relation between logic and grammar. The former considers speech only indirectly as a vehicle of thought: it is concerned with meanings only, and demands that those meanings shall have a certain amount of definiteness and stability. Relations of words are of interest only as affecting meaning. Grammar, on the other hand, is only indirectly concerned with meanings. It explicitly deals with the relations of words. So that what is primary in logic is secondary in grammar, and what is secondary in logic is primary in grammar. It follows that the distinctions of grammar have no necessary connexion with those of logic, nor those of logic more than an indirect and subordinate bearing on grammar.

CHAPTER II.

THE LAWS OF THOUGHT.

1. General Character of the Laws.—*The Laws of Thought, Regulative Principles of Thought*, or *Postulates of Knowledge* are those fundamental, necessary, formal, and *à priori* mental laws in agreement with which all valid thought must be carried on. They are *à priori*, that is, they are assumed in all the processes of reason exercised upon the facts of the real world. They are *formal*; for, as involved in all thinking, they cannot by themselves ascertain the definite properties of any particular class of things. They are *necessary*, for no one can either conceive them reversed or knowingly violate them, because no one ever accepts a contradiction which presents itself to his mind as such.

It is true that fallacious reasoning is common enough, but this springs from a misapprehension of meaning, or from a confused use of terms, for which the ambiguities of language give abundant scope. Especially in long and involved reasonings, the force of terms is often unconsciously modified, and even entirely changed, or inconsistent judgments are accepted, with the result of invalidating the argument; but, at no stage of the process does the reasoner *consciously* accept a contradiction.

As always operative in thought, the Laws of Thought are laws in the scientific sense of uniformities; when applied practically to govern and test arguments, they are laws in that other sense of the word in which we speak of laws of the land. They are Postulates of Knowledge because they are involved in all attempts at interpreting experience, that is, they are assumptions without which thought cannot even begin the work of reducing to order

the chaos of sense impressions. Into the justification of these postulates logic does not enter. It assumes them because it finds them assumed in every piece of correct thought, and it aims at expressing them as perfectly as possible.

With regard to their number, *formal* logicians have generally recognised only three such laws of thought—the Principles of Identity, of Contradiction, and of Excluded Middle, for they only are implied in thought considered narrowly as simply consistent with itself. But for the wider view now adopted in which logic deals with the attainment of truth about reality there is needed a fourth principle—that of Sufficient Reason.

2. **The Principle of Identity.**—The simplest statement of this law is the formula *A is A*, or, as Leibniz put it, "*Everything is what it is.*" It demands that, during any argument, we use each term in one unvaried meaning.

No difficulty can be experienced in understanding, and assenting to, such propositions as *A is A*, *B is B*. But they convey no real information. To say a thing is itself tells no more about it than does the bare mention of its name. Identity must be interpreted in such a way as to cover such propositions as *A is B*, which we are continually making, and which experience tells us are justified by facts. We say "Gold is yellow," "Lions are fierce," and such statements are capable of conveying real information. No doubt such propositions imply the form *A is A*. "Gold is yellow," does not mean that all yellow things are gold—that is, that gold and yellow are convertible terms; nor yet that gold is any yellow, but only gold-yellow. But this analysis is not actually made in thought, nor is it necessary. Identity is really expressed in the proposition *A is B*, viz. the identity of the things to which *both* names, *A* and *B*, can be applied.

But this identity is expressed amidst a diversity of meaning; the two names have not the same signification, and, hence, the proposition, in which they are conjoined, is capable of giving real information. In truth, it is only

amidst some diversity that we know identity at all. I am the identical person I was ten years ago, and yet I have changed; individual men all differ from each other in many points, yet all share in the common nature of humanity.

When, then, we say *A is A*, we mean that a thing remains itself even amidst change, and that a common nature is manifested in different individual instances. In this sense the principle is the fundamental justification of the affirmative judgment.

3. **The Principle of Contradiction.**—This Principle, which would be better named **The Principle of non-Contradiction,** is most simply expressed by the formula *A cannot both be B and not be B*.

On this axiom, together with that of Identity, is based all immediate inference from affirmative propositions. It denies that the same thing can, at the same time, both possess a certain attribute and not possess it; and, as thought must be self-consistent, that we can conceive a thing as at once both possessing and not possessing the same attribute. The same statement cannot be, nor can we conceive it as being, at the same time both true and untrue; nor can the same thing at once be strong and yet not be strong. Different parts of the same object may, of course, possess incompatible attributes; one end of a bar of iron may be hot and the other, in common parlance, cold, but the *same* end cannot at once both be hot and not be hot to the same person; and our propositions must refer to the same end, as otherwise, not being made of identically the same subject, they would not be contradictory of each other. Similarly, the same end of the bar may at one time be hot, and, at another time not be hot; but there would be no contradiction in asserting this, for judgments referring to the same subject at different times are not the same judgment.

A judgment does not change with time, but once true is always true. Contradictory judgments, therefore, must refer to identically the same subject at identically the same time; they must assert incompatible attributes as standing

in the same relation, including that of time, to the same subject. Of course, there must be perfect sameness of sense both in the single terms of the contradictory propositions and in their affirmation and negation; the propositions must be contradictories not merely apparently and in words but in reality and meaning.

4. **The Principle of Excluded Middle.**—The Principle of Excluded Middle between two contradictory judgments is most clearly expressed by saying *A either is, or is not, B*.

This principle of thought has been questioned, and even denied, by writers who have confounded contradiction with other forms of incompatibility, especially contrariety, as *e.g.* hot and cold, happy and miserable. But, while contrary terms mark the utmost possible divergence, contradiction is simple negation. There are, of course, many intermediate stages of grey between the contrary attributes, black and white; and many varying degrees of warmth between the contraries, hot and cold. There are, then, many alternatives besides the propositions, This paper is white—this paper is black, This water is hot—this water is cold. But there is no third alternative whatever between the contradictory assertions, This paper is white—this paper is not white, This water is hot—this water is not hot.

It has been urged, as proof that contradiction is not thus exhaustive, that there is a mean between *plus* and *minus*, viz. *zero*; but here again, we have contraries, not contradictories. A mathematical quantity must either be positive, or not be positive; and, if the latter, it may be either zero or negative. Similarly, one given thing need not be either greater or less than another given thing, because 'greater' and 'less' are not contradictories, and there is a mean, 'equal to,' between them; but a thing must either be greater or not be greater than another given thing, and, if it be not greater, it may be either equal to it or less than it.

In short, great care is necessary to avoid the confusion of judgments whose predicates are contrary terms with

those whose predicates are contradictories; it is so easy to make the negation, which should only deny a strict agreement in all points, imply a thorough-going and complete divergence. If a man is declared not guilty of a certain crime people are inclined, thereupon, to attribute to him perfect innocence; whereas there may have been any degree of approximation to full guilt which yet fell short of it. The denial of guilt *as the accusation puts it* leaves open the possibility of some less degree of guilt; in many cases, further enquiry is invited rather than barred.

The Axiom of Excluded Middle is necessary, in addition to those of Identity and Contradiction, to form a basis for some forms of immediate and mediate inference. It expresses that our power of thought is limited in that we cannot help accepting one or other of two really contradictory statements.

Of course, the same limitation to a definite point of time holds here as in the Principle of Contradiction. So, by the Principle of Contradiction we are forbidden to think that two contradictory attributes can be together present in the same subject; by that of Excluded Middle we are forbidden to think they can both be, at once, absent; but no help is given us to decide which must be present and which absent.

From the point of view of language the three principles above discussed may be summed up by saying that whenever we use a term we must be understood to use it in its full meaning both (1) positively and (2) negatively, and (3) it must either be given or denied to everything whatever. That is, the use of a term asserts all the attributes it implies, and denies all others which are incompatible with those; and everything must either possess all those attributes or be without some, or all, of them.

5. **The Principle of Sufficient Reason.**—The Principle of Sufficient Reason may be expressed in the words of Leibniz, who first distinctly formulated it:—"*Whatever exists or is true must have a sufficient reason why the thing or proposition should be as it is and not otherwise.*"[1]

[1] Cf. *Monadologie*, §§ 31-39.

Here are two quite different things: (1) the reason why an event occurs, (2) the reason why we make a statement, and these by no means always coincide. Science attempts to explain the former and to account for the occurrences in the natural world. This it does by finding what body of conditions produces the event; and the relation thus established is known as Causation. But a statement of an event may be justified in other ways. For example, if I find the ground covered with snow in the morning, I say it has been snowing during the night; the reason for my *statement* is the presence of the snow. But this is the effect of the physical causation, and the reason for the presence of the snow is to be found in certain meteorological conditions of humidity and temperature.

Thus the physical *effect* is often the *cause* of the statements we make, though, of course, in other cases the two causes coincide, and the cause of my statement is also the cause of the physical event.

CHAPTER III.

TERMS.

1. **Analysis of a Proposition.**—The simplest element of thought is the judgment, and the verbal expression of a judgment is a proposition. When a proposition is expressed in its perfect logical form it is seen to consist of three parts—

(*a*) Something of which the assertion is made, called the *Subject*.

(*b*) Something affirmed or denied of the subject, called the *Predicate*.

(*c*) The verb *is*, either alone or accompanied by *not*, by means of which the assertion is made, called the *Copula*.

The Subject and Predicate are called the Terms (from Lat. *terminus*, or boundary) **of the Proposition.** They are the verbal representatives of the things, and of our concepts of them, between which the judgment affirms a relation. Both, therefore, must be names of objects or of attributes. Names may consist of a single word, as 'horse,' 'London,' or of a combination of several words, as 'The First Lord of the Treasury,' 'The House of Commons.' Logic considers names only as actual or possible terms of a proposition.

2. **Table of Divisions of Terms.**—Terms may be divided in various ways according to the point of view from which we regard them. The following Table sets forth these different divisions, and the principle upon

which each is founded. Of course, each group is exhaustive and independent; every term must fall under one or other of the members of each. It may be remarked that (i), (ii), and (iii) are the only divisions which are logically important, for they alone are founded on logical considerations.

(i) *Individual* and *General*—as names of individuals or of members of classes.

(ii) *Connotative* and *Non-connotative*—as names capable or incapable of definition.

(iii) *Positive* and *Negative*—as names implying the presence or the absence of some quality.

(iv) *Concrete* and *Abstract*—as names of objects or of attributes and relations.

(v) *Absolute* and *Relative*—as names implying or not implying a mutual determination of meaning.

A further division into *Univocal* and *Equivocal* terms is sometimes made. But this is entirely a matter of language. Whenever the same word serves as name for two or more distinct classes of things—as, *e.g.*, sleeper, which may mean either an individual asleep or the support of rails on a railroad—we have logically a plurality of terms, for the word in each of its meanings is a distinct and separate term, representing a distinct and separate concept.

3. Individual and General Terms.—(i) **An Individual Term is one which can be affirmed in the same sense of only one single thing.**

Thus, 'London' can be used in the same sense of only one place, though more than one place may have this same name; 'honesty' denotes only one quality, though it may be possessed by many individuals; 'this book' is limited to one single volume, and can only be understood by a person who knows what particular book the speaker is indicating. So, 'The present King of England,' 'The richest man in the world,' 'The longest river in Europe,' are all Singular or Individual names.

But an examination of these and similar examples will show that, though they are all names of individuals, yet they differ from each other in that, while some of them tell us of some quality possessed by the thing they denote, others do not. Of the latter kind are 'Honesty' and 'London.' Such terms as honesty will be discussed under the head of Abstract Terms.

(*a*) 'London' belongs to that subdivision of Singular Terms called *Proper Names*, which may be thus defined—

A Proper Name is an arbitrary verbal sign whose sole function is to indicate an individual object.

It may be thought that such names tell us a great deal about individuals; that 'London,' for instance, tells us that the object spoken of is a large city, situated on the Thames, the Capital of the British Empire, and many other particulars about it with which we may happen to be acquainted; but this is to confuse our knowledge of the thing, obtained from all kinds of sources, with the meaning implied by the name. The *word* 'London' informs us of none of these things; it may *suggest* them by association of ideas, in the same way as hearing a song which we have heard before may suggest the room in which we first heard it or the person who then sang it; but 'London' no more *means* these suggested particulars than the melody of the song means a place or person.

Care must then be taken to distinguish between implication and suggestion. Suggestion is purely a psychological fact. Logically, the point is that a proper name is not given as implying a certain meaning, that is, on account of the possession of certain attributes, but as a mark of recognition.

The fact that more than one object may receive the same Proper Name does not disprove the assertion that all such names are singular or individual. Thousands of men may be named Brown, and the same name may be borne by many dogs, horses, and other things; for instance, a town could be named Brown as appropriately as Washington, Gladstone, or Peel, all of which names are thus employed. But the name is given to no two of these objects in the same sense. As it is simply a mark of

identification, it does not matter logically to how many people or things it is applied.

(*b*) *Significant Individual Terms.* Proper Names are the simplest singular or individual terms. But the individual things we may wish to refer to are too numerous for us to give each of them a Proper Name of its own, and, sometimes, when a Proper Name has been given it is unknown to us. We are, therefore, often driven to use a General Name with a limiting word to make definite its applicability to only one object.

The simplest means of doing this is to use a demonstrative word—as *This* pen is bad; Let us go for a walk by *the* river. Here we are, in both cases, referring, in a perfectly determinate sense, to only one object, and the name is, therefore, singular. No doubt, in the latter case, the river has a Proper Name of its own; but in speaking of very familiar objects we often use such a limited General Name in preference to the Proper Name.

Again, we may use a many-worded name because it is our only means of indicating definitely the object to which we wish to refer, as its Proper Name may be unknown to us; thus, if we speak of 'The inventor of the Mariner's Compass,' 'The writer of the Letters of Junius,' or 'The man in the Iron Mask,' we may be using the only means in our power of designating the person we mean.

In other cases, such a many-worded name may be used because there can be no doubt as to its application; as when we say 'The leader of the House of Commons,' or 'The present leader of the House of Commons.' Regarded from a point of view limited in respect of time this name can only refer to one definite person, and is, therefore, individual. Had we said simply 'Leader of the House of Commons' the name would not have been singular, but general; for it could then be applied in the same sense to many individuals; the prefixing 'The' or 'The present' limits its application so long as we restrict ourselves to one point of time. In all such cases, however, it is the office, not the individual who holds it regarded simply as a man, to which the term primarily refers, and such holding of office it really implies.

It is evident that singular names of this second kind have meaning; they are *significant*—for they not only point out one member of a wider class, but, at the same time, inform us that it does belong to that class, and has at least one attribute which marks it out from every other member of the class. In short they are from the point of view of *meaning* very like class-names, but the class can contain only that one member.

(ii) **A General, Common, or Class Term is one which can be applied in the same sense to each of an indefinite number of things;** as book, man, dog.

Subjectively considered, a General Term is the verbal sign of a General Notion or Concept. A Proper Name indicates an individual directly, but a General Name does so indirectly, for such a name is given because the individuals to which it is applied, and from an examination of which the concept is formed, possess some attribute or attributes in common. The name, then, implies the possession of certain common qualities by every individual object which bears it, and, thus, has a meaning in itself. This likeness constitutes the similar objects a class, and, hence, a General Term is often called a *Class Term.*

It is not necessary for a true General Term that it should be *really* applicable to a plurality of objects, or indeed to any real physical object at all; it is sufficient for it to be *potentially* thus applicable; that is, for it to represent a possibly real, or even an absolutely imaginary, class of things, because of their possession of some common quality or qualities. For instance, 'Conqueror of England,' 'Emperor of Switzerland,' and 'Centaur' are true General Terms; though the first is *really* applied to only one historical individual—William I., and the second is not applicable to any individual at all in the present or in the past, though both may, conceivably, have an actual application in the future; whilst the third is the name of a purely imaginary being.

It is this potentiality of application to a class which distinguishes General Terms from the second class of Singular Terms; for the latter, though they are significant—that is,

have implication—are not applicable, even potentially, to more than one individual. There is thus an antithesis between Individual and General Terms, and every term must be one or the other.

Collective Names are sometimes treated as a separate division of terms, co-ordinate with Singular and General, but this is not desirable; for, as has been said, every term must, of necessity, be either singular or general. Collective Terms are found in each class, and there is, therefore, no opposition between them on the one hand and either Individual or General Terms on the other. A short examination of such terms will make this clear.

A Collective Name is one given to a group of similar units. It thus implies a plurality in unity; as an army, a flock, a library. No group of individual objects can receive a Collective Name unless the constituents of the group have some bond of resemblance to each other; thus, an alphabet is composed of letters, a navy of ships, a library of books, a museum of objects of interest. We could find no use for a name denoting a group composed partly of ships, partly of books and partly of men, or any other fortuitous concourse of heterogeneous objects simply as such. No doubt we do at times class together very unlike objects, as when we speak of the 'contents' of a curiosity shop or museum. But the force of the Collective Term is then small, for its only implication is that the objects in question are artificially brought together for a particular purpose. They have in themselves no common nature. The greater the common bond the more useful is the Collective Name. If no common bond can be found a Collective Term is impossible.

As a rule, Collective Terms are not Proper Names, but a few instances may be found, chiefly among geographical names, in which they are. Thus we speak of the Alps, the Pyrenees, the Himalayas, the Hebrides, the Marquesas, the Antilles, the Orkneys, all of which are true Proper Names, for they imply no qualities of the groups of natural objects to which they are applied, and are yet Collective, for they denote a group of similar units.

When the application of an ordinary Collective Term is

limited—in the way illustrated in (i) (*b*) of this section—to one particular instance of the groups it denotes, it becomes a Significant Individual Term. Thus we can speak of 'The German Navy,' 'The Greek Alphabet,' 'The Bodleian Library,' 'The British Museum,' 'The French army which fought at Waterloo.'

Without such limiting words a Collective Term is general with regard to the class of which it denotes a member, as well as collective in respect of the units of which the group is composed. Thus 'navy' is collective as regards the ships which form it, but general, as denoting a member of the class 'navies'; 'alphabet' collective as indicating a group of letters, general as the name of a member of the class 'alphabets.' We have as true concepts, in fact, of navy and alphabet as we have of ship and letter, and the former terms imply attributes equally with the latter.

The group denoted by a Collective Term may even itself be a unit in a larger group which bears a collective name of wider generality; so we may have a series of terms, each, except the first, collective as regards the preceding one, and each, except the last in generality, forming a constituent of the group denoted by the following one: *e.g.* soldier, company, regiment, brigade, army.

Thus the term Collective is relative in its meaning. At the same time, a General Term, which taken by itself is not collective, may, if in the plural number, be used in a collective sense by the prefixing of such a word as 'All' in the sense of 'All together,' as 'All these books weigh several tons.' The true antithesis is, therefore, not between Collective and General Terms, but between the

Collective and Distributive Use of Terms.—When we use a term collectively our assertion will only apply to the group as a whole; when we use it distributively we assert something about each member of the group individually. Thus, if we say 'Half the fleet was lost in a storm,' 'The regiment was decimated by fever,' 'All the novels of Thackeray would fill a small bookcase,' 'The books filled six large boxes,' we are evidently using the terms which form the subjects of our propositions, whether

they are 'Collective' or 'General' in a collective sense; and, equally clearly, if we say 'The fleet separated,' 'The army fled in all directions,' 'All the men were fatigued,' 'All the novels of Thackeray can be read in a day,' we are using the terms distributively. The full sense of the separate words is seen to depend on the context.

4. Connotative and Non-connotative Terms.

(i) **What names are Connotative.**—In the last section terms were considered according to their applicability to one or more objects, but bound up with that division was another fundamental distinction. It was shown that, while an individual name may be a mere indicative sign, implying no attribute, all names which are applicable to a plurality of objects are essentially significant, and imply some attribute or attributes possessed in common by those objects. This distinction between significant and merely indicative names is expressed by the terms Connotative and Non-connotative, which may, therefore, be thus defined—

A Connotative Term is one which denotes a thing and implies an attribute or attributes.

A Non-connotative Term is one which merely denotes a thing.

When we speak of *a thing* in this connexion we mean anything which can possess an attribute; while under *attribute* we include all that belongs to a thing, not only the outward marks by which it is known—as its shape, size, colour, weight, etc.—but all its properties and relations whatsoever. Thus a 'thing' or 'object' is in logic anything of which we can make an intelligible statement. Whether it is a 'thing' in the colloquial sense of an object we can perceive by the senses, a topic of thought, a wish or desire, matters not. So long as we can think and talk about it we may call it a 'thing' or an 'object,' and whatever we may think or say about it we may speak of as its attribute.

From what has been already said, it is evident that all General Terms are connotative, for they all denote—or are

applicable to—certain things, and imply that those things agree in possessing some attribute or attributes in common; in fact, it is the possession of these attributes which entitles any particular thing to bear the name. Thus, if we use the name 'horse,' we not only refer to an indefinite number of animals which are so styled, but we imply that they all agree in possessing certain well-defined characteristics; we should, without hesitation, call a horse any new animal brought under our notice which possessed those attributes.

All those Collective Names which are not Proper Names are also connotative, for they are general when viewed as members of a class; for instance, 'army' implies the attributes of being composed of soldiers, armed, trained, and maintained for warlike purposes, as well as denotes each collection of men which possesses these attributes.

When any General Name, whether Collective or not, is restricted in its application by some limiting word or phrase, of course its implication is not lost. Indeed, it is increased, and thus we have the class of Significant Individual Names, which, though they denote only one object, yet imply the possession of many attributes by that one object. Thus, if we speak of 'a mountain' we imply the attributes 'height' and 'composition of rock'; if we add 'in Asia,' we increase the number of characteristics, though we limit the number of things to which the name applies; by adding 'high' we carry both these processes a step further; and if, finally, we make the term singular, and speak of 'The highest mountain in Asia,' we, manifestly, retain all the attributes previously implied, and add to them uniqueness. All these attributes are implied by the name, and anybody using the name must be supposed to intend to convey them to his hearers.

With Proper Names, as we saw, it is not so. They indicate individuals, but do not imply any of their qualities. No doubt many Proper Names were originally significant, and implied attributes. Thus, 'Avon' in old English meant water; 'Jacob' meant a supplanter; 'Smith' or 'Butcher,' one who followed a certain trade.

But to deduce connotation from this original descriptive character is to confuse connotation with etymology. With surnames there is a very strong suggestion, amounting almost to implication, of family relationship. But as a surname can be changed at will, it seems clear that now, at any rate, its true function is merely to distinguish the individual, and that it has no necessary implication of meaning.

When the Proper Name of some typical person is used descriptively—as when we speak of 'a Nero,' 'a Napoleon,' 'a Cicero'—it implies by a metaphor the possession in a marked degree of certain qualities. It is, therefore, logically no longer a Proper Name. It has become General.

When, then, we grasp the distinction between implication and suggestion, we see that the definition given in the last section of a Proper Name as 'an arbitrary verbal sign' is strictly accurate, or, in other words, that Proper Names are non-connotative.

The only class of names which remain to be examined in this connexion is that of Abstract Names, and it will be more convenient to postpone our consideration of the nature of those terms.[1]

(ii) **Limits of Connotation.**—All the attributes *directly implied* by a name form its *Connotation*. This does not include all those which are common to all the members of a class denoted by a General Name, but *only those on account of the possession of which the name is given, and wanting any of which it would be denied.*

To include in the connotation of a name all the attributes common to the members of the class of which it is the name, whether these attributes are known or not, would have many logical inconveniences. It would divorce connotation from definition, and make connotation a matter, not of knowledge, but entirely of objective existence, and it is only with such existence as known that logic is concerned. If, to avoid this objection, connotation were made to embrace all the *known* attributes common to a class, then it must be pointed out that some of these cannot be

[1] Cf. pp. 35-37, 48-49.

regarded as essential; for instance, an animal which chewed the cud would be regarded as a ruminant, even though it did not agree with most known ruminants in possessing cloven feet. The name cannot, therefore, be said, strictly speaking, to imply the possession of that attribute.

Again, some attributes are derivative from others. Thus, that an equilateral triangle is equiangular is an attribute derivable from those primary ones which the name directly implies. It is most convenient, however, not to regard such attributes as forming part of the connotation, as that would be to confuse primary with secondary implication.

It must be granted that the limits of connotation are conventional. But it is essential that terms should have as fixed and definite a value as possible at any given time. This value is not determined by the knowledge of any particular person of the qualities common to the class, but by agreement between competent students of the particular subject.

(iii) **Difficulties of assigning Connotation.**—This, however, is somewhat vague and promotes occasion for dispute. Such disputes deal with what ought to be demanded of an object before the class-name is given to it, or, in other words, before it is recognised as that particular kind of thing. In a sense they are verbal, as they are occupied with the right use of words. But in a yet deeper sense they are much more: for such right use depends on accurate knowledge of reality. Ultimately agreement is reached, and the result is expressed in that form of brief statement we know as a definition. So we are all quite clear as to the connotation of 'square,' 'triangle'; many botanists are equally clear as to the implication of the names of many classes of plants. But there are many terms about which agreement has not yet been reached. As the decision of connotation thus rests on growing knowledge, it is plain that at all times the connotation of terms must be subject to revision should occasion arise.

(iv) **Denotation of Terms.**—Evidently significant names may be viewed in two lights—their implied mean-

ing or connotation and their range of application to a number of objects. This latter aspect is called their *Denotation*, which may, therefore, be defined as *the number of things to which the term is applicable in the same sense.*

From what has been already said it is clear that the denotation is logically fixed by the connotation; objects receive a certain name, and so form part of the denotation of that name, because they agree in its connotation. Nevertheless, practically each helps to determine the other. The connotation expresses the concept which is formed after an examination of part, at least, of the denotation; and, at all times, not only is the connotation likely to be modified by an increase in the denotation, but also conversely, making the connotation more definite or more elastic may decrease or enlarge the denotation. In truth, neither is absolutely fixed, though, for the purposes of Formal Logic, it is necessary to regard the connotation as strictly invariable throughout the same argument, or the Law of Identity would be violated.

As all terms must refer to something they must all have denotation whether they have connotation or not, though in the case of Proper Names and of some Abstract Names the denotation is reduced to the least possible limit—the unit.

When we speak of creatures purely fabulous, as dryads, centaurs, or griffins, their denotation must be sought in the appropriate sphere of existence—that of mythology, fable, or heraldry, as the case may be. This is, of course, using the word 'existence' in a somewhat wider sense than is common in ordinary speech, but it does no violence to it, and the extension is necessary to enable it to include entities having an existence only in thought or fancy, such as the characters of romance. As has been said, a 'thing' is anything we can think or talk about.

In speech a term is modified and generally limited by the context expressed or understood, and thus both speaker and hearer often tacitly restrict the application of the term to some portion only of its possible denotation when taken alone. If, for instance, we say "Everybody says so," we

certainly do not intend the term 'everybody' to be taken in its full extent so as to embrace all the inhabitants of the world; we probably refer, and are understood to refer, to a very few persons. It is important, therefore, to bear in mind that terms are continually joined together into propositions in a sense narrower than the words taken by themselves would warrant, and that such propositions are only intended to apply within this limited sphere. This sphere is called the **Universe of Discourse.**

(v) **Relation between Connotation and Denotation.**—As connotation implies attributes, and denotation refers to the individual objects which possess those attributes, and which usually form various sub-classes, it is evident that, as a general rule, an increase in either will cause a decrease in the other. As we augment the number of attributes implied by a name we diminish the number of things to which that name is applicable, for we exclude some of the sub-classes; there are, for instance, fewer white horses than horses. Conversely, if we wish to include under a name a group of things not before included under it, and so to enlarge the borders of the class which the term denotes, we can, usually, do so only by removing from the implication of the name those attributes which before marked the difference between the two classes, or, in other words, by decreasing its connotation.

For instance, if we unite the classes, 'white men' and 'not-white men' we must omit from the connotation of the common term all specification of colour; similarly, if we wish to include both sailing-ships and steam-ships under one common name, we must omit the points of difference, 'sailing' and 'steam,' and retain only the term 'ship,' which will be applicable to all the members of both classes but which implies less than the separate name of either. In short, generally speaking, the less a name implies, the more groups of things it is applicable to, and the more it implies the narrower is its range of application. It was shown in contrasting Significant Individual Names with Proper Names how the continued addition of attributes increases the connotation and decreases the denotation of a term, till at length

the latter is reduced to unity, and the former has become the fullest which that term is capable of bearing, so that connotative singular terms are the most significant of all names.

It cannot, however, be said that connotation and denotation vary in inverse ratio to each other; such a mathematical conception is quite inappropriate. We can speak intelligibly of halving or of doubling the denotation of a term, but it is meaningless to talk about doubling or halving its connotation; and even could we do so there would be no ratio maintained in the variation of the two aspects of the term. The application of a term is limited by the addition of some attributes much more than by that of others; thus, to add 'white' to man would not limit the denotation nearly so much as to add 'red-haired,' for there are many more white men than there are red-haired men.

Moreover, it is not true that an addition to the connotation of a term will always cause a decrease in its denotation; for as a name does not usually connote every attribute common to a class, the addition to the connotation of any number of these common attributes not included in it will not affect the denotation. There are, for instance, as many mortal men as there are men; so, though 'mortal' is not part of the connotation of man, yet to speak of 'mortal men' does not narrow the limits of the class 'men.' Attributes of things are in nature very often found in groups, so that where one is found others are found too; and it is evident that, when this is the case, the addition of any of these attributes to the connotation of a term will not limit its denotation so long as the one member of the group with which they are all connected already forms part of that connotation. To add to 'right-angled triangle' the attribute 'having the square on the hypotenuse equal to the sum of the squares on the sides' brings in no fresh limitation, for that attribute is one of a group necessarily found wherever the property 'right-angled' is joined to triangle. There may, thus, be many additions to the connotation of a word which will have no effect on its denotation.

It is, perhaps, scarcely necessary to point out that the idea of an opposite variation of connotation and denotation is only applicable to classes which can be arranged in a series of varying generality, so that each smaller class forms a part of the next larger; such as, figure, plane-figure, plane-rectilineal-figure, plane-triangle, plane-isosceles-triangle, plane-right-angled-isosceles-triangle; vehicle, carriage, railway-carriage, saloon-railway-carriage, first-class-saloon-railway-carriage, first-class-dining-saloon-railway-carriage. It would be absurd to say that an increase or decrease in the number of members of any one class affects the connotation of the class name; that, for instance, the birth of every baby must decrease the number of attributes implied by the term 'human being,' and that the death of each man, woman, and child, must increase that number. It is only when we add an attribute not common to the whole class that we exclude some members of the class from participation in the class name and so decrease the denotation; or when we introduce into a class some things not possessing all the attributes connoted by the class name, that we have to omit part of its meaning, that it may cover the whole of this more extended class; and thus we decrease the connotation. In short, increase of connotation decreases denotation by excluding a whole sub-class: increase of denotation affects connotation only when the increase is by the addition of a group before excluded, and which now becomes a sub-group under the term the meaning of which must be widened to include it.

The terms Intension instead of Connotation, and Extension instead of Denotation, are used by many logicians.

5. **Positive and Negative Terms.**—The formal distinction of Terms into Positive and Negative is a particular case of the *Incompatibility of Terms. All Terms whatever which imply attributes which cannot co-exist in the same subject are incompatible.* This incompatibility may be expressed either by Contradictory or by Contrary Terms. The division into Positive and Negative is the formally logical means of marking the first of these.

(i) **Contradiction—For two terms to be contradictories it is necessary that they be mutually exclusive and at the same time collectively exhaustive in denotation**; that is, they must be incapable of being predicated at the same time about the same subject, and between them they must embrace everything in the Universe of Discourse.

Formally, *contradiction* is expressed by prefixing **not-** or **non-** to the term—thus 'not-happy' simply excludes 'happy,' 'not-white' shuts out 'white,' 'not-man' removes 'man,' and in each case the two contradictories evidently include between them all things in the universe of discourse. The limitation implied by the last phrase is important. Thus 'white' and 'not-white' are not supposed to embrace *all* existing or conceivable things: for 'not-white' is meant to apply only to all colours except white, and not to such things as sounds, tastes, hymn-tunes, half-holidays, etc., etc. In other words 'white' and 'not-white' taken together comprise everything within the limited universe of colour only.

Now the connotation of terms formed by prefixing **not-** or **non-** in this way is simply negative; they imply nothing but the absence of the attributes connoted by the term to which they are prefixed. We may say then that **a Positive Term implies the presence of an attribute or group of attributes,** and **a Negative Term simply implies the absence of the attributes connoted by the corresponding positive term.**

We may, however, have two contradictories in fact, each of which is expressed by a distinct name, and then it is necessary to examine the facts in order to discover the contradictory relation between them, since it is not formally indicated by the names themselves. Thus 'British' and 'Foreign' are contradictories in the realm of material things, 'male' and 'female' in that of living organisms. Such pairs of terms are comparatively few in number. They all have a connotation which implies the possession of certain attributes, and are, therefore, positive.

Care must be taken not to class as contradictories terms which allow of an intermediate idea. For instance, 'happy'

and 'unhappy' are not contradictories, because they leave an intermediate state of indifference between them; we are often neither happy nor unhappy, for the latter word does not simply imply the absence of happiness, but, in addition, the presence of positive misery. True contradiction exists only where no intermediate idea is possible, *e.g.* equal—unequal.

(ii) **Contrariety.**—While logical contradictions simply negate each other, common speech can do more than this; it can express degrees of divergence, as we saw in the case of the terms 'happy' and 'unhappy' as contrasted with happy and not-happy. **When two terms express the greatest degree of difference possible in the same universe they are said to be Contrary or Opposite Terms:** thus black—white; wise—foolish; strong—weak; happy—miserable are pairs of contraries.

The idea of contrariety rests on the assumption that we do not simply divide our universe into two classes as in formal contradiction, but into a series of groups which have no sharply defined boundaries, as pleasant, indifferent, unpleasant, painful, where the extreme terms are contraries.

Under this head may be included what are sometimes called *Privative Terms*, which *signify "the absence of an attribute in a subject capable of possessing it."*[1] These include most of the words formed with negative prefixes or affixes, as unkind, unhappy, and also such terms as deaf, blind, dumb, lame, etc.

6. Concrete and Abstract Terms.

Relation between Concrete and Abstract Terms.—The division of terms into concrete and abstract is founded upon psychological and grammatical rather than upon logical reasons. It is, however, usual to consider it as part of the logical doctrine of terms. The following definitions express the difference—

[1] Cf. Stock, *Deductive Logic*, pp. 35 37.

A Concrete Term is the name of a whole object.

An Abstract Term is the name of an attribute considered by itself.

In the above definition the word 'object' is used widely to denote anything, whether material or not, which can be regarded as having a more or less separate existence as a whole whose parts or elements are in essential relation to each other and to the whole which comprises them. It thus includes such 'things' as Logic and Ethics; and such as point, line, etc., in their strict mathematical sense.

Abstract Terms are formed by the process of abstracting the attention from all the qualities of a thing, except some particular one, or group, to which the name is then given. Thus, by attending to one quality only of a tree, we form the idea of greenness; by considering only the moral quality of a number of good actions we gain the concept of virtue. But it is evident that all general terms represent concepts which are formed by abstraction; we must not, therefore, regard this process as a sufficient ground for calling a term abstract. If we did, we must include in that class all terms whatever except Proper Names. A term can only be called abstract when it denotes a quality which, though it can only exist in some object, may yet be thought of apart from all objects whatever. Thus, we can think of 'strength' by itself, although we know there can be no strength except as an attribute of strong things; or, of virtue, though it cannot exist apart from good actions

If it be borne in mind that an Abstract Name is not simply the name of a quality, but of a quality considered by itself, and apart from the objects which possess it, it will be immediately seen that adjectives are not abstract terms; for they name qualities only indirectly, and considered in connexion with the things to which they belong. If we say 'Gold is yellow,' we do not mean that gold is a colour, but that it is a thing which possesses a certain colour. It is the colour of gold which we call yellowness, not gold itself. 'Yellowness' is, then, the name of the colour or quality; 'yellow' is the name of all *objects* which possess that quality.

Whether a name is abstract or concrete will often depend on the sense in which it is used; for the same word may be concrete in one sense and abstract in another. The importance of the distinction between concrete and abstract, however, from a logical point of view, lies in those pairs of terms wherein one is the abstract and the other the related concrete, as strong, strength; man, humanity; square, squareness; etc., and in these cases there can never be any doubt as to which is concrete and which is abstract.

7. **Absolute and Relative Terms.**—This division of terms is based on the fact that the relations of things differ from their other attributes in that they involve direct reference to more than one object. If, for instance, we speak of a man as strong we can confine our attention to that one individual, but if we speak of him as a friend we must at once extend our view to include some other person who stands to him in the relation of friendship. We may say, then, that

> **An Absolute Term is a name which in its meaning implies no reference to anything else.**
>
> **A Relative Term is a name which, over and above the object which it denotes, implies in its signification another object which also receives a name from the same fact or series of facts which is the ground of the first name.**

Each one of such a pair of terms is called the *correlative* of the other. In some cases each correlative has the same name, as friend, companion, partner, like, equal, near; in other cases the names are different, as parent, son; king, subjects; governor, governed; cause, effect; greater, less; north of, south of. But they are always found in pairs, and they always owe their names to the same fact or series of facts.

It must be remembered that it is the fact that the *terms imply* the relation in which the objects stand to each other which makes them Relative, not the mere *existence* of the relation; thus a king governs men, but king and man

are not correlatives, for the terms do not imply this relation; king and subject are correlatives because they do imply it. Each member of a pair of correlatives connotes the same fact viewed from a different standpoint; paternity and sonship are not two different facts but the same fact viewed from two different sides, and connoted both by parent and by son. So, rule and subjection imply the same condition of things regarded from the point of view of the ruler and of the subject respectively.

The fact or series of facts which is the basis of the relation is called the *fundamentum relationis.*

CHAPTER IV.

THE PREDICABLES.

1. **Definition of Predicable.—The Predicables are a classification of the relations of the predicate to the subject of a logical proposition.**

They do not express what a term is by itself, but only what relation it bears to the subject of the proposition of which it forms the predicate. We cannot refer any general term taken by itself to one definite predicable; for the same term must be assigned now to one and now to another of the predicables according to its relation to the subject of which it happens, in any particular proposition, to be predicated.

2. **Porphyry's Five-fold Scheme of Predicables.** The traditional classification of predicables is that of Porphyry, and is closely connected with the subjects of logical definition and division. It is as follows—

$$\text{Predicables are}\left\{\begin{array}{l}1.\ \textit{Genus.}\\ 2.\ \textit{Species.}\\ 3.\ \textit{Differentia}\\ 4.\ \textit{Proprium}\\ 5.\ \textit{Accidens}\end{array}\right\}\text{of the Subject.}$$

We will now briefly define these five Heads of Predicables or 'Five Words,' as they are frequently called, and will then consider them more in detail.

A Genus is a wider class which is made up of narrower classes.

A Species is a narrower class included in a genus.

A Differentia is the attribute, or attributes, by which one species is distinguished from all others contained under the same genus.

A Proprium is an attribute which does not form part of the connotation of a term, but which follows from it, either as effect from cause or as a conclusion from premises.

An Accidens is an attribute which neither forms part of the connotation of a term nor is necessarily connected with any attribute included in that connotation.

3. **Genus and Species.**—These are not absolute terms, but purely correlative. A genus has no meaning apart from the two or more species into which it is divided; nor has a species apart from the containing genus. The same term may be, at the same time, a species of the next more general class, and a genus to the less general classes it contains; no term by itself can be styled a genus or a species. Thus, in the example quoted on a previous page[1] to illustrate the relation between connotation and denotation of terms, each intermediate term is a species to the preceding term, and a genus to the succeeding.

The nearest genus to every term, of which that term is itself a species, is called the *Proximum*, or *Proximate, Genus*. If a term is so general that it is not a species of any more general term it is called a *Highest Genus* or *Summum Genus*; and if it cannot be further divided into species, but only into individuals, it is a *Lowest Species* or *Infima Species*.

When a general term is predicated of another general term, it is a genus and the subject is a species. For example, in the proposition 'The dog is a domestic animal,' 'dog' is a species of the genus 'domestic animal,' and both are general terms. But when a general term is predicated of a singular term, it is a species, for it is under *infimae species* that individuals are directly included. Thus, if we say, 'This triangle is an equilateral triangle,' the subject is a singular term, and the general term 'equilateral triangle' predicated of it is an *infima species* since it cannot be further divided except into individual equilateral triangles.

4. **Differentia.**—It has been pointed out above that a species is wider in connotation than the genus under which, in denotation, it is contained. *The excess of the connotation of a species over that of its proximate genus is called the Dif-*

[1] See p. 33.

ferentia or Difference of that Species. Thus, in connotation, the sum of genus and differentia gives species.

It is plain, however, that there can be no such thing as an absolute genus or differentia, for the same attribute may be differentia in one case and part of the connotation of the genus in another. Thus, if we have three classes of things with the respective connotations *ab*, *ac*, *bc*, while *a* is the genus of the first two, and *b* and *c* differentiae; *b* is the genus of the first and third, and *a* and *c* differentiae; and *c* is the genus of the second and third, and *a* and *b* differentiae.

As the connotation of a general name only embraces those of the attributes possessed in common by the things denoted by the name which are explicitly stated in its accepted definition, it is evident that a comparison of the definition of the genus with that of any species gives the differentia of the latter. If *ab* symbolises the definition of a given genus, and *abc*, *abd*, *abe*, the definitions of species under it, then *c* is the differentia of the species *abc*, *d* that of *abd*, and so on.

5. **Proprium.**—*Those attributes which are common to every individual which bears the class name, and which are not included in its connotation, though necessarily connected with it, are called its Propria* or *Properties.* Propria need not, however, be peculiar to the members of this class, for they may flow from a part of the connotation which is also part of the connotation of some other class name.

The distinction between differentia and proprium is rather founded on the conventions of language than on the nature of things; for there is often no imperative reason why some, rather than others, of the common attributes of a class should be implied by the class name. Thus, with the common definition of a triangle, the attribute 'three-sided' is the differentia which distinguishes that species of plane rectilinear figures from others, and 'three-angled' is a proprium; but if we defined a triangle—as the etymology of the name, indeed, suggests—as a 'three-angled figure,' then the attribute 'three-angled' would become the differentia, and 'three-sided' the proprium.

This is not so, however, in every case; and, always, propria are attributes which flow from the whole, or part, of the connotation, either as effect from cause or as a conclusion from premises. Thus, that man is a tool-using animal flows from his rationality, as effect from cause—the attribute 'tool-using' is therefore a proprium. That the 'square on the hypotenuse of a right-angled triangle is equal in area to the sum of the squares on the sides containing the right angle' is also a proprium, because it is an attribute common to all right-angled triangles, which can be shown, by reasoning, to be a necessary consequence of the connotation.

6. **Accidens.**—In this class are included all those attributes which are neither connoted by a term nor are necessarily connected with its connotation; that is, which are included under neither of the heads Genus, Differentia, or Proprium.

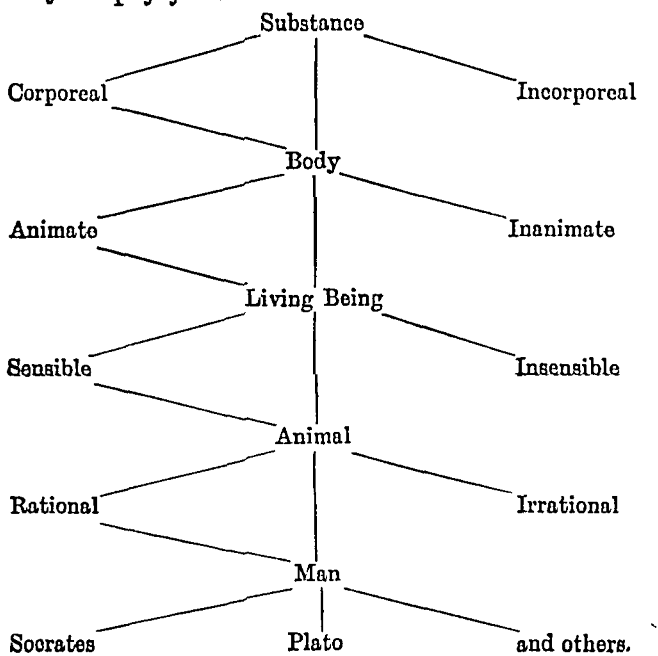
We have no real definition of what an *Accidens*, or *Accident*, is; we can only say what it is not. An accidens may be described as an attribute which can be removed from the class, or individual, without necessitating any other alteration; while to remove a proprium or a differentia would be to destroy the individual, or class, or at least fundamentally to change its character.

There may be accidentia of a class or of an individual, and in both cases they may be separable or inseparable. An *Inseparable Accidens of a class is one which belongs to every member of the class.* It is, of course, difficult to distinguish such accidentia from propria, and a more extended investigation into the nature of things is always likely to remove an attribute from the former class to the latter. But, where there is no known reason why the attribute should always be found in the individuals of a class, it is called an accidens. Thus, that all European ruminant animals are cloven-footed appears to be an invariable rule, but, as there is no apparent connexion between chewing the cud and having a cloven hoof, we regard having a cloven hoof as an inseparable accidens of the class European ruminant. White was long regarded as an insepar-

able accidens of swans, but the discovery of black swans in Australia has shown that it is only a *Separable Accidens*, that is, *one not common to every member of a class.*

When we come to individuals the words separable and inseparable have, necessarily, a somewhat different meaning. An *Inseparable Accidens of an individual is one which belongs to him at all times and can never be changed*, as the date and place of a man's birth, while a *Separable Accidens is one which is sometimes present and sometimes absent or which can be changed*, as a man's trade, his acts or postures. These individual accidentia are of no logical importance.

7. The Tree of Porphyry.—An example of a portion of this scheme of Predicables is furnished by a table known as the *Tree of Porphyry* because it was first set forth by Porphyry. It is as follows—



Here we have the summum genus—Substance, and the infima species—Man, which cannot be divided into any narrower species, but only into individuals. The intermediate terms down the centre of the 'tree'—Body, Living Being, Animal—are subaltern genera and species; that is, each is a genus as regards those below it in the list, and a species with respect to those above it. The attributes Corporeal, Animate, Sensible (*i.e.* able to feel), Rational, are differentiae which divide each genus into species. Of course, the corresponding negative attributes are also differentiae, but the species to which they would give rise are omitted for the sake of simplicity; their existence must not, however, be forgotten, for every genus must be divisible into at least two species.

8. **General Remarks on the Predicables.**—With regard to this five-fold scheme of Predicables it should be noted that no provision is made in it for singular terms as predicates. In fact, by the older logicians singular terms were never regarded as predicates, and such propositions as 'George V is the second son of Edward VII' were looked upon as outside the scope of logic. A predicable was only another name for a universal: one and the same term regarded in denotation was a predicable, as being applicable to several different things; considered in connotation it was a universal, as the attributes implied were to be found in several other and different notions.

When a genus or differentia is predicated, the proposition is said to be Analytic or Verbal as the predicate only states explicitly part of what is implicitly contained in the subject; but *when a proprium or accidens is predicated the proposition is synthetic or real,* as the predicate then asserts an additional fact, which no analysis of the implication of the subject would reveal.

Strictly speaking, a species is only predicated of an individual; when the individual is denoted by a Proper Name the proposition is, of course, synthetic, as the Proper Name implies nothing; but when the subject is a Significant Singular Name such a proposition is often analytic,

as the Significant Singular Name frequently contains the species in its connotation. Thus 'Socrates is a man' is a synthetic proposition, but 'This great Greek philosopher is a man' is an analytic proposition; for, 'philosopher' implies 'man,' but 'Socrates' does not.

There is much that is valuable in this scheme, for all classification depends on the formation of genera and species, and one of the chief aims of science is to classify accurately, and to decide what attributes are essential to the inclusion of any individual in a given class.

CHAPTER V.

DEFINITIONS OF TERMS.

1. **Functions of Definition.—Definition is the explicit statement of the connotation of a term,** that is, of all the attributes, and of those only, which are recognised by common agreement of competent thinkers as implied by the name.

Every definition is, therefore, an analytic proposition, or, rather, a series of analytic propositions, as a new proposition is required to affirm each separate attribute, and in every one of these propositions the predicate simply states in so many words what was already implicitly contained in the subject. A complete definition will exhaust the total number of analytic propositions that can be made with the defined term as subject, for it will state its whole connotation. Moreover, each one of these propositions must be universal, that is, the predication must be made of every one of the things denoted by the subject term; for the definition must necessarily be applicable to each object which bears the class name.

Were the logical assumption that all words have exactly the same distinct meaning for all who use them universally true, definition would be unnecessary. But our ideas are often *clear* without being *distinct* or *adequate*; that is, we can apply a name accurately enough to the things denoted by it without having distinctly present to our minds all the attributes on account of which it is bestowed upon them. The use of definition is to give distinctness to these clear ideas and to make them adequate—to enable us not only to use the name accurately as regards its denotation, but to employ it with an intelligent apprehension of its exact implication.

It is evident, then, that to form a good definition is a work of no small difficulty, and one calling for no small sagacity. It involves careful observation, comparison and analysis of the things observed, abstraction of the mind from their differences, and generalisation, besides the power of distinguishing primary from derivative qualities. In short, the definition is the perfecting and completion of the process of conception.

Moreover, the preliminary process of *seeking* for a definition is often more important than the *finding* of it. "What we gain by discussing a definition is often but slightly represented in the superior fitness of the formula that we ultimately adopt; it consists chiefly in the greater clearness and fullness in which the characteristics of the matter to which the formula refers have been brought before the mind in the process of seeking for it. While we are apparently aiming at definitions of terms, our attention should be really fixed on distinctions and relations of fact. These latter are what we are concerned to know, contemplate, and as far as possible arrange and systematise. . . . And this reflective contemplation is naturally stimulated by the effort to define; but when the process has been fully performed, when the distinctions and relations of fact have been clearly apprehended, the final question as to the mode in which they should be represented in a definition is really—what the whole discussion appears to superficial readers—a question about words alone."[1]

Definition is thus essentially practical, and is, therefore, a part of Applied Logic. Its uses are twofold. The first and most obvious is to give precision to indefinite knowledge; the second is to make clear the exact sense in which in any given exposition or discussion a term is to be used. The former may be called its use to the learner; the latter is its logical function.

2. **Definition per Genus et Differentiam.**—In unfolding the complete connotation of a name it is often practically impossible to express it in terms which denote

[1] Sidgwick, *Principles of Political Economy*, pp. 49-50.

simple attributes only; and, in nearly every case, to do so would make the definition needlessly long and involved. It is always allowable to employ terms expressive of groups of attributes. Hence we have the time-honoured rule that definition should be *per genus et differentiam*. In mentioning the genus we use a term which implies all the attributes common to the species whose name is the term to be defined and to all other co-ordinate species of that genus; by adding the differentia, we complete the statement of the connotation by giving those attributes which differentiate that species from all such co-ordinate species. In other words, when we have to define a term, we first decide what class of things it belongs to, and then we mark the attribute, or group of attributes, which distinguishes it from other members of that class. The name of the class is the genus, the distinguishing attribute, or group of attributes, is the differentia.

The genus selected must be a proximate genus,[1] as, otherwise, our definition will omit part of the connotation of the term we are defining. If, for instance, we defined 'man' as 'rational being' we should omit the attributes connoted by the word 'corporeal,' and our definition would allow the name to be applied to other possible beings. Or, symbolically, if we define a class term whose connotation is *abcd* by referring it to the genus *a* (instead of to the proximate genus, *abc*), and adding the differentia *d*, we plainly omit the attributes *bc* from our definition.

It must be remembered that when definition *per genus et differentiam* is spoken of, it is not meant that the differentia must be a single attribute; it may be a group of attributes. Each species when referred to any one particular genus can have but one differentia, that is, one set of attributes to distinguish it from the co-ordinate species.[2]

3. Limits of Definition.—As definition is the unfolding of the meaning implied by a name it follows that every significant name can be defined, and that the only terms incapable of definition are Proper Names which have no

[1] See p. 40. [2] See pp. 40-41.

signification, and singular Abstract Terms which as the names of the simple attributes that form the ultimate limit of our analysis cannot be expressed in terms more elementary than themselves. No words can enable one who has never experienced pain or whiteness to conceive what either is.

Some terms are manifestly much more easily defined than others. Those in which the connotation is the more important element—such as technical terms, whose sole value lies in an exact meaning—are much more easily defined than those in which the denotative element predominates. Examples of the latter are the names of most common objects—*e.g.* chair, horse, dog—where we learn to apply the names without any distinct idea of the attributes connoted by them. That this is the case anyone may easily discover who contrasts the ease with which the connotation of such a term as 'rectangle,' for example, can be stated, with the difficulty of writing down the essential attributes of 'dog.'

But, though all significant names can be defined with more or less difficulty, a great number of such definitions can only be regarded as provisional. Fresh advances in knowledge may alter our estimate of the relative importance of attributes, may change propria to differentiae, or differentiae to propria, and so may revolutionise the connotation of the term, and thus necessitate a revised definition. In fact, the growth of knowledge *must* cause modifications of the definitions of many scientific terms; were they all fixed, science would cease to advance. Discovery and definition must go hand in hand, and universal finality in the latter is not to be looked for; it could only be possible with complete and perfect knowledge.

Not only the growth of knowledge, but a change in the point of view from which a term is regarded may lead to a change in the accepted definition. Examples of this are most common in mathematics. Thus an ellipse was originally defined as a conic section with the differentia that the cut goes quite across the cone, not at right angles to the axis. But in modern works it is defined as the line traced out by a point so moving that its distance from a

fixed line bears always a certain ratio to its distance from a certain fixed point. Then the fact that such a curve is a conic section is deduced by a long and intricate argument; it is, in fact, degraded from forming part of the connotation to the position of a proprium.[1]

Again, changes in the denotation of a term caused by its application to new classes of objects result in a certain vagueness of connotation, which, of course, reacts on the denotation, and gives rise to an indefinite zone, of the contents of which it is difficult to say whether they have a right to the name or not. This is especially so in the use of such words as 'capital,' 'wealth,' 'rent,' 'labour,' etc., which are of importance in the sciences which deal with social phenomena. Indeed, most common words will, if carefully examined, be seen to be more or less vague as to the boundary line both in their connotation and in their denotation.

4. **Principles of Definition.**—A good definition should satisfy the following conditions—

I. *It should state neither more nor less than the connotation of the term defined.*

II. *It should not be expressed in vague or figurative language.*

III. *It should not be mere tautology.*

IV. *It should not be negative unless the whole meaning is negative.*

Or, to sum the rules into one—

A definition should in its content be (i) *adequate and precise; and in its expression* (ii) *clear, and neither* (iii) *tautologous nor* (iv) *negative.*

We will now discuss each principle in some detail.

Principle I.—If the definition embraces more than the connotation of the term defined it must include either some of its propria or some of its accidentia.

In the cases where either propria or inseparable accidentia are added to the connotation, the denotation covered

[1] Cf. Dr. Venn's *Empirical Logic*, pp. 284-285.

by the definition remains the same as that of the name defined; but the very fact of adding these extra attributes suggests that they are *necessary* to the true definition; and that, therefore, other objects exist which possess all the attributes mentioned except these very ones; which is, in fact, not the case. If, for example, an equilateral triangle is defined as a triangle which has three equal sides and three equal angles, this, though perfectly true of all equilateral triangles and of no other figures whatever, is yet a faulty definition; for it *suggests* that there are triangles which may have three equal sides and yet not have their angles equal. In all these cases the definition is redundant, and, therefore, wanting in conciseness and precision.

If a separable accidens is added to the connotation of a name as part of its definition, a graver fault is committed. In this case the definition does not refer to the whole denotation of the name defined, for some only of the things which correctly bear the name possess the attribute in question. The definition in this case is *too narrow.* If, for instance, a triangle is defined as a plane rectilinear figure having three *equal* sides the definition is too narrow; for it applies only to a section of the figures correctly called triangles. The attribute 'equal-sided' is the differentia which marks off the species 'equilateral triangles' from the other co-ordinate species included in the genus 'triangle,' and is, therefore, only a separable accidens of that genus. Or, if a labourer is defined as one who performs manual work for wages the definition is again too narrow, as, by the addition of the separable accidens 'for wages,' it excludes all slaves from the class labourers of which they indubitably form a part.

If, on the other hand, the definition contains less than the connotation of the name it is *too wide*, for evidently it is applicable to a greater number of things than are included in the denotation of the term defined. If, for instance, an equilateral triangle is defined as a plane rectilinear three-sided figure the definition includes *all* triangles. In other words it refers to the genus instead of to the species only, and is inadequate.

In all cases, then, the denotation of the definition must be exactly the same as that of the term defined, and this can only be secured with certainty by its stating all the connotation of the term and nothing else.

Principle II.—The violation of the demand for clearness in a definition is known as defining *ignotum per ignotius* or *per aeque ignotum*—explaining the unknown by the more, or equally, unknown. Dr. Johnson's definition of a net as "a reticulated fabric, decussated at regular intervals" is an amusing instance of this. To say that "Eccentricity is peculiar idiosyncrasy" or that "Fluency is an exuberance of verbosity" is, in each case, to give a definition which is certainly not clearer than the term defined.

The so-called definitions which are expressed in figurative language are a variety of this fault. To say "The lion is the king of beasts," "Bread is the staff of life" or "Necessity is the mother of invention" gives no explanation of the meaning of the terms 'defined.'

This rule, however, is not violated if a name is defined, for the purposes of a special science, in terms which to one who is not a student of that science would be less clear than the name itself; as, for instance, if for the purposes of Conic Sections a circle were defined as a section of a cone parallel to the base. Definition from its nature assumes sufficient pertinent knowledge for the comprehension of its terms.

Principle III.—The violation of the rule against tautology in a definition is called *circulus in definiendo*, or *a circle in defining*.

It is evidently no addition to our knowledge of fact to 'explain' a term by itself or by a synonym. To say that "Truth is veracity in speech and act" is simply to affirm that "Truth is truth," and this, though it may give knowledge of the meaning of a hitherto unknown term, gives none of the nature of the thing named. Thus, logically, it is perfectly useless. The great number of synonyms in English, due to the presence in the vocabulary of words derived from both Teutonic and Latin sources, offers many opportunities for committing this fault; and, it may be added, these opportunities are by no means sparingly used.

But it is by no means confined to English. Ueberweg quotes the following example from the German writer, Maass: "'A feeling is pleasant when it is desired because of itself.' 'We desire only what we in some way represent to be good.' 'The sensibility takes that to be good which warrants or promises pleasure, and affects us pleasantly; —the desires rest on pleasant feelings.' The pleasant feeling is here explained by the desire, and the desire again by the pleasant feeling."[1]

It is in cases of long and involved definitions, such as the above—where the three sentences are taken from different parts of the book—that a 'circle' is most frequently found. Still, it is by no means uncommon to meet with such 'definitions' as "Life is the sum of vital functions," "Force is a motive power," "Man is a human being." The definition once given by a Church dignitary that "An archdeacon is one who exercises archidiaconal functions" is a very neat example—in this case, intentionally humorous—of *circulus in definiendo.*

There is no tautology when the name of the genus is repeated in defining a term which denotes a subordinate species which has no distinct name but is specified by the addition of some limiting attribute to the name of the genus; as, for example, in defining an equilateral triangle as a *triangle* which has three equal sides. For the species 'equilateral triangle' has no separate name, and is distinguished from the species of the genus triangle which are co-ordinate with it merely by the limiting adjective 'equilateral.' Euclid, before giving this definition, has, of course, defined the name of the genus, 'triangle.' This word when it occurs in the definition of equilateral triangle is simply the name of the genus, not that of the thing defined at all; and the definition is strictly one *per genus et differentiam.*

From this principle it follows that a term which is the name of a simple and elementary attribute cannot be defined, as it can only be explained by a synonym or by itself; for instance, "White is that attribute of sensible

[1] *Logic*, Eng. Trans., p. 175.

objects which occasions us to experience the sensation of whiteness." We can only, in truth, *describe* such terms by analysing the conditions under which the sensations they denote are produced.

Principle IV.—Negative definitions are always less satisfactory than those expressed in positive terms, unless they are definitions of the names of negative notions, in which case they are to be preferred.

It is, for example, simplest to define an alien as one who is not a citizen of the British Empire; for the name 'alien' represents a notion whose sole differentia is just this negative attribute. Any definition of Accidens must also, necessarily, be of a negative character.[1] But, to 'define' virtue as the opposite of vice, or liquid as that which is neither solid nor gaseous, is not to say in either case what attributes constitute the class notion, and are, therefore, connoted by the name; but to state attributes which are not so connoted. This, besides giving no positive information, is certain to lead to indefiniteness. Merely to say what a thing *is not* gives no clear indication of what it *is*.

Many of the objections to the indefiniteness of thoroughgoing negative (or infinite) terms apply to negative definitions.[2] Euclid's definition of parallel straight lines as "those which lie in the same plane, and which, being produced ever so far both ways, never meet" offends against this principle, as does his definition of a point as "that which has no parts and which has no magnitude." This rule of expression is really involved in the first principle of definition, for the connotation of a positive term cannot be expressed negatively.

5. **Formation of Definitions.**—Every definition is the explicit unfolding of connotation, and therefore is an analytic proposition in which the denotation of the subject and predicate may be equated together. But, as we have seen, there are difficulties in the assignment of connotation, and we may ask on the one hand how far the definition explains the nature of a thing, or on the other, how far it

[1] Cf. p. 42. [2] Cf. p. 34.

represents a general agreement as to the application of a name.

This distinction, however, is more apparent than real. It arises from the imperfection of our knowledge. Definition is in a sense the end of science, and so long as science has yet to discover the essential nature of things its definitions cannot be more than approximations to truth.

Consider the difficulty of determining what shall be regarded as *differentia* and what as *propria*. In mathematics the case is comparatively simple. Such a definition may be given as will suffice to ensure the construction of the figure, and all that can be demonstrated from that under the fixed conditions of space may be regarded as properties. The same procedure is not open to us in regard to natural objects, whether organic or inorganic. The process of seeking for a definition involves an examination of instances which seem to be of the same kind, and the recognition that these are instances already implies at least an implicit definition, for otherwise they could not be recognised as such.

Ideally then it would be necessary to enumerate all the characteristics of a species, and to determine which were essential, and which were accidental. It is needless to dwell on the impossibility of carrying out such a process; yet without it we can never be sure that a characteristic of fundamental importance has not been omitted. The discovery of such an omission may revolutionise a science.

Assuming, however, that the enumeration of attributes has been made as complete as the state of knowledge makes possible, how are we to sort out those which most truly represent the nature of a species? The elimination of accidents will not theoretically present much difficulty, though in practice it is not always easy. But we need some principle on which to divide the essence from the property. That which is available in mathematics is not available here: we cannot demonstrate the properties of a natural object from certain given fundamental characteristics. The most that we can do is to choose for the purposes of definition those attributes which seem to carry with them the largest number of other attributes, whether we can trace

a causal connexion or not. Now such grouping is to a large extent arbitrary, and when a name is given to a group of attributes so determined, and things are called by that name according as they possess or do not possess those attributes, the definition is rather a statement of the sense given to the name than an expression of the nature of the thing.

This is seen even more clearly in the case of terms relative to human relations; as capital, wealth, rent, education, right, justice. These are used in so great a variety of ways that for any specific purpose it is necessary, if we would avoid ambiguity and misunderstanding, to state explicitly the range of application we propose to give them, that is, the sense in which we are going to use them. Then, of course, our definition is primarily one of the word—adapted to a special purpose.

Yet, even in these cases, we do not define the word arbitrarily, but always in view of the exact relation it is to bear to facts. So the traditional distinction between *nominal* and *real* definitions—that is, definitions of *words* and definitions of *things*, is a matter of no logical significance.

6. **Genetic Form of Definition.**—A change in the aim of definition has resulted from the theory of descent formulated by Darwin in his *Origin of Species*. Species are no longer regarded as natural kinds with permanent and definite but diverse characteristics. They are acknowledged to be the results of a long descent from a common ancestry in which successive modifications have produced all degrees of divergence. Could these modifications be exhibited completely it would be possible to construct a genealogical tree in which the species would be shown to be members of one great family, and their affinities with one another would appear at a glance. This gives an important clue to those attributes which are essential to the constitution of a species, and which will, therefore, be in its definition. They will not necessarily be those which are important or even essential to the present life of the animal or plant, but rather those which best exhibit its origin. Definition

which sets forth the process by which a thing comes to be what it is, and which is, therefore, based on the idea of causation, is termed *genetic*.

We may also describe as genetic those definitions in mathematics which indicate a process by which we may form a concept of a geometrical figure: *e.g.* "A cylindrical surface is the surface marked out by a straight line which revolves at a fixed distance round another straight line to which it is parallel"; or in chemistry, those which indicate the formation of a compound from its elements, *e.g.* "water is a fluid formed by combining one part of oxygen with two parts of hydrogen."

7. **Descriptions.**—Definitions should not be confounded with descriptions. In the latter, inseparable accidentia are often used, with or without some of the propria, to enable us to recognise the objects denoted by the name. Such a proposition is no more a definition than would be the act of pointing out a member of the class in question and saying "I mean something like that." Still, description is by no means useless, though it is not logical definition. It has the valuable function of enabling us to identify easily anything which bears the name. The main object of definition is to make distinct our concepts of things, and so to lead to a greater clearness and definiteness of thought and language; that of description is to furnish a rough and ready means of recognition.

Descriptions are in place in a dictionary where logical and scientific definitions would be of little use. A scientific definition of a crocodile would not enable anyone to form an idea of its appearance—for definition does not deal with appearance but with qualities and relations essential to that kind of animal, which are not generally closely connected with its appearance. The function of the dictionary is to enable anyone who has not seen a crocodile to realise what it looks like, so it describes it and aids its description by a picture. Description, in a word, appeals directly to the eye: definition to thought and previous knowledge.

CHAPTER VI.

FALLACIES OF DEFINITION.

1. **General Nature of Fallacy.**—The word 'Fallacy' is, like many other words, very loosely used in the common speech of the present day. Thus, any false statement is, by some persons, included under the term; "that men are in the habit of walking on their heads, they would say is a very obvious fallacy."[1] Others include under it any false belief, or any mental confusion whatever, no matter what its origin. It seems preferable, however, to use the term in a more definite manner. It is better not to call a prejudice or a mere inaccurate statement, a fallacy, but to confine the term to offences against logical principles more or less perfectly concealed. We will, therefore, give as our definition:

A Fallacy is a violation of logical principle disguised under a show of validity.

Wherever there is a logical principle there is a possibility of offending against it. There are, thus, fallacies incident to conception, including invalid definition and division, to judgment, to method, and to each valid form of inference. It must be remembered, however, that the difference between these processes is not a fundamental one, but rather one of emphasis. The essence of inference is implicit in judgment, even as judgment is implicit in conception, and is involved in definition and classification. All violations of logical principles, therefore, involve faulty inference, either implicitly or explicitly, and when those in which this fault is implicit are employed in fully stated arguments they lead to further violations of the rules of explicit inference.

We will consider the most usual forms of logical error—or fallacy—in connexion with each department of our

[1] De Morgan, *Formal Logic*, p. 238.

treatment. To take first, then, those which may lurk in our concepts, and in the use of the terms which express them.

2. **Faults in Definition.**—Any violation of the principles of definition leads to fallacy through faulty or imperfect conception of the force of the terms employed. Thought wanting in clearness and definiteness is extremely likely to become, at some point or other, actually erroneous. All fallacies due to any kind of ambiguity in the terms employed are, thus, at bottom faults of definition; once make the meaning of every term employed clear and such fallacies become impossible.

Under the general head of faulty definition we may include several fallacies traditional in logic since Aristotle, and which it is still customary to designate by their mediaeval Latin names, as well as one which is specially incident to vague and indefinite conception and which we will consider first.

(i) **Concept embracing incompatible attributes.**—Whenever the meaning of a term is explicitly and clearly conceived it is, of course, impossible to include in it incompatible attributes. But when one's ideas are not clearly and definitely set forth, this is by no means impossible. Thus we find people arguing about "an indivisible portion of matter," where the attribute 'indivisible' is incompatible with 'matter' which implies extension, and, therefore, divisibility. The expression is self-contradictory, and can only be used by one whose thought on the subject is vague and confused, for no one can hold together in clear thought attributes which destroy each other. A logical concept of indivisible matter is as much an impossibility as one of unequal-equality or unjust-justice.

Again, all the numerous and varied attempts which have been made at 'squaring the circle' may be set down to a faulty conception of 'incommensurable,' a conception which, indeed, embraces the idea of 'commensurable in terms of an indefinitely small unit.'

Similarly many people seem to use the word 'infinite' as meaning finite at a very extended limit. Others make

'eternal' signify enduring throughout a very long time, whereas the very notion of time, embracing, as it does, the ideas of before and after, and consequently of beginning and end, is incompatible with that of 'eternity,' whose true meaning is existence out of time.

(ii) **Æquivocatio** or **Homonymia.**—By a fallacy of equivocation is meant one due to the use of a word capable of two or more meanings. Traced to its origin, this implies that the concept corresponding to the word is wanting in clearness and accuracy. In other words it is a failure in definition. Such failure may, of course, be really made by him who uses the ambiguous term. At other times its use is merely a sophistical device, the person who uses the term in two different senses trusting that the change of meaning may escape detection. An example frequently given by the old logicians is—"The end of a thing is its perfection; death is the end of life; therefore, death is the perfection of life" where 'end' is used in the two senses of *aim* and *finish*. Evidently what is true in the one sense has no pertinence to the other. Similarly in "Knowledge is power; perception is knowledge; therefore, perception is power," we have an ambiguous use of the term knowledge.

In such examples as these the error is obvious and, consequently, not likely to be committed. But it must be remembered that a writer on fallacies is bound to choose some, at least, of his examples of such a character that every reader will see at once where the fallacy comes in. Moreover, when an argument is, as it were, reduced to its lowest terms, and stated in strict logical form, it is easy to see an error, which might well escape detection if it occurred in a long and more or less involved disquisition, with the premises, perhaps, far apart.

Fallacies due to ambiguity of words are, indeed, amongst those most commonly committed. As Bacon well says: "Men believe that their reason rules over words; but it is also the case that words react, and in their turn use their influence on the intellect."[1]

[1] *Novum Organum*, I. 59.

One of the chief sources of ambiguity is that a word which has changed its meaning still retains some of the associations which were connected with its original meaning. De Morgan gives some good examples, one or two of which we will borrow.

"The word *publication* has gradually changed its meaning, except in the courts of law. It stood for *communication to others* without reference to the mode of communication, or the number of recipients. Gradually, as printing became the easiest and most usual mode of publication, and consequently the one most frequently resorted to, the word acquired its modern meaning; if we say a man publishes his travels, we mean that he writes and prints a book descriptive of them. I suspect that many persons have come within the danger of the law, by not knowing that to write a letter which contains defamation, and to send it to another person to read, is *publishing a libel*; that is, by imagining that they were safe from the consequences of publishing, as long as they did not print. . . . A similar change has taken place in the meaning of the word to *utter*, the sense of which is to *give out*, but which now means usually to give out of the mouth in words. As yet I am not aware that any person charged with the *utterance* of counterfeit coin has pleaded that no one ever uttered coin except the princess in the fairy tale: but there is no saying to what we may come, with good example, and under high authority."[1]

Confusion between the etymological and the currently accepted meaning of a word is, indeed, always liable to lead to fallacy. In a discussion as to the meaning of a word, an appeal to etymology is really out of court; current usage alone must decide the question.

Nothing is more helpful, and even necessary, for scientific —i.e. *exact*—thought than a precise set of terms. The moral sciences have no such well-established and exact system of technical terms, and it follows that in those sciences fallacies due to ambiguity are most easily committed.

Especially is this the case with Economics. Mill gives

[1] *Formal Logic*, p. 243.

a good example of such a fallacy which is still quite common: "The mercantile public are frequently led into this fallacy by the phrase, 'scarcity of money.' In the language of commerce 'money' has two meanings: *currency*, or the circulating medium; and *capital seeking investment*, especially investment on loan. In this last sense the word is used when the 'money market' is spoken of, and when the 'value of money' is said to be high or low, the rate of interest being meant. The consequence of this ambiguity is, that as soon as scarcity of money in the latter of these senses begins to be felt—as soon as there is difficulty of obtaining loans, and the rate of interest is high—it is concluded that this must arise from causes acting upon the quantity of money in the other and more popular sense; that the circulating medium must have diminished in quantity, or ought to be increased. I am aware that, independently of the double meaning of the term, there are in the facts themselves some peculiarities, giving an apparent support to this error; but the ambiguity of the language stands on the very threshold of the subject, and intercepts all attempts to throw light upon it."[1]

Another ambiguous word is 'government,' which is used both to denote the system of laws established in a nation, and the body of men charged with the carrying out of those laws. Loyalty to the government in the former sense may involve resistance to the government in the latter sense, though a tyranny would not be willing to acknowledge this, and would trade on the ambiguity of the word, trusting that the ignominy which may rightly accrue to resistance to law will attach to resistance to the men whose mal-administration of the law may have become intolerable.

'Nature' is another fertile source of fallacy. What, for instance, is "education according to nature"? With the seventeenth century writer Comenius it meant seeking to base educational method on fanciful analogies drawn from the physical world, as, for instance, when he argued that because the sun does not occupy himself with objects one by one—a tree or an animal—but illumines and warms

[1] *Logic*, V., vii., § 1.

the whole earth, therefore, there should be only one teacher for each class.[1] With Rousseau the phrase meant a return as far as possible to the condition of uncivilised man, 'nature' being regarded as the condition from which man starts. The same idea is present in the excuse so often given for childish faults that they are 'natural.' On the other hand, Plato and all modern idealists would seek man's true nature in the ideal towards which civilisation can only be regarded as a slow progress, and with them "education according to nature" means guiding the child in the path of this development in the order and to the extent to which a study of psychology shows to be possible.

One frequent cause of the fallacy of equivocation is when a writer or speaker uses a word in a definite and perhaps special meaning without taking steps to secure that it is understood in that meaning. "It is very difficult," says De Morgan, "to avoid this form of the fallacy, without giving the meaning of the most essential terms, on the first occasions of their occurrence. It is not uncommon to meet with a writer who appears to believe, at least who certainly acts upon, the notion that the right over words resides in him, and that others are wrong as far as they differ from him. . . . The writers of whom I speak . . . treat words as absolute images of things by right of the letters which spell them. 'The French,' said the sailor, 'call a cabbage a *shoe*; the fools! why can't they call it a cabbage, when they must know it is one?'"[2]

This last example leads us to notice that all puns are logically instances of this fallacy. Lamb in the *Essays of Elia* quotes the following from Swift's *Miscellanies*: "An Oxford scholar meeting a porter who was carrying a hare through the streets, accosts him with this extraordinary question: 'Prithee, friend, is that thine own hare or a wig?'" Similarly, the proof that every cat has three tails, because no cat has two tails and every cat has one more tail than no cat, turns on ambiguity of terms.

It is probably impossible to enumerate all possible

[1] *Great Didactic*, Ch. 19. [2] *Formal Logic*, pp. 246-247.

sources of ambiguity, and the reader will readily suggest many others to himself. We will here only notice one more in which the ambiguity is rather in a phrase than in the separate words of that phrase. Words when united frequently have together a compound meaning which a mere union of their separate meanings will not give. That is to say, the force of a word depends partly on the context in which it occurs. Thus, "a person undertakes to cross a bridge in an incredibly short time: and redeems his pledge by crossing the bridge as one would cross a street, that is, by traversing the breadth. Now, though it is true that, in general, to cross is to go over the breadth, or shorter dimension, yet, in the case before us, the phrase is elliptical and signifies crossing *the river* upon the bridge. Nor can it be said that this common meaning is incorrect; that which is common and well known is, in language, always correct."[1]

(iii) **Figura Dictionis.**—The *Fallacia Figurae Dictionis* or *Fallacy of Figure of Speech* denoted with Aristotle a sophism which might arise from supposing words similar in form to be similar in meaning; *e.g.* that *poeta* is of the feminine gender because most Latin words with the same termination are so. The following invalid argument from Mill's *Utilitarianism* seems to fall under this head. "The only proof capable of being given that an object is visible, is that people actually see it. The only proof that a sound is audible, is that people hear it. . . . In like manner, I apprehend, the sole evidence it is possible to produce that anything is desirable, is that people do actually desire it." Here Mill assumes that the meaning of 'desirable' is analogous to that of 'visible' and 'audible.' But 'visible' simply means 'able to be seen,' and 'audible' 'able to be heard'; while 'desirable' does not mean 'what can be desired' but 'what should be desired.'[2]

The fallacy has been extended to cover other perversions of grammar, as in the traditional example: "What a man walks on he tramples on; this man walks on the whole day; therefore, he tramples on the whole day." This, it

[1] *Ibid.*, p. 246. [2] Cf. Mackenzie, *Manual of Ethics*, pp. 98-99.

will be seen, is only another form of ambiguity in words, and the fallacy does not differ in essence from *æquivocatio.*

The most important cases under this head are those in which error arises from the use of paronyms or conjugate words, as different parts of speech derived from the same root. As Dr. Davis points out, "These have by no means similar meanings, *e.g.* 'Artist, artisan, artful'; 'Pity and pitiful'; 'Presume and presumption'; 'Project and projector'; What is 'imaginary' is unreal, but an 'image' formed of wood or stone is real; To 'apprehend' is to lay hold on, or to come to a knowledge of, while 'apprehension' often signifies fear or dread."[1] So, to be 'artful' is not the same as being filled with artistic appreciation; an artful lover is not necessarily a lover of art.

Mill instances "the popular error that *strong* drink must be a cause of *strength*" as an example of this fallacy, and one which involves the further fallacy of supposing that an effect must be like its cause—using the words in their popular sense. People who fall into this error should, as Dr. Davis suggests, try strong poison; which is, perhaps, exactly what many ardent teetotallers would say they are doing. This fallacy, like the last, is an offence against the rules of definition, and when employed in syllogistic argument gives rise to the fallacy of four terms.

(iv) **A dicto simpliciter ad dictum secundum quid. A dicto secundum quid ad dictum simpliciter.**—These fallacies are converses of each other, and their essence is the confusion of an absolute statement with one limited or qualified in some way pertinent to the matter in hand. In other words, the essence of the fallacy is a failure to determine definitely the force of our terms, that is, it is a fallacy in definition.

The traditional example of the first is: "What you bought yesterday you ate to-day; you bought raw meat yesterday; therefore, you ate raw meat to-day," where the fallacy lies in not making clear that the 'rawness' is not regarded in the major premise as a relevant circumstance, and then assuming it to be relevant in the conclusion. On

[1] *Theory of Thought*, p. 270.

this example De Morgan remarks: "This piece of meat has remained uncooked, as fresh as ever, a prodigious time. It was raw when Reisch mentioned it in the *Margarita Philosophica* in 1496; and Dr. Whately found it in just the same state in 1826."[1]

An example of the converse form is: "Everything which is harmful should be forbidden; wine is harmful; therefore, its use should be prohibited," where the conclusion omits the tacitly understood, but very relevant, condition in the given minor premise of immoderate use.

Under the same head must be included arguments which illicitly conclude from a statement qualified in one way to one qualified in another way. These may be called *fallacia a dicto secundum quid ad dictum secundum alterum quid.* An example would be to conclude from the assertion —whether true or not—that to take life in sport is cruel, therefore, to eat flesh from which life has been taken by others is to show a cruel disposition. This is to infer from one special case to another special case differing from it in circumstances distinctly relevant to the question of intention, which is the point under discussion. "All the fallacies which attempt the substitution of a thing in one form for the *same thing* (as it is called) in another, belong to this head: such as that of the man who claimed to have had one knife twenty years, giving it sometimes a new handle, and sometimes a new blade."[2]

An amusing example of arguing *a dicto simpliciter ad dictum secundum quid* is contained in the following story: "A servant who was roasting a stork for his master was prevailed upon by his sweetheart to cut off a leg for her to eat. When the bird came upon the table, the master desired to know what had become of the other leg. The man answered that storks never had more than one leg. The master, very angry, but determined to strike his servant dumb before he punished him, took him next day into the fields where they saw some storks, standing each on one leg, as storks do. The servant turned triumphantly to his master; on which the latter shouted, and the birds put

[1] *Op. cit.*, p. 251. [2] De Morgan, *op. cit.*, p. 252.

down their other legs and flew away. 'Ah, sir,' said the servant, 'you did not shout to the stork at dinner yesterday: if you had done so, he would have shown his other leg too.'"[1]

More serious instances of the same fallacy are found in applications of abstract general rules to particular concrete cases without taking account of any modifying circumstances that may exist. It is easy to fall into errors of this character in the application to special cases of general rules of either social or individual life. Hence arise many wild assertions on economic and social questions. The argument, for instance, that because employment of labour is beneficial to the community, therefore, unemployed workmen may wisely be set to do work of an entirely useless character, merely to find them employment, is an example of reasoning *a dicto secundum quid ad dictum simpliciter*, as it omits in the conclusion the relevant condition without which the given premise is false—that the work must be productive of some utility.

So too, the application of the 'common sense of our ancestors' embodied in proverbs is very liable to involve the fallacy of arguing *a dicto simpliciter ad dictum secundum quid.* "What man has done man may do" we are told for our encouragement; but it scarcely seems to follow that each one of us is capable of becoming a Shakespeare or a Newton. So "a rolling stone gathers no moss" scarcely justifies us in inferring that every commercial traveller must starve.

De Morgan points out that "the law, in criminal cases, demands a degree of accuracy in the statement of the *secundum quid* which many people think is absurd."[2] He then gives two instances which he discusses at length.

In the first, a man tried for stealing a ham was acquitted on the ground that what was proved against him was that he had stolen a piece of a ham. In the second, a man was committed for perjury "in the year 1846" and the judge admitted the objection that it ought to have been "in the year of our Lord 1846." De Morgan argues—and it seems

[1] Boccaccio: *The Decameron.* [2] *Op. cit.*, p. 252.

to us conclusively—that in the latter case the *secundum quid* is unessential, for "as things stand, there is no imaginable difference: for there is only one era from which we reckon."[1] But in the former case the difference between the two descriptions is material. For suppose that the two descriptions were put before two different persons. "One is told that a man has stolen a ham; another that he has stolen a part of a ham. The first will think he has robbed a provision warehouse, and is a deliberate thief: the second may suppose that he has pilfered from a cook-shop, possibly from hunger. As things stand, the two descriptions may suggest different amounts of criminality and different motives."[2]

(v) **Compositio and Divisio.**—The fallacies of *Composition* and *Division* are converses of each other, and are most conveniently considered together. The error consists in joining together things which ought to be kept separate, or in separating those which ought to be kept conjoined. In other words, there is confusion between the collective and the distributive use of terms.[3] To take Aristotle's example: "Two and three are even and odd; two and three are five; therefore, five is even and odd"; which commits the fallacy of composition. Similarly, three and five are (together) four and four (together); but neither three nor five is four; and such an inference would be vitiated by the fallacy of division.

Such examples are obvious, but they do not differ in essence from the argument given by Mill in support of utilitarianism. He says: "No reason can be given why the general happiness is desirable except that each person, as far as he believes it to be attainable, desires his own happiness. This, however, being a fact, we have not only all the proof which the case admits of, but all which it is possible to require, that happiness is a good: that each person's happiness is a good to that person, and the general happiness, therefore, a good to the aggregate of all persons."[4] It would be difficult, as Professor Mackenzie

[1] *Ibid.*, p. 254. [2] *Ibid.*, p. 253. [3] Cf. pp. 25-26.
[4] *Utilitarianism*, p. 53.

says, "to collect in so short a space so many fallacies as are here committed."[1] There is a fallacy of equivocation in the word 'desirable,' here used by Mill in two different senses. But it is with the last part of the argument that we are now concerned. It is an obvious example of the fallacy of composition. Put symbolically it is— A desires the happiness of A, B that of B, C that of C, etc., therefore A desires the happiness of $A + B + C$, so does B, and so does C, which is about equivalent to arguing in mathematics that $ax + by + cz = (a + b + c)(x + y + z)$.

The converse fallacy of division seems to lurk in many of the arguments brought forward in support of an encyclopaedic curriculum for all schools. That a knowledge of this, and that, and the other subject is necessary to the community, therefore, that all those subjects should be taught to each member of that community, is the line of reasoning frequently adopted, as, apparently, by Mr. Herbert Spencer in the First Chapter of his book on *Education*.

The most common form of the fallacy may be reduced to an implicit confusion between a disjunctive and a copulative proposition. Thus, the spendthrift, falling into the fallacy of composition, argues "I can afford a or b or c or . . . z, therefore, I can afford a and b and c and . . . z." On the other hand the converse fallacy of division is often found lurking in the argument by which a miserly person refuses to subscribe to any charitable object. "I cannot afford to subscribe to a and b and c and . . . z, therefore I cannot afford to subscribe to a or b or c or . . . z."

The ambiguity of the word 'all' is a frequent occasion for this fallacy. As De Morgan says: "It must be remembered that the word *all*, in a proposition, is not necessarily significant of a universal proposition: it may be a part of the description of the subject. Thus, in 'all the peers are a House of Parliament,' we do not use the words *all the peers* in the same sense as when we say 'all the peers derive their titles from the Crown.' In the

[1] See *Manual of Ethics*, 3rd Ed., p. 219.

second case the subject of the proposition is *peer*; and the term *all* is distributive, synonymous with each and every. In the first case the subject is *all the peers*, and the term *all* is collective, no more distinguishing one peer from another than one of John's fingers is distinguished from another in the phrase 'John is a man.' The same remarks may be made on the word *some*; as in 'some peers are dukes,' and 'some peers are the committee of privileges.'"[1]

The unhappy and ungrammatical custom which has grown up of late years of using a plural verb after a collective noun, even when the action is that of the corporate body, as "The Senate recommend that such a course be pursued," tends to confuse the important distinction between statements made distributively and those which are only true collectively.

[1] *Op. cit.*, p. 248.

CHAPTER VII.

DIVISION AND CLASSIFICATION.

1. Logical Division.

(i) **General Character of Logical Division—Logical Division is the analysis of the denotation of a term.**

By this is not meant an enumeration of the individuals which form the class of which the term is the name, but a statement of the sub-classes into which that class can be divided. In other words, it is the splitting up of a genus into its constituent species.

The genus which is divided is called the *totum divisum*, or *divided whole*; the species into which it is analysed are styled the *membra dividentia* or *dividing members*. In dividing a genus we think of an attribute which is possessed by some of its members and not by others, and this suggests the *fundamentum divisionis*, or *basis of the division*.

The same genus may obviously be divided on several different bases into different sub-classes, according to the attributes on which the division is founded. Thus, triangles may be divided into equilateral, isosceles, and scalene, where the basis is the relation of the sides to each other in length; or into right-angled, obtuse-angled, and acute-angled, where it is the size of the angles. So, the various divisions of terms are analyses of the same genus on different bases. When the same genus is thus divided in different ways the process is called *Co-division*; and the classes obtained by co-division more or less overlap each other, for every member of the genus must fall into one class in each division, and the classes obtained on one basis will not correspond exactly, if at all, with those resulting

from another. If they did coincide the supposed different bases would really be one base.

When the classes resulting from an act of division are themselves again divided into their sub-classes we perform an act of *Sub-division.* These sub-classes may be again sub-divided, and so the process may go on till we reach *infimae species*—classes, that is, which are only capable of being split up into individuals.

Of course, in every step of a sub-division, we must have a new basis of division, for each step exhausts its own basis. Thus, having divided triangles into equilateral, isosceles, and scalene, it is evident that we cannot sub-divide any of these classes on the basis of the relative lengths of their sides. But if we take a new basis we may continue the analysis; for instance, we may sub-divide both isosceles and scalene triangles on the basis of the size of their angles into right-angled-isosceles, obtuse-angled-isosceles, acute-angled-isosceles; right-angled-scalene, obtuse-angled-scalene, and acute-angled-scalene. Or, if our original basis was the size of the angles, then we may sub-divide acute-angled triangles into acute-angled-equilateral, acute-angled-isosceles, and acute-angled-scalene; while right-angled and obtuse-angled triangles may be divided into right-angled-isosceles, right-angled-scalene; obtuse-angled-isosceles and obtuse-angled-scalene.

Every division should be progressive; it should proceed one step at a time, and omit no intermediate species. Hence the old logical rule *Divisio non faciat saltum*—Division must not make a leap. If this rule is broken we need not be surprised to find that some of the members of the whole we start to divide find no place at all in any of the members into which we have divided it; for in omitting an intermediate class the distinctive marks of that class will probably at the same time have been overlooked, and thus, individuals having those marks, but not possessing the distinctive marks of the lower species contained in the division, will have been omitted.

(ii) **Logical Division is indirect and partially material.**—But few words are needed to show the utility of division. Every subject is treated more easily and com-

prehended more thoroughly when its various parts are arranged in an orderly way. Division, in fact, adds clearness to our notions, as definition makes them distinct. As compared with definition, however, logical division is a secondary and indirect process, for it is a necessary assumption of formal logic that the connotation of a class term determines its denotation. We do not select a number of objects indiscriminately, and then seek for some attributes common to them all, which may form the connotation of the class-name we affix to them. We start from the class-names of ordinary speech and from the more or less definite meaning attached to them, and then include or exclude individuals from that class in accordance with their possession of, or want of, that meaning.

As knowledge advances the meaning becomes more definite and exact, and the application of the term is in consequence made more precise. This is *practically* much less the actual process in some instances than in others, but, *formally*, we must assume the class to be always determined by the connotation and not by the denotation. Besides, division must presuppose more or less complete definitions of the names of the species into which a given genus is to be divided; for it is only by appeal to such definitions that we can determine a *fundamentum divisionis*; while every definition of a species-term *per genus et differentiam* suggests such a *fundamentum*.

Hence, we also see that no division can be purely formal, that is, involve no appeal to knowledge outside the matter given. If we are simply given a genus we cannot even begin to divide it; for, of necessity, the attributes which separate one species from another can form no part of the connotation of the genus. Every such attribute must be a separable accidens of the genus, and can only be known by an appeal to sources of information other than the connotation of the name of that genus. Moreover, for the division to be of any practical use, this appeal must be to the objects themselves which are included in the genus; for only thus can we be sure that we are dealing with really existing classes of things. Thus, every division contains, at least to some extent, a material element.

That element, without which the formal process cannot start, gives the basis of the division. Formal logical division, therefore, is inadequate for the purposes of modern science, which has developed elaborate schemes of classification. These, however, rest on and assume the formal principles of the earlier process, just as all thought assumes the fundamental postulates of knowledge we discussed in Chapter II. We shall do well, then, to consider division before going on to deal with classification, and in doing so we shall be following the order of historical as well as logical evolution.

(iii) **Operations somewhat resembling Logical Division.**—As a logical division is the analysing of a genus into its species it follows that only general terms can appear in it. A singular term cannot be divided, for it is a name applicable in the same sense to one individual only; and the logical meaning of an 'individual' is that which is incapable of logical division. And the division must stop at *infimae species*; for to go further would be to enumerate individuals, and this, as has been already pointed out, is not logical division. Hence, *Logical Division* must be carefully distinguished from

(*a*) *Physical Partition*, which is the splitting up of an individual into its constituent parts; as, for instance, a ship into hull, masts, sails, rigging, etc.

(*b*) *Conceptual Analysis*, or the enumeration of the attributes of a class or of an individual; as when we name whiteness, ductibility, malleability, etc., as the attributes of silver.

(*c*) *Distinction of the various meanings of an equivocal term*; as when we distinguish between 'vice' meaning a moral fault, and 'vice,' a mechanical tool.

In a logical division the genus can be predicated of each of the species, and of each individual member of those species. This follows necessarily from the fact that the definition of the species involves the genus. For instance,

if we divide animals into men and brutes, we can predicate of each man and of each brute that he, or it, is an animal. In none of these other processes, however, can the whole be predicated of the parts. We cannot say 'A mast is a ship,' or 'Whiteness is silver.' In the case of Distinction, of course the same *verbal symbol* can be predicated of each of the meanings: 'This tool is a vice,' or 'This fault is a vice.' But the same definition—that is, the same connotation—cannot be so predicated, which shows that it is not the *same logical term* which is predicated in each case.

This illustrates anew what was said before—that an equivocal term is really and logically two or more terms.[1]

2. Principles of Logical Division.—That a valid division satisfies the following conditions may be gathered from the preceding section—

I. *Each act of division must have only one basis.*

II. *The sub-classes must be together co-extensive with the whole.*

III. *If the division be a continued one* (i.e. *embrace more than one step*), *each step must be, as far as possible, a proximate one.*

Or, more briefly: *The division must* (i) *avoid cross division, be* (ii) *exhaustive, and* (iii) *step by step.* Evidently only the first two principles apply to a single act of division; the third refers to the relation of one division to another in the same series.

Principle I.—This principle is fundamental and really implies the other two. A division made on more than one basis would be worthless. It would be nearly certain to include some individuals in more than one sub-class, so that the total denotation of the species would be apparently greater than the denotation of the genus. If, for example, triangles were divided into isosceles, scalene, and acute-angled, every possible triangle would fall into one or other of these classes (for every equilateral triangle is acute-angled), but some would fall into more

[1] Cf. p. 20.

than one; viz. those which are acute-angled-isosceles and those which are acute-angled-scalene.

But we have no guarantee that the opposite fault will not be committed and the division be made *too narrow* by the exclusion of some individuals from every sub-class. If we divided triangles, for instance, into equilateral, obtuse-angled, and right-angled, we should not, indeed, include any individual twice, but we should exclude all acute-angled-scalene and acute-angled-isosceles triangles. Our division is too narrow. Very probably both faults will be committed; some individuals will be included more than once and others omitted altogether. Thus, if we divide triangles into equilateral, isosceles, and right-angled we include right-angled-isosceles triangles twice, and exclude obtuse-angled-scalene and acute-angled-scalene triangles altogether.

It may, indeed, occasionally happen that a division may be made on two ostensible bases and yet be practically accurate. But it is only in the exceptional instances when one attribute solely involves, and is solely involved by, another that this can occur. And then, as has been said, the two apparent bases are really one. For example, a division of triangles into equiangular, isosceles, and scalene would be both exclusive and exhaustive; but that is simply because all equilateral triangles, and they only, are equiangular, and so the division coincides with one made on the single basis of the relative lengths of the sides. Only when we have but one basis of division can we be sure that our sub-classes are necessarily exclusive of each other—that no individual can be placed in more than one.

Principle II.—We have seen that a violation of the first principle is apt to lead to a too narrow division, that is, to the exclusion of part of the denotation of the whole from each of the sub-classes. In practice the same fault may be committed, even if that principle be rigidly adhered to; for it is possible to omit one or more of the sub-classes in any division. We should get too narrow a division, for example, by dividing triangles into equilateral and scalene, and omitting isosceles.

Other examples of too narrow divisions are of men into

good and bad, of books into instructive and amusing, of objects into useful and ornamental. In such simple cases as these the fault is not likely to be committed, but when we are dealing with matter as complex as nature continually presents to us it requires great care to ensure that we have made a *complete* enumeration of all the species contained under a genus. This, of course, depends on our knowledge. It is only when that is adequate that our division really coincides with the distinctions existing in the world. And often we do not know the degree to which our knowledge is complete. So we see that logical division really is an arrangement of our thoughts according to our knowledge—an arrangement which may be of any degree of coincidence with the system of nature. It is here that we see both its inadequacy for science and its basic relation to the classifications of science, which are of necessity arrangements not only of things but of men's thoughts of things.

The opposite fault is to make the division *too wide*; that is to include among the species some objects not denoted by the genus. This, again, is not likely to occur in simple cases; few, for example, would think of dividing coins into gold, silver, bronze, and banknotes. But an indistinct apprehension of the connotation—that is, of the definition—of any of the terms we employ in our division may lead to this fault when we are dealing with complex matter.

It is plain that if this rule is broken we have not really divided the genus at all; but either only a part of it—when the division is too narrow; or the genus and something else as well—when the division is too wide. In a true division the sum of the denotations of the species exactly coincide with the denotation of the whole; and only when this is the case has the genus given been really and accurately divided.

These two related principles may be summed up symbolically.

If a genus, G, is divided on the basis, D, into the species, S_1 S_2 - - - S_n, then D appears in each species in a special form, d_1 d_2 - - - d_n.

Then the first principle gives

$$G(D)$$

$$\begin{array}{cccc} S_1 & S_2 & & S_n \\ (= Gd_1) & (= Gd_2) & \ldots\ldots & (= Gd_n) \end{array}$$

And the second demands that in denotation

$$S_1 + S_2 + \; - \; - \; - \; S_n \;=\; G.$$

Principle III.—This has been already discussed when it was remarked *divisio non faciat saltum*, and it was pointed out that a violation of it usually leads to a division being too narrow.

3. **Division by Dichotomy.**—A Logical Division can be assured of absolute *formal* validity when it proceeds by dichotomy, that is by division at every step into a positive term and its corresponding negative. This process is formally perfect, for it is wholly based on the Principles of Contradiction and Excluded Middle.

A strictly dichotomous, or bifid, classification can always be thus formed, and it can never violate any of the principles just considered. But in relation to real thought about real things it lies open to the objections—

(*a*) That, at each step, one of the sub-classes—and that frequently the largest; viz. that denoted by the negative term—is entirely undefined in its extent; and, no matter how far the process of sub-division is carried, the last term must always be formally left thus indefinite.

(*b*) That, in so far as it is formal, it is entirely hypothetical; the division does not guarantee the existence of any of the sub-classes.

(*c*) That it is excessively cumbrous. It seems absurd to divide a genus into two classes when it evidently falls naturally into some other, and equally definite, number of species, and to do so obscures the fact that these species are coordinate.

We may illustrate the process by the following examples:—

(i)

- Men
  - Europeans
    - Frenchmen
      - soldiers
      - non-soldiers (*if any*)
    - non-Frenchmen (*if any*)
  - non-Europeans (*if any*)

(ii)

- Men
  - Europeans
  - non-Europeans (*if any*)
    - Asiatics
    - non-Asiatics (*if any*)
      - Africans
      - non-Africans (*if any*)
        - Americans
        - non-Americans (*if any*)
          - Polynesians
          - non-Polynesians (*if any*)[1]

It is evident that these, like all other divisions, in so far as they are not hypothetical, have material bases. It is by appeal to the *matter* that we know that some men are Europeans. Even then, the existence of non-Europeans is hypothetical unless we make a further appeal to outside matter. Similarly, 'Frenchmen' is not part of the connotation of Europeans—we must again appeal to fact. Every step is, therefore, either partly material, or wholly hypothetical, and it is evident that a purely hypothetical division is of no practical use.

[1] Cf. the Tree of Porphyry, pp. 43-44.

In all strictly dichotomous division we must, at least, finish with a hypothetical term of whose existence, or non-existence, the division leaves us absolutely ignorant. Every division may be reduced to dichotomy, but, as was said above, it is absurd to do this when we know definitely the number of sub-classes to which our basis of division will give rise. Thus we may make the division

- Triangles
  - equilateral
  - non-equilateral
    - isosceles
    - non-isosceles
      - scalene
      - non-scalene (*if any*)

We know that the last class does not exist, for equilateral, isosceles, and scalene form a complete enumeration of the species of the genus triangle on this basis of division, and these species are co-ordinate. This shows, however, that every division on one basis into more than two sub-classes may be expanded into several successive divisions on slightly different bases. But to do this obscures the fact that they are co-ordinate in relation to the original genus.

Dichotomy may at times be found a useful mode of testing the validity of a division—particularly in discovering whether it is exhaustive—and in finding the position of any assigned class. Thus, in the Analytical Key prefixed to Bentham's *British Flora*, which is intended to enable anyone who has a specimen of a certain plant before him to discover its species and its technical name, the arrangement is nearly entirely dichotomous, and, for such a purpose, this form is the most useful. But to adopt dichotomy as a *final* arrangement would be absurd. A botanist, for example, starts at once with three classes of the *summum genus* 'plant,' viz. exogens, endogens, and acrogens, and each of these is sub-divided

into varying numbers of orders, and these again into still further varying numbers of genera, and so on, with little or no regard to dichotomy, the object being to make the classification agree with the distinctions existing in the plants themselves.

It may be added that every definition *per genus et differentiam* suggests a division by dichotomy, and, conversely, every such division supplies us with a definition of that kind.

4. **Nature of Classification.**—We have seen that logical division is the analysis of the denotation of a term as far as possible on formal grounds, though all reference to material considerations cannot be avoided. Such a doctrine can be of little utility by itself: it limits thought to its own content, and thus does not prompt it to advance in mastery of the real world. But just as definition builds on the old formal basis of genus and difference, so scientific classification is founded on the principles of formal division. It goes beyond it, in that its essential interest is in the actual things of the world and their relations to each other, but it does not discard its principles, for if it did the organisation of reality it presented would be nullified by internal inconsistency. Every classification seeks its material in the world, but it only represents that material truly if it can also stand the test of formal validity.

Classification is systematic arrangement, and is thus at once the aim of science and, even in an incomplete stage, a great help to its advance. It reduces the bewildering variety of nature to some semblance of order, seeking to group together things according to their likenesses and to separate them according to their differences, and that in such a way that the grouping may be profitable for the purpose in hand. There is no question for logic of an actual arrangement of real things in space and time. That, of course, may be made—as of the objects in a museum; but such physical arrangement is only the expression in fact of a system already constructed in thought. It is with classification in the sense of mental systematisation that science, and consequently logic, are concerned.

The work of classifying the real things of the world is evidently one of extreme complexity, which can only be accomplished little by little. The most obvious divisions have been made in the past and are embedded in language. Every general name marks a classing of like things. But these classes are not only in many cases somewhat indeterminate in their extent and vague in their outlines, but they generally stand in no definite relations to each other. A lexicon of common nouns is in no sense a classification of things; it gives us names of unrelated groups.

If, then, we take as a starting-point the name of a wide class of objects, say 'plants,' how shall we proceed to group the innumerable plants which exist in the world? Here again the initial work has been largely done for us. Language gives us many groupings of individual things of very different degrees of generality. So we can speak of cabbages as well as of vegetables, of roses as well as of flowers, of oaks as well as of trees. These were the first rough genera and species. The subsuming of them all under the highest genus 'plant' was a great and fruitful step in generalisation.

Such groupings were originally based on external resemblance; and the history of science presents many instances in which more exact knowledge has removed whole classes from one wider group to another; even from one highest genus to another, as when sponges were found to be animals and not plants as had previously been assumed from their general appearance.

Moreover, such groupings have been made without any particular object. The things are massed together first because their similarity in some respect has been noticed. But if we set out to think exactly, it is always in reference to some special purpose. So we want to group the things about which we think in reference to that purpose. If we want to classify plants according to their medicinal qualities we shall take as our basis of division quite different qualities from those we should choose were it our object to arrange them according to their values as foods for man and beast. The former of these would interest the doctor, the latter the farmer and market gardener. Neither would

satisfy the botanist, who would want to think the various plants in those relations to each other which would indicate their origin and descent. This last is, obviously, the classification which takes plants just as constituents of organic nature; and it, therefore, best shows their place in the general scheme of things. So it would be called the General Classification of plants.

There are, of course, as many such general classifications as there are sciences, and they often overlap. A medicinal classification will, for example, include objects from both the plant and the mineral world; a chemical classification covers from its own point of view every material object. The old idea of a world made up as it were of water-tight compartments, to each of which corresponds a science, has had to yield to the view of one comprehensive system, which, however, man can only study in separate aspects. His list of sciences is only a classification of his own modes of thought and investigation, in which he singles out certain classes of forces and relations for examination: but this singling out corresponds to no separation in reality. A plant is a subject for chemistry, for physics, for geology, as well as for botany. But in botany only is it looked at essentially as a plant; in the other sciences its unity as a plant is disregarded. So for plants as plants the botanical is the General Classification; that is to say, the highest genus for botany is 'plants,' just as for medicine it might be 'drugs.'

None of the sciences has yet a perfect classification, though some are more advanced than others. So it is plain that the systematising of them into a comprehensive thought of the world as a whole is not yet in sight.

5. **Special or Artificial Classifications.**—In addition to our attempts to organise our thoughts of things for purposes of exact and comprehensive knowledge, we often require to make temporary arrangements of an essentially artificial character. These are intended as a rule to enable us to find or identify any object amidst a number of others. That attribute which renders identification as easy and certain as possible is chosen as a basis, whether

it has, or has not, importance in itself. A very familiar instance is the alphabetical arrangement which is found to be most serviceable in dictionaries, library catalogues, and indexes of books, the sole purpose of which is to indicate where a certain word, book, or statement, is to be found. At each step the alternatives are quickly narrowed down until we are led with certainty to the object of our search.

Useful as such an arrangement is as a key, there its validity ends. It is founded on a conventional distinction; its value is confined to the language in which it is written; it does not admit of any general statement being made about any of the groups which are formed. The *M* group, for instance, may—like Monmouth and Macedon—have practically nothing in common except their initial letters. Nor does it form a permanent part of the results of the investigation and research to which it is subsidiary.

An alphabetical arrangement is impossible where the number of objects to be classified is quite indefinite. Then a less simple method is needed to ensure a speedy identification. Thus the animal and vegetable kingdoms comprise innumerable individuals, and it is desirable for the purposes of study to be able to identify and name any given specimen. Since, from the nature of the case, there are no conventional marks, it is necessary to choose as distinguishing characteristics some property or properties of the objects themselves. Any easily recognisable attributes which are sufficiently distinctive will answer the purpose: they are not chosen for their intrinsic importance, but solely as convenient descriptions of the objects specified. Some of the classifications of the earlier naturalists were of this kind. Linnaeus, for example, arranged plants into twenty-four classes distinguished by the number or by some other simple characteristic of the stamens. This was of great service to the student, who by a brief examination of the stamen and pistil was able at once to assign a new plant to its proper place.

Similar service is rendered by such an index to a more scientific classification as the analytical key prefixed to Bentham's Flora. The object is still to enable us to find

the name of a plant of which we have a specimen before us. As an arrangement with only two alternatives at every step is found to offer the most ready means for attaining this object, that is the plan almost exclusively adopted. Thus, flowering plants are first divided into those whose flowers are compound, and those which are not compound. Then the former are sub-divided into those with one seed, and those with more than one; and the latter into those in which the perianth is single, and those in which it is double; and so on. Here we see a direct application of the most formal kind of division—that by dichotomy.

A classification which serves as a key to a more general classification is generally called a *Diagnostic Classification*. Such classifications have been called *Artificial* as opposed to those based on deep knowledge of nature, which have been termed *Natural*. The distinction originated in a doctrine, now exploded, that 'natural kinds' existed in nature which were sharply and definitely separated from one another by an unknown and indefinite number of permanent differences, and that a natural classification should correspond in its divisions to such natural groups. All other classifications were regarded as arbitrary and, therefore, artificial. But no such fixed divisions do exist in nature, and the distinction as originally conceived consequently falls to the ground. The names, however, are still often retained, and then they correspond broadly to the two divisions we have made.

The objection to the terms is that the distinction they indicate by their meaning is not clear. For in a sense all classification is artificial: it represents that mental grouping most convenient for our purpose, and is based on qualities selected with reference to just that purpose. And if it be said that an alphabetical arrangement is purely artificial because it is not based on attributes of the objects themselves, an analytical key will scarcely be called natural because it has such a basis. It remains a device or artifice for a special purpose in spite of the fact that the marks of identification are sought in the nature of the object, and are not imposed from without. The distinction between

The difficulties in the way of classification are great. Logic, of course, can never tell us what attributes should be chosen as the basis of a division. Wide and intimate acquaintance with the department of knowledge concerned will alone enable a wise selection to be made. For the problem is the same as that of framing a scientific definition. Just as in common speech every general name marks a class, so, but with greater accuracy, does the definition which results from scientific research. But we have seen how intractable to exact determination are physical objects, whether organic or inorganic. In the last resort we are left, say in the case of a chemical element, with the co-existence of certain properties, and in the case of a living thing, whether plant or animal, with the co-existence of certain characteristics without any apparent connexion between them. Now we desire to hold together in thought those things which really resemble one another, and it becomes essential to determine which collocation of attributes most truly represents their nature. In other words, we have to ask what attributes are the most important, and on what principle, if any, are they to be selected.

The history of classification shows that men of science have long been groping their way towards answers to these questions, and that only in comparatively recent times has a satisfactory principle been suggested.

At first only the most obvious resemblances were noted. Thus we have the ancient division of plants into trees, shrubs, and herbs. And for a long time the tendency was to seek out the obvious and the superficial, and to count rather than to weigh characteristics that appeared alike. But in time it was found that this procedure was misleading, and that species were often akin although externally very dissimilar. This stimulated men to probe deeper. Some characteristics were found to carry with them many others, and though often no reason could be given why this should be the case, they were regarded as dominant characteristics and made the basis of classification. Again, the organs which subserve the processes of life are of such obvious importance and utility to the individual and to the species that they were pressed into the same service,

Now both these bases are extremely serviceable in the practical work of science, but both are defective. The characteristic which is of the first importance in one case, and which seems to dominate the whole being of an animal or plant, is in another of altogether minor import. This led to divisions on many bases. "So with plants, how remarkable it is that the organs of vegetation on which their whole life depends are of little signification excepting in the first main divisions, whereas the organs of reproduction with their product the seed, are of paramount importance."[1]

The object of the search throughout was to reach divisions in which things having the most affinity should be placed together. The doctrine of evolution taught that affinity should no longer be regarded as a metaphorical term implying resemblance, but should be taken as implying that family relationship which is the primary meaning of the word. It thus supplied the principle of community of descent as the binding thread for scientific classification, and made those characteristics important which indicated a common origin.

Under the influence of this conception the aim of classification has been completely revolutionised. Whereas formerly the realm of organic life was assumed to consist of a definite number of species which could be separated from one another by definite marks until the whole number of species had been enumerated and described, now, as it exists at any given moment, it is considered rather as the result of descent from a common ancestry with the modifications which successive ages have witnessed. The ideal for the older classifications was that of a formal division in which the highest genus was so divided that the sub-classes were mutually exclusive and at the same time co-extensive with the whole; the aim for the newer is rather the construction of a genealogical tree which will just as surely include every member of the family. Were it possible, for instance, in the animal kingdom, existing species would be traced back from im-

[1] Darwin, *Origin of Species*, Ch. 13.

mediate to remote ancestors until their common origin appeared. At each step the relationship between one class and another would be sufficiently evident for the degree of affinity to be observed, and those which were nearest of kin would be classed together. Thus species which are derived from the same genus would come nearer to one another than to those derived from another genus, and genera would be more nearly related than the species to which they give rise. Indeed at each remove upwards the relationship would become closer, until the endless variety of species was seen to issue from an ancestry common to all. Such a table would have the great advantage of being drawn up on a common principle throughout.

It is not implied that the actual procedure is always from the species upwards. The reverse process is also used in practice. Indeed the construction of such a genealogical tree is beset with difficulties on all sides. For it must embrace not only existing species, but many of which fossil remains are now the only evidence. Ideally, too, it should contain others of which the existence can only be conjectured. Gaps in the relationship exist, and in many cases all certain evidence of affinity is lost. Then the scientist has to fall back upon other characters to constitute his classes.

Nevertheless the doctrine of descent with modifications is a clue of the greatest importance, and indicates the kind of evidence which should be sought. We are guarded, for instance, from the natural mistake of supposing that characters essential to the maintenance of life are good guides to natural affinity. They may be useful, but resemblances in this respect may be merely the result of adaptation. External resemblance is, indeed, of no scientific significance. "It might have been thought (and was in ancient times thought) that those parts of the structure which determined the habits of life and the general place of each being in the economy of nature would be of very high importance in classification. Nothing can be more false. No one regards the external similarity of a mouse to a shrew, of a dugong to a whale, of a whale to a fish, as of

any importance."[1] The possession of organs now useless often gives a better indication of descent.

The process of evolution is not yet exhausted. Present species are liable to modification or even to extinction, and there is the possibility of new forms being evolved. The classifications of to-day, therefore, are in a sense transitory. No doubt they are permanent so far as they are correct. Past history shows that vast epochs of time have been necessary for the transformations which are already known, nor is there any reason to suppose that the changes which the future may hold will be effected any the more rapidly. The error in taking as fixed what has been described as a cross-section through a continuous process is negligible. Error, of course, may arise through defects of observations, or through mistaken affiliation, but this does not affect the validity of the principle employed.

Since a direct relation is established between the several classes of such a nature that we can pass from one to another through different degrees of descent, one class may be regarded as subsumed under another because of its affinity. The classification is then called *subsumptive.* Moreover it is found convenient, in order to describe the classes so fixed and determined, to use in each case, in addition to the terms 'genus' and 'species,' such terms as 'kingdom,' 'class,' 'order,' 'family.' Thus the animal kingdom has the vertebrates as one 'sub-kingdom,' and this again is divided into the five 'classes,' mammals, birds, reptiles, batrachians, fishes: these again are sub-divided into orders, families, genera, species, and of the last there may be a further sub-division into varieties.

The theory of evolution has not involved a radical alteration of the classes established before its first enunciation by Darwin in 1859. In Botany, for example, most of the orders and families distinguished by Jussieu towards the close of the eighteenth century are still recognised. Other able workers in the same field contributed to the establishment of classes based on real affinities which were little

[1] Darwin, *Origin of Species*, Ch. 13.

understood. Out of wrong principles, or principles only partially correct, arose much valuable work. The classifications thus blindly but brilliantly developed received their key and explanation in the doctrine of descent: this afforded a clue in doubtful cases, and a norm by which to correct the past and to control the labours of the future. "Community of descent is the hidden bond which naturalists have been unconsciously seeking, and not some unknown plan of creation or the enunciation of general propositions and the mere putting together and separating objects more or less alike."[1]

7. Limits of Classification.—The advantages of classification for the purposes of study lie on the surface. Whatever introduces order into the material under consideration makes it easier to grasp. Where the objects are so numerous that the mind is bewildered by their variety, a system which arranges them in groups on a recognised principle, especially if it be accompanied by a scientific nomenclature, is an invaluable aid to the memory. Only in this way do the species of the organic world, running in number to hundreds of thousands, become at all manageable.

Moreover, when a specimen is assigned provisionally to its place amongst those most closely allied to it, properties other than those noted for the purposes of classification are suggested, comparisons may be made, and inferences by analogy may be drawn. And so the work of discovery and investigation is aided.

Nevertheless it must be admitted that only in the so-called natural sciences of Botany and Zoology has classification been developed at all extensively. Organic kinds seem especially susceptible of being classed according to their origin. If we seek a similar principle in inorganic nature there is nothing corresponding to a genealogical tree. The physical sciences are concerned very largely with processes. These have a history, but it is not their history which is principally important for their explanation, other-

[1] Darwin, *Origin of Species*, Ch. 13.

wise the method of experiment would be inapplicable. The guiding thread in such investigations is the principle of causation. Fruitful as this has been, it has not so far resulted in broad classifications at all comparable with those of the organic world.

At the same time the principle is proving of increasing value in this direction. "There is the physical classification into Solids, Liquids, and Gases. But these states of matter are dependent on temperature; at least it is known that many bodies may, at different temperatures, exist in each of the three states. They cannot, therefore, be defined as solid, liquid or gaseous absolutely, but only within certain degrees of temperature and therefore as dependent on causation. Similarly, the geological classification of bodies according to relative antiquity (primary, secondary, tertiary, with their sub-divisions) and mode of formation (igneous and aqueous) rests upon causation, and so does the chemical classification of compound bodies according to the elements that enter into them in definite proportions."[1]

It is only, then, in a few of the sciences that classification has wide scope. In itself, moreover, it is only a preparatory stage. It determines its classes but does not answer the question *why* they are as they are. That is the task of explanation which can only be considered at a later stage.

8. Scientific Nomenclature and Terminology.

A Nomenclature is a system of names for the groups of which a classification consists.

No classification could long remain fixed without a corresponding nomenclature, and every good nomenclature involves a good system of classification. The two are indissolubly connected. It follows that only those sciences which have a fairly complete and generally received classification possess a true general nomenclature—the sciences, that is, of botany, zoology, and chemistry. As the classifi-

[1] Read, *Logic Deductive and Inductive*, pp. 267-268.

cation must be the ground of the nomenclature it follows that the latter is a consequence rather than a cause of extended knowledge.

Whenever a science admits of comprehensive heads of classification a good nomenclature recalls both the resemblances and the differences between classes. Such a nomenclature prevents our being overpowered and lost in a wilderness of particulars. The number of species of plants, for example, is so enormous that if each had a name which expressed no relation with any other, memory would find it impossible to retain more than a very small fraction of the whole number. The nomenclature should, therefore, be so constructed as to suggest these relations. There are two main ways of doing this—

(1) The names of the lower groups are formed by combining names of higher and lower generality.

(2) The names indicate relations of things by modifications of their form.

The former method is that which, since the time of Linnaeus, has been adopted in botany and zoology. In botany, for instance, the higher groups have distinct names, *Dicotyledon*, *Rosa*, *Geranium*, etc. The species is marked by adding a distinctive attribute to the name of the genus, as *viola odorata*, *orchis maculata*, etc. These distinctive attributes are not the logical differentia of the species, so the specific name is not a definition. They are, on the contrary, formed from all kinds of more or less important considerations; for example, the *Anemone Japonica* was named from the country in which the plant was first observed. The names of varieties, sub-varieties, etc., are formed on the same principle as those of species.

The second method of constituting a nomenclature is that employed in chemistry. This system of names is founded on the oxygen theory. It "was constructed upon . . . the principle of indicating a modification of relations of elements, by a change in the termination of the word. Thus the new chemical school spoke of

sulph*uric* and sulph*urous* acids; of sulph*ates* and sulph*ites* of bases; and of sulph*urets* of metals; and in like manner, of phos*phoric* and phos*phorous* acids, of phos*phates*, phos*phites*, phos*phurets*. In this manner a nomenclature was produced, in which the very name of a substance indicated at once its constitution and place in the system."[1]

We require, however, not only a system of names to designate classes, but **a collection of terms which enables us to describe individual objects.** This is a **Terminology,** and it embraces names of the properties—shape, colour, etc.—and of the parts of the objects recognised in the science. Terminology is, in brief, the language in which we describe objects, and without description there can be no classification. All the names which form a terminology are general names; though, by their combination, we can describe individuals.

Botany is the only science which, as yet, possesses a complete terminology; this, as well as its nomenclature, it owes to Linnaeus. "The formation of an exact and extensive descriptive language for botany has been executed with a degree of skill and felicity, which, before it was attained, could hardly have been dreamt of as attainable. Every part of a plant has been named; and the form of every part, even the most minute, has had a large assemblage of descriptive terms appropriated to it, by means of which the botanist can convey and receive knowledge of form and structure, as exactly as if each minute part were presented to him vastly magnified . . ."

"Thus the flower was successively distinguished into the *calyx*, the *corolla*, the *stamens* and the *pistils*; the sections of the corolla were termed *petals* by Columna; those of the calyx were called *sepals* by Neckar."[2]

If a term used in common life is needed for a scientific description, its meaning must be exactly determined. *Apple-green*, for example, must refer to one specific shade of green. Otherwise the term is ambiguous, and, therefore, useless for the purposes of scientific terminology.[3]

[1] Whewell, *Novum Organon Renovatum*, p. 275.

[2] *Ibid.*, p. 315

[3] Cf. *ibid.*, pp. 111-113.

9. **Fallacies in Division.**—A violation of any of the principles of division involves fallacy. We may thus enumerate three classes of fallacy of division, each violating one of the fundamental principles, viz.—

(i) Changing the basis of division.
(ii) Omitting part of the genus to be divided.
(iii) In a continued division, not proceeding by proximate steps.

These faults have been sufficiently discussed and illustrated in our positive treatment of the principles of division.[1]

[1] See pp. 75-78.

CHAPTER VIII.

DEFINITION AND KINDS OF PROPOSITIONS.

1. Definition of Proposition.

A Proposition is the verbal expression of a truth or falsity. In other words, a proposition is a statement which claims assent, but may meet with denial. It is thus distinguished from such forms of speech as commands or questions, with which, as such, logic has no concern, as they cannot enter into the structure of knowledge. They do not claim truth, nor can they be rejected as false. Only statements can be true or false, and in no other way can truth or falsity be expressed. A fact is not true; it simply exists. It is when we make a statement about it that we challenge assent. To deny a proposition is to assert its falsity; to assent to it is to affirm its truth. When a statement is false it may be so intentionally or unintentionally. In the former case it is a falsehood as well as a falsity. But with the ethical intention logic has no concern. The only question for it is whether the statement is in itself true or false.

2. **Kinds of Propositions.**—The forms in which statements are actually made are very various, but they all express one of three fundamental forms of assertion. The assertion may be absolute or **categorical,** as 'Lions are fierce'; or it may be **hypothetical** when the truth of the main assertion is given as dependent on the fulfilment of some condition, as 'If the season is wet the harvest will be spoilt'; or lastly, it may be **disjunctive** when the statement is of several alternatives, as 'Triangles are either equilateral, isosceles, or scalene.'

In the next place there is the fundamental distinction between assertion and denial—the *Quality* of the proposition as it is called. In other words a proposition may be either **affirmative** or **negative**.

Lastly, propositions differ in the scope and definiteness of their reference. We may say 'All lions are fierce'; 'Some children are easily frightened'; 'This town is healthy.' In actual speech the number of such variations is very large, but logically the important distinction is whether the statement is made definitely about every object of which it is made, when the proposition is called **Universal**, or whether its scope is left indeterminate, when it is styled **Particular**. This distinction is said to be one of *Quantity*.

3. Categorical Propositions.

(i) **A Categorical proposition makes an absolute statement**, as 'Gold is yellow,' 'The quality of mercy is not strained.' No matter in what form such an assertion may appear in ordinary speech, logical analysis reduces it to the simple expression which may be symbolised by *S is P*, or *S is not P*, where *S*—the subject—stands for that of which the assertion is made, and *P*—the predicate—for that which is affirmed or denied of it. Not a little ingenuity is sometimes required to make this reduction, but till it is made, formal logic cannot deal with the judgment. Such an exclamatory statement as 'Fire!' must, for example, be expanded into some such proposition as 'That property is on fire.' Similarly, the impersonal assertion 'It rains' becomes for logic 'Rain is falling.'

Sometimes the statement is qualified by subordinate clauses embedded in it. In such cases the main predication must be carefully distinguished from the subordinate assertions. This, in itself, is a matter of grammatical construction. In other cases two logical propositions are united in one statement, as 'Gold and silver are precious metals,' or 'He is poor but honest,' where it must be recognised that there are really two predications which it may be necessary to consider separately. In such cases as

'Graduates alone are eligible' the two-fold predication is abbreviated; the full force of the proposition can only be represented by the simultaneous assertion of two propositions, 'Some graduates are eligible' and 'No non-graduate is eligible.'

In every such re-statement care must be taken not to change the judgment—that is, the assertion intended to be made—but only its verbal expression. In other words, logic claims the right to translate the forms of ordinary speech, but not to change their significance.

(ii) **Analysis of the Categorical Proposition.**—If we examine this form we see that it consists of two **terms** joined by *is* or *is not*, which is, therefore, called the **Copula.** Of the two terms that of which the assertion is made is the **Subject**; from that the thought starts; it is its fixed centre. The other is the development of that thought by the assertion of something further—this is the **Predicate.**

Logic writes the subject first, *S is P*, but it is not always so in common speech. 'Blessed is the man who has loyal friends' is an example of a very common inversion. When only a single sentence is before us it is easy to decide which is the predicate by asking which term expands and amplifies the meaning. But in continuous speech this often involves a reference to the context.

Nor does the logical distinction necessarily coincide with the grammatical. The latter is always the noun or pronoun in the nominative case. But the former depends upon what is assumed to be known or granted, and what is the amplification made. Often it is best detected by asking what question the whole sentence may be regarded as answering. The answer is the predicate. Thus, "This is an orchid" may be an answer to the question "What is this?" when 'orchid' is the predicate; or it may answer the question "Which of these flowers is an orchid?" when 'this' and the actual pointing out which 'this' implies is the predicate. Thus while the grammatical distinction can be determined without going beyond the individual sentence concerned, the logical distinction in the consecutive sentences which compose real speech cannot. Doubtless the two often coincide even in such con

tinuous speech, and when an isolated statement is all that is before us, we have no alternative but to assume that the grammatical is also the logical analysis, that is, that the distinctions of thought are directly expressed in speech. So in the case supposed, if we have *only* the sentence, 'This is an orchid,' we must assume that 'orchid' is the predicate, as that term, on the face of it, defines and amplifies the meaning of 'this.'

The **copula** merely marks the act of assertion. Doubtless 'is' sometimes implies existence, as when we say "Much evil is in the world." But then 'is' is not the copula: logic must translate the statement into "Much evil is existent in the world."

The copula is always in the present tense. Any assertion of past or future requires a separate predication. In the logical proposition we, as it were, put ourselves at the point of time to which it refers. This is necessary, as the present is the only unambiguous tense. Propositions relating to the past or the future—as *S was P*, *S will be P*, cannot be conjoined in a formal argument, because there would be no assurance that they had any real point of identity. From *S was M*, and *M was P*, we can infer nothing, for we do not know whether the connexion of *S* with *M* synchronised with the union of *M* and *P*.

Moreover the use of the present tense enables us to express the coexistence of attributes in the same subject. We recognise that gold is *at once* yellow, and heavy, and malleable, though we can only test those qualities successively.

Though we can thus analyse a proposition we must not forget that all the time the proposition is a unity and expresses one single thought. Subject and predicate do not exist as separate thoughts, but as distinguished elements in one thought. This unity of the judgment the copula also expresses.

(iii) **Quality of Categorical Propositions.**—Every statement either affirms or denies the predicate of the subject. So we have the ultimate distinction between **Affirmative** and **Negative** Propositions. In the facts there is no mere negation. The denial that *S* is *P* can

only be justified by the fact that S has some attribute which is incompatible with P. If gold is not blue it is because it has some other colour. Were blue denied of that which cannot have colour at all there would be no real assertion. That 'vice is green and virtue red' is an unmeaning jumble of words masquerading as a proposition. But though reality is thus always positive, yet thought may only be concerned in denying some statement about it, without specifying the reason for the denial. Hence the negative form of proposition, which simply denies, is correctly recognised by logic as equally ultimate with the affirmative form.

(iv) **Quantity of Categorical Propositions.**—Though the assertions of ordinary life are of many degrees of definiteness, the only really formal distinction is between explicit reference to *all* the subject and indefinite reference to some unspecified part of it.

The former propositions are called **Universal,** the latter **Particular.** Symbolically the former are expressed affirmatively by *Every S is P, Each S is P, All S's are P*; or negatively by *No S is P*: the latter by *Some S's are P*; *Some S's are not P.*

It will be noted that 'Particular' here has a different meaning from that given to it in common speech. There a 'particular' proposition would naturally mean a statement about a particular person or thing, as 'This boy is clever'; 'Mr. Asquith is the premier.' These in logic are called *Singular* Propositions. Their chief function is to bring a special case under a general rule, as if we say 'All Secretaries of State receive high salaries; Mr. Churchill is a Secretary of State; consequently Mr. Churchill receives a high salary.' From the point of view of logical quantity they are regarded as akin to universals, as they evidently assert definitely that their predicates are true of the whole denotation of their subjects.

Similar to these are propositions made about a collective subject as a whole—as 'The Romans conquered Gaul.' In a true universal the predication is made of every individual which bears the class-name which stands for the subject. It is, therefore, better to write *Every S is P* or *Each S is P.*

than *All S is P*. For 'all' is ambiguous in this respect, and propositions true of *All S* taken collectively are not true of each *S* taken distributively. It is well to mark the distinction between the distributive and the collective use of terms with which we have already dealt by using '*All S is P*' only when *P* is asserted of the whole group *S* taken together.

When we wish to express a universal negation the only unambiguous form is *No S is P*. To write *Every S is not P* or *All S's are not P* is ambiguous. Strictly interpreted, indeed, these propositions are justified if a single exception can be found to the rule that *S's* are *P*. 'Every statement in a newspaper is not true' does not imply that the whole contents of the press are to be rejected. If we wish to assert that we must say 'No statement in a newspaper is true.'

In form the universal proposition suggests that it is justified by a complete examination of all the instances. This, however, is seldom the case. The real basis of the judgment is such a knowledge of *S* and *P* as makes it evident that wherever *S* is found, *P* will be present; or on the other hand that *P* is incompatible with *S*.

This would be expressed by saying *S as such is P*; or *S as such is not P*, and such judgments are called *Generic*. For example, 'Right-angled triangles are by their nature inscribable in semi-circles.' For the purposes of formal logic, however, these judgments must be translated into the ordinary universal forms. If we simply say *S is P* we do not make it clear whether we affirm *P* of every *S* or only of an indefinite portion of the *S's*.

It is this indefiniteness which marks the **Particular** Propositions, with *Some S's* as their subject. Of course no sensible person says 'Some *S*' when he knows 'Every *S*' would be true. So in common speech 'Some' is often meant to imply 'Some, but not all.' Similarly it is not used when we *know* the proposition to be true of only *one*. But even in common speech these limitations are not always implied, and in logic they are always excluded. *Some S's are P* expresses imperfect knowledge. Certainly in fact each *S* is either *P* or is not *P*. But our knowledge

does not always cover all the facts, and so we cannot say which is which. We cannot, however, rest satisfied with this imperfect knowledge in any matter about which we desire exact thought. So our researches must be pushed further. Either we must find that in reality the *P* which we already know to belong to some of the *S's* really belongs to them all; or we must make among the *S's* two classes—one which is *P* and one which is not *P*—and discover what other differences separate them. Then to each of these we give a special name and so reach two universals—*Every SM is P, No SN is P.*

When no sign of quantity is given the proposition is formally *indesignate*, as "Birds are feathered." Sometimes extraneous knowledge tells us that this is a generic judgment and therefore really universal. Wanting this, it must be interpreted as particular.

If an Indesignate Proposition is negative, the predicate must evidently be either a separable accidens of the subject or an attribute never found in that subject. In the former case, the propositions are particular; as 'Englishmen are not cowardly.' In the latter case they are, of course, universal, as 'Englishmen are not negroes.'

Logic reduces to 'some' all the signs of quantity of ordinary speech, such as 'any,' 'a few,' 'few,' 'most,' 'two-thirds.' They are all indefinite in reference. There is this peculiarity about 'few,' that it really states by implication a judgment of opposite quality to itself. 'Few poets are greater than Shakespeare' is really a rhetorical mode of saying that 'Most poets are not greater than Shakespeare,' and is, therefore, best represented by the particular negative form—*Some S's are not P.* This, however, is again only a case of translating the language of common life into the form demanded by formal logic.

(v) **The Four-fold Scheme of Propositions.**—If we combine the divisions under Quality with those under Quantity we get a four-fold Scheme of Categorical Propositions, viz. Universal Affirmative, Particular Affirmative, Universal Negative, Particular Negative. These it is customary to indicate by the letters **A, I, E, O,** respectively, those letters being the first two vowels of the Latin verb

affirmo (I affirm), which represent the Universal Affirmative (**A**) and the Particular Affirmative (**I**); and the vowels of the Latin verb *nego* (I deny) which stand for the Universal Negative (**E**) and the Particular Negative (**O**). By writing these letters between S and P we obtain a brief symbolic mode of expressing propositions. Thus—

A	-	*Every S is P*	-	$S\ a\ P$.
I	-	*Some S's are P*	-	$S\ i\ P$.
E	-	*No S is P*	-	$S\ e\ P$.
O	-	*Some S's are not P*	-	$S\ o\ P$.

(vi) **Distribution of Terms.** In every *affirmative* proposition, whether universal or particular, we assert that a certain subject possesses an attribute P, but we make no assertion as to the full extent of the denotation of P. We do not consider whether or not other objects exist of which P can also be predicated. In some cases there are such objects—as when we say 'All lions are fierce,' for there are certainly other fierce animals; in other cases there are not—as when we say 'All diamonds are pure crystallised carbon.' But in no case is any explicit reference made to the full denotation of P. In every affirmative proposition, therefore, the predicate is left indefinite as to its denotation, and is, therefore, *undistributed.*

In a *negative* proposition, on the other hand, when we look at the predicate in denotation we find that, in every case, it is *distributed.* For if the subject is not definitely separated from the whole extent of P, it may at least partially agree with it, and then there is no negation. And this is independent of the extent of the subject. If we deny P of only one individual—as when we say 'This S is not P' or 'An S is not P'—yet we must deny that *any and every* P is the S in question, or we have evidently denied nothing at all.

If we now sum up our results as to the distribution of each of the terms in each kind of categorical proposition when read in denotation, we have—

(1) *Universals* (**A** and **E**) *distribute their subjects; Particulars* (**I** and **O**) *do not.*

(2) *Negatives* (**E** and **O**) *distribute their predicates; Affirmatives* (**A** and **I**) *do not.*

Thus, **E** distributes both subject and predicate.
A distributes its subject only.
O distributes its predicate only.
I distributes neither term.

4. Hypothetical Propositions.

(i) **Nature. A Hypothetical Proposition is one in which the predication made in one proposition is asserted as a consequence from that expressed by another.** The proposition containing the condition is called the **Antecedent** or **Protasis,** and is introduced by some such word as *If*; that containing the result is termed the **Consequent** or **Apodosis.** For example, in the sentence 'If all prophets spoke the truth, some would be believed,' the antecedent is 'If all prophets spoke the truth,' and the consequent is 'some (prophets) would be believed.'

The most general symbolic expression of the hypothetical proposition is *If* **A** *then* **C**, where **A** and **C** represent not terms but propositions. Other forms frequently given may be included under this general expression. Of these one of the most common is *If A is B, C is D*, but the one which most truly represents the nature of the judgment expressed by the proposition is *If S is M it is P*, where both the protasis and the apodosis have the same subject. This form indicates that the essence of the judgment is the explicit assertion that the ground of the attribution of *P* to *S* is found in the fact that *S* is *M*.

When the proposition contains four terms, and, therefore, falls at once under the form *If A is B, C is D*, analysis of the meaning frequently shows that this is a mere accident of expression, and that the real subject of thought is the same in both antecedent and consequent, so that the judgment may be equally well expressed by a proposition of the form *If S is M it is P*. Such reduction from one form to another is always allowable when it does not affect the meaning of the proposition, *i.e.* the real judgment. For

example, the judgment involved in the proposition 'If the government of a country is good, the people are happy' finds perfect expression in 'If the people of a country are well governed, they are happy.' So, 'If the barometer falls, we shall have rain' is reducible to 'If the state of the atmosphere causes a fall of the barometer, that atmospheric state will bring rain.' 'If we ascend a mountain, the barometer falls' is equivalent to 'If a barometer is taken up a mountain, it falls.' 'If a child is spoilt, its parents suffer' may be resolved into 'If a child is spoilt, it brings suffering on its parents.' 'If you take a large dose of arsenic you will die' is expressed by 'If arsenic in undue quantity is taken into an animal organism, it causes death in that organism.' 'If patience is a virtue, some virtue may be painful' is the same as 'If virtue includes patience, then virtue may be painful.'

In other cases the translation is not so easy; and links have to be supplied which are not explicitly stated in the original proposition. In other words, a hypothetical expressed with four terms conceals the essential unity of the judgment it expresses, as there is in the symbolic statement no obvious point of union between the antecedent and the consequent. But the union is always there in thought when the proposition is expressed—as all real judgments always are—in significant words and not in mere empty symbols, and is generally found without difficulty. For example the point of unity involved in the judgment 'If some agreement is not speedily arrived at between employers and workmen, the trade of the country will be ruined' is the *recognition of the injurious effect of* strikes on trade, and the whole judgment may be expressed 'If trade continue to be injured by strikes, it will soon be ruined.' Sometimes the union is found in the recognition that the subject of the apodosis is a species under the wider subject of the protasis, as in 'If demagogues are mischievous, this stump-orator is mischievous,' 'If violent emotion is followed by a reaction, your fit of anger will lead to a reaction'; 'If all savages are cruel, the Patagonians are cruel.' In other cases, both are recognised as species under the same genus, as in 'If virtue is voluntary, vice is volun-

tary.' But, in every case, where the judgment is really hypothetical—that is, asserts the consequences of a supposition—such unity is present. No doubt, the hypothetical form of proposition is occasionally used when no such judgment is really involved, as when Mr. Grimwig in *Oliver Twist* said, "If ever that boy returns to this house, sir, I'll eat my head"; which was only a forcible mode of asserting disbelief in the realisation of the supposition stated in the protasis; and was, therefore, in its essence, categorical. Such propositions are of but small value in a theory of knowledge.

In discussing the universal categorical proposition we found that its justification must be sought in a relation of content which is most appropriately expressed in the Generic Judgment, *S is P*. If such a judgment is true, it is because there is something in the nature of *S* of which *P* is the necessary consequence. If we make this explicit we have the hypothetical judgment *If S is M it is P*, where the sufficient ground for the predication of *P* is found in the presence of *M*. Such a relation is nearly as explicitly stated in a Generic Judgment of the form *S which is M is P*, a fact which shows that the categorical and hypothetical forms are not separate and distinct species of judgment, but merge into each other, and are distinguished chiefly by the more abstract character of the latter. In the hypothetical judgment we have got away from the concrete; our proposition is an abstract universal, and deals with only one element in a complex whole. The judgment, if true, is necessarily and universally true, and yet may be incapable of concrete realisation. This, indeed, is so with geometrical judgments, such as 'If a triangle is right-angled, it is inscribable in a semi-circle,' for no concrete diagram is ever a perfect right-angled triangle or a perfect semi-circle. Still more clearly, perhaps, is this seen in such a judgment as 'If a body is given a certain movement, and if no counteracting conditions are operative, it will continue for ever to move in the same direction and with the same velocity.' This is impossible of realisation in actual fact, and yet it is a fundamental law of physics; that is, a necessary element in our mental construction of the material world.

The hypothetical judgment is essentially abstract, and, as such, states connexion of content. But as the generic judgment finds an enumerative or denotative expression in the universal categorical proposition, so many hypothetical judgments can be represented by what may, perhaps, be called concrete conditional propositions, whose general symbolic expression is *If any S is M that S is P*, or *Whenever an S is M that S is P.* The denotative form has a reference to occurrence in time and space; it expresses connexion of phenomena, and is, therefore, only appropriate when such occurrence is possible. In other words, it contains a distinctly categorical element, and is practically equivalent to the proposition *Every S which is M is P.* Its form, like the proposition *Every S is P*, suggests that it is based on enumeration of instances, but its real justification is connexion of content expressed by the pure abstract hypothetical.

(ii) **Relation to Categorical Propositions.**—We have seen that as judgment becomes less occupied with concrete and complex phenomena regarded as wholes, and concerns itself more and more with abstract relations of content, it tends to pass from the categorical to the hypothetical form. But the fact that the generic judgment mediates this transition, while the denotative conditional form mediates a transition in the opposite direction, shows that no strict line of demarcation can be drawn between them as modes of thought. With their form as propositions, the case is, of course, different; here language makes fixed and definite a distinction which is far from being so fixed in thought. Sometimes, it is an accident whether a judgment is expressed in the hypothetical or the categorical form; for instance 'Right-angled triangles have the square on the hypotenuse equal to the sum of the squares on the sides' really gives the ground for attributing the predicate to the subject, and would appropriately take the hypothetical form 'If a triangle is right-angled, the square on the hypotenuse is equal to the sum of the squares on the sides.' But, in all cases, it should be considered whether the categorical or the hypothetical form is the more appropriate, and this

depends upon the degree of abstraction involved in the judgment.

(iii) Quality and Quantity.

(*a*) **Quality.**—Hypothetical Propositions admit of distinctions of quality. Of course, a negative antecedent does not make a hypothetical proposition negative; for the consequent is still asserted to follow as a result of the antecedent. Thus *If S is not M it is P* is affirmative—'If a swan is not white, it is black.' It is when the connexion of the apodosis with the protasis is denied that the proposition is negative. The most general symbolic form is *If* **A** *then not* **C**, and the most expressive *If S is M it is not P*, for example, 'If a man is honest he will not deceive his fellows.'

(*b*) **Quantity.**—The essence of true hypothetical judgments is their abstract, and necessarily universal character. But cases may arise in which though a connexion is established between *P* and *M*, yet *M* may not be the full ground of *P*, or, though it is the complete ground, may not be universally operative, or may be liable to be counteracted by other conditions. In such cases the appropriate proposition takes the general form *If S is M it may be P*, and negatively *If S is M it need not be P*. The corresponding denotative forms—which in these cases can always be found—are *Sometimes when an S is M, it is* [or *is not*] *P*. As examples we may take 'Sometimes when men are much worried, they commit suicide'; 'If a man is punished for a crime, perhaps he will not transgress again'; 'Sometimes when a target is aimed at, it is not hit'; 'Although a man tries his hardest, he may not succeed.' The last three examples are particular negative, the first particular affirmative. The 'Sometimes' in the denotative examples, it must be remembered, is as purely indefinite as is 'Some' in a particular categorical; it must not, therefore, be regarded as excluding the possibility in fact of 'always.'

The great characteristic of all particular propositions is their imperfect and incomplete character. They, on their very face, therefore, challenge completion. They are but

stepping-stones on the way to that more exact and complete knowledge which finds expression in the true and universal hypothetical.

5. Disjunctive Propositions.

(i) **Nature.—A Disjunctive Proposition is one which makes an alternative predication.**—The most general symbolic form is *Either* **X** *or* **Y**, where **X** and **Y** represent propositions. But as the most expressive form of the hypothetical is that which makes explicit the unity of the judgment, so here the symbolic form *S is either P or Q* representing the prescription to the same subject of an alternative between a definite number of predicates, most truly expresses the nature of the judgment. In the simplest cases these alternative predicates are known to be contained under a wider predicate *M* which can be asserted of *S*. For example 'He is either a doctor, a lawyer, a clergyman, or a teacher' may be expressed in the simple categorical proposition 'He is a member of a learned profession.' So, we may say 'Any swan is white or black,' where the wider predicate is the possession of colour.

Such subsumption is always theoretically possible, and so we see that the function in a theory of knowledge of the disjunctive judgment is to express the arrangement and content of a system. In other words the subject is a genus, and the alternative predicates are the species which together compose that genus. In such a case it is evident that those predicates are mutually exclusive and collectively exhaustive of the denotation of the subject. The disjunctive judgment is the form appropriate to the statement of a complete and perfect classification of the denotation of its subject.

Our knowledge of the systems of the real world is, however, generally imperfect. It is but seldom that we can in concrete matters set out the whole body of species which necessarily complete the genus. We can say that 'Graduation at the University of London is in either arts or science or law or medicine or music' or any other faculties which we can find recorded in the Calendar. But we should be

hard put to it to give an equally complete and satisfactory statement of many of the alternatives which yet find a place in our knowledge. Moreover we often use the disjunctive form loosely, especially in reference to concrete cases. We may read that all the candidates for a certain post must be graduates of Oxford, Cambridge, London, or some other British University. Evidently a graduate of two or more of these universities would not be thereby excluded. But the system on which the proposition is based is really that the Universities in the British Empire are Oxford, Cambridge, London, etc., and that graduation must have taken place at one of these. Then we see that the act of graduation at one is not the act of graduation at another. It is these acts which are the real alternatives, and if Jones has performed two of them, he performed them independently and at different times. He is, so to say, a separable accidens of the acts.

Such an example—and many others could be given—as *e.g.* 'Brown is either drunk or mad'; 'Smith is either a liar or a forger'; 'Robinson is either a knave or a fool'—shows us that we cannot assume mutual exclusion among the alternative predicates from the mere empty form of the proposition. It is true that the logical ideal of the disjunctive judgment is such exclusiveness and comprehensiveness as set forth the articulation of a system. But we have to deal with propositions couched in the language of common life, and often they not only represent very imperfect knowledge, but express with considerable looseness the knowledge actually possessed. So to the question —much disputed among logicians—as to whether alternative predicates are mutually exclusive we must reply that ideally they should be, but that actually in many cases they are not, and so we cannot assume them to be so in any. In other words, taking the use of language as it is, alternative predicates do not formally exclude each other.

(ii) **Relation to Hypothetical Propositions.**—As every disjunctive proposition prescribes an alternative between a definite number of different predicates, one or more of which is, therefore, affirmed of the subject, it follows that, if some of these alternatives are denied, the others

are affirmed; either categorically—if only one is left, or disjunctively—if more than one remains. Thus, if we start with the assertion *S is either P or Q*, and then deny that *S is P*, we must necessarily proceed to affirm that *S is Q*. Similarly, if the original assertion is *S is P or Q or R or T*, and this is followed by the denial that *S is either P or Q*, the affirmation that *S is either R or T* is a necessary result. As this affirmation of one, or more, of the alternatives is an inference from the denial of the rest of them, it follows that all disjunctive propositions involve a hypothetical judgment of the general form *If S is not P it is Q*, or, *If S is not Q it is P*. These propositions are exactly equivalent to each other, each being, in fact, derivable from the other.

But the disjunctive judgment makes explicit a categorical element which is wanting to the hypothetical. Were we confined to the latter, thought would be condemned to an endless regress. For though *If S is M it is P*, gives us in *M* the ground of *P*, yet we must go on similarly to ask for the ground of *M*. This regress can only be avoided by assuming that the judgment refers to a more or less self-contained system. It is such a system that the disjunctive judgment in its ideal form makes explicit in its enumeration of the sub-species under the subject genus. It is in the exhaustive character of this enumeration that the sufficiency of the hypothetical as a statement of a condition is found. Hence, we find in the disjunctive the mode of expressing that systematic connexion which is the only form in which we can think reality.

(iii) **Quality and Quantity.**

(*a*) **Quality.**—It follows from the very nature of disjunctive propositions that they can only be affirmative; for they must give a choice of predicates, one or other of which must be affirmed of the subject. Propositions of the form *S is neither P nor Q* give no such choice, nor do they increase the scope of the predicate as do propositions of the form *S is either P or Q*. They are essentially Compound Categorical Propositions. It is true we can have a disjunctive proposition involving negative terms—as *S is*

either P or non-Q—but the disjunction is as affirmative as if both terms were positive.

(*b*) **Quantity.**—The ideal disjunctive judgment is always both abstract and universal, and expresses relation of content. But like the Generic Judgment it can be expressed in terms of denotation, and in this case we get distinctions of quantity. Thus we get propositions of the form *Every S is either P or Q*; 'Every idle man is either incapable of work or morally blameworthy,' and *Some S's are either P or Q*; 'Some laws are either oppressive or are rendered necessary by an abnormal state of society.' It is evident, however, that such particular propositions are of practically no logical value.

CHAPTER IX.

IMPORT OF CATEGORICAL PROPOSITIONS. DIAGRAMS.

1. **Predication.**—The whole treatment of logic must depend upon the view held as to the nature of the predication made in a categorical proposition, and the consequent import of that proposition. The first point to be settled in considering this is whether such a proposition expresses a relation between words only, or between ideas, or between things. The view adopted in this work is that a proposition interprets an objective fact, by stating a relation which is apprehended in thought and expressed by language. It is not necessary to say more on this fundamental point. We may pass on to consider the nature of the relation expressed in predication, and, as a consequence of this, the aspect in which the terms should be regarded. Of this there are two chief interpretations according to whether the emphasis is placed on the connotation or on the denotation of the predicate.

2. **The Predicative View.**—The predicative view regards the relation expressed between the terms of a formal categorical proposition as that between subject and attribute. It makes the element of denotation in the subject, and that of connotation in the predicate, the more prominent.

The subject is thought as the name of certain objects, and though it is true they are indicated indirectly—that is, as members of a class to which they belong solely because they possess certain attributes—yet the attention is fixed on them as things, not on the attributes which

their names connote. It is of the *things* which possess the attributes that the assertion is made; the attributes themselves are not directly and explicitly before the mind, but are symbolised by the name.

But in the case of the predicate we are thinking of the attributes which we affirm of the objects denoted by the subject; for our whole purpose is to predicate such a qualification. We think, not of two sets of objects which we compare, but of one set of which we assert an attribute. This is most obvious when the predicate is a directly attributive word, as when we say 'All metals are fusible,' 'The dog is barking'; but it is equally true when the predicate is a substantive. For instance, in the proposition 'All the candidates for the appointment are graduates,' if we examine the meaning we shall find it to be that the candidates in question possess certain qualifications which are conveniently summed up in the word 'graduates'; we do not think of graduates as individuals, but predicate the connotation of the name. And the same holds in every case; the predicate asserts a qualification of the subject, and this qualification consists of the attributes implied by the predicate.

This is the natural interpretation of a categorical proposition whose subject is expressed with a sign of quantity, though it must be borne in mind that its foundation is to be found in the Generic Judgment whose essence is that it deals with content of both subject and predicate. It is also fully consistent with the four-fold scheme of propositions; for, if the predicate names an attribute and the subject indicates certain objects, we must either affirm or deny the former of the latter, and in each case the assertion must be made either of a definite or of an indefinite number of individuals.

3. **The Class-inclusion View.**—On the class view the relation between the subject and predicate is that of inclusion in a class. Both terms are said to be read in denotation, and the proposition is held to assert that the objects denoted by the subject are to be found among those denoted by the predicate. Whether the subject is

used collectively or distributively is of no importance; in each case it forms part of the predicate. The predicate, however, is necessarily regarded as a whole or class—that is, it is used collectively. 'All owls are birds' means that each owl—and, therefore, the whole class of owls regarded collectively—is to be found within the whole class of birds. This collective interpretation of the predicate is the only permissible one; for to take it distributively would give no real meaning at all;—each owl is certainly not *any* bird. The only possible meaning is that the total class composed of birds contains every individual which can be called an owl; or, what is exactly the same thing, that it contains the whole class of owls. Similarly, a negative proposition means that every individual denoted by the subject is excluded from the whole class of things denoted by the predicate, and that the two classes are, therefore, entirely separate.

With respect to this view it may be pointed out, first, that though it is, of course, possible to attend to the denotation of the predicate, yet in forming a judgment it is more natural and common not to do so. No doubt, as every general term can be considered both in denotation and connotation, it is possible so to interpret propositions, and such a mode of interpretation lends itself readily to the exposition of formal reasoning and to its illustration by diagrams.

It is, indeed, the only one practicable for the more complex developments of formal reasonings with more than three terms. It expresses a truth implicit in every judgment, though it does not represent the primary purpose with which judgments are made. Indeed *to adopt* this interpretation as the fundamental import of judgment would be to fall into the error of basing knowledge upon a supposititious possibility of a complete enumeration of instances, instead of upon an investigation directed to establish connexion of content. Moreover, this view of predication obscures the essential unity of the judgment and regards it as stating a relation between two independent objects rather than as expressing an interpretation of one element or aspect of reality.

Further, it must be pointed out that if both subject and predicate are regarded as classes—and, as was said above, on this view, the subject *may*, and the predicate *must*, be always so regarded—then the four-fold scheme of propositions is not an exhaustive statement of the relations which may exist between them. We require a five-fold scheme; for if we have two classes, *S* and *P*, it is evident—

(*a*) They may exactly coincide and so be identical wholes.

(*b*) *S* may be included in but not form the whole of *P*.

(*c*) *S* may include *P* and not be wholly exhausted.

(*d*) *S* and *P* may partially include and partially exclude each other.

(*e*) *S* and *P* may wholly exclude each other.

To express these in ordinary language we must give some' the meaning 'some but not all.' We then have—

(*a*) All *S* is all *P*.

(*b*) All *S* is some *P*.

(*c*) Some *S* is all *P*.

(*d*) Some *S* is some *P*.

(*e*) No *S* is any *P*.

But it should be remembered that we have here a statement of the actual relations which must hold, in fact, between two classes, not of our knowledge of those relations. To express that knowledge it may frequently be necessary to give two or more of these propositions as alternatives. This will be seen more clearly when we consider the representation of the four-fold scheme of propositions by diagrams which primarily indicate these relations of classes.

4. **Nature and Use of Diagrams.**—Diagrams are intended to make obvious at a glance the relations between the terms expressed in a proposition. That any scheme may do this satisfactorily, it is essential that—

(1) The diagrams employed should be self-interpreting immediately the principle on which they are constructed is understood.

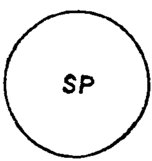
(2) Each diagram should be capable of one, and only one, interpretation; and, conversely,

(3) Each proposition should be representable by one, and only one, diagram.

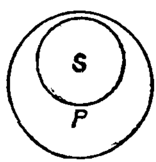
The value of every scheme of diagrams must, therefore, be estimated by the perfection with which it fulfils these requirements.

Diagrams in logic often help a beginner to grasp the exact scope of a proposition, and to see more readily the immediate inferences which can be drawn from it.

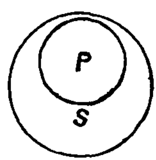
5. **Euler's Circles.**—The best known and most commonly used scheme of diagrams is that of Euler, a distinguished Swiss mathematician and logician of the eighteenth century. It is based on the actual relations between two classes, each of which is represented by a circle. This necessitates the following five diagrams to express all the possible relations—



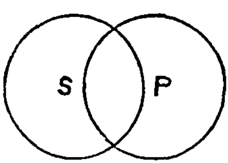
I.



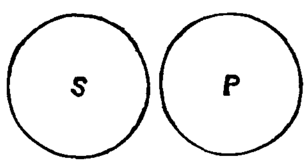
II.



III.



IV.



V.

This scheme admirably satisfies the first criterion of excellence given in the last section—there can be no doubt

as to the information given by each of the above diagrams. But, as it is founded, not on the predicative view of propositions, but on an analysis of the possible relations which may subsist between classes, it is not surprising that the diagrams do not satisfactorily represent the four-fold scheme of propositions. They correspond, in fact, to the five elementary forms of proposition which are necessary to express all possible actual class relations. Thus—

I. represents that *S* and *P* are coincident—*All S is all P*.
II. that *S* is included in, but does not form the whole of *P*—*All S is some (only) P*.
III. that *S* includes *P*, but is not wholly exhausted—*Some (only) S is all P*.
IV. that *S* and *P* partially include and partially exclude each other—*Some (only) S is some (only) P*.
V. that *S* and *P* are mutually exclusive—*No S is any P*.

If, however, we try to fit in this scheme of diagrams with the ordinary four-fold analysis of simple propositional forms, we find that only in the case of **E** have we an adequate expression in any one diagram. Bearing in mind the absolute logical indefiniteness of 'Some,' it is plain that every other form of proposition can be fully represented only by a combination of diagrams. Thus, for **A** we require I and II; for **I** we need I, II, III, IV; and for **O** we must have III, IV, V. If, on the other hand, we are given either of the Figures I, II, III, or IV, we cannot say with certainty what proposition it is meant to represent. The scheme, then, cannot represent simply the ordinary forms of proposition; and when propositions are united into syllogisms, it becomes so complex as to be practically unworkable. Thus, when applied to represent **A**, **E**, **I**, **O** propositions, the scheme does not satisfy either of the two last criteria of excellence given in the last section. To attempt to escape this complexity by representing **A** by II alone and both **I** and **O** by IV alone—as is often done—is misleading, insufficient, and inaccurate.

But even were it not open to these objections, we should still have an ambiguous diagram; for IV would represent indifferently **I** and **O**. To attempt to avoid this difficulty, as Euler apparently did, by writing the *S* in the part of the *S*-circle which is excluded from the *P*-circle (as is done above) when the proposition intended is **O**, and in the part of the diagram common to both circles when it is **I**, is not satisfactory; for this assumes that we already know what proposition is intended. The diagram itself still remains ambiguous; and, if it is given in the empty and unlettered form, we do not know what predication is intended.

Nevertheless, this very want of coincidence between the diagrams and the scheme of propositions will perform a useful service if it makes more emphatic the imperfections of the latter as a mode of interpreting reality. The diagrams express facts of existence. If, then, we need more than one diagram to represent any judgment we may make about actual things, that very need is a warning to us that our knowledge of the matter is wanting in precision and at the same time an indication as to where the vagueness lies.

CHAPTER X.

FALLACIES INCIDENT TO JUDGMENT.

1. Judgment involving Self-contradiction.—The fallacies incident to judgment are not essentially different from those incident to conception, as, indeed, might be anticipated from the fundamental identity of mental process underlying the two forms of thought. In a clearly apprehended judgment there can, of course, be no incompatibility between predicate and subject. But when the force of the judgment is not clearly apprehended such incompatibility is by no means impossible. The careless use of language, which is so common nowadays, makes this particularly easy. Especially dangerous is the habit of making universal assertions when there is only justification in fact for particular ones. We thus have such self-destructive assertions as that 'Every rule has an exception.' This statement, being itself a rule, contradicts itself. Thus we may put it syllogistically: 'Every rule has exceptions, This statement is a rule, Therefore, this statement has exceptions, that is, Some rules have no exceptions'; when the contradiction implicit in the major premise is made explicit. Similar considerations solve the old logical puzzle involved in 'Epimenides the Cretan says that all Cretans are always liars; therefore Epimenides can only speak the truth if he lies, and lies if he speaks the truth.' Here also the major is self-contradictory, for every judgment claims to be true, and this proposition at the same time asserts its own falsity. Quite similar is the scientific conundrum as to what would happen if an irresistible force were to impinge on an immovable body, where two incompatible conditions are assumed to be simultaneously operative.

2. **Misinterpretation of Categorical Propositions.**—Even when the meaning of each individual word in a sentence is clearly apprehended, there is still a possibility of misunderstanding the statement as a whole, owing to some ambiguity of construction or of interpretation. This, of course, results from the dependence of terms upon the context for their full meaning. The two chief cases were noticed by Aristotle.

(i) **Amphibolia.**—The *Fallacia Amphiboliae* originates in ambiguity due to the construction of a sentence. It is in essence a misinterpretation of a proposition. It does not, therefore, differ fundamentally from fallacies of ambiguous terms, for an ambiguous term employed in a proposition necessarily leads to that proposition also being ambiguous. However, in amphiboly the ambiguity lies in the general structure of the proposition rather than in the terms it contains. Latin with its construction of accusative with infinitive in indirect narration lent itself very readily to this form of ambiguity. Thus, the oracle given to Pyrrhus: "Aio te, Aeacida, Romanos vincere posse"—"Pyrrhus the Romans shall, I say, subdue"—left it entirely in doubt on which side victory was to lie. This is very similar, as Shakespeare makes the Duke of York point out, to the witch's prophecy in *Henry VI.*—"The Duke yet lives that Henry shall depose."[1] Open in the same way to double interpretation is the line in W. R. Spencer's poem *Gelert*: "The noble hound the wolf hath slain." The walls of Windsor Castle still bear the ambiguous sentence "Hoc fecit Wykeham," whose capability of a double reading is said to have averted the King's displeasure from the bishop, who, like a true courtier, explained it as meaning that the tower made him.

One has not far to seek to find numerous instances of amphiboly in modern English, generally due to want of precision in the order in which the words are arranged, a point of great importance in an analytic language. A few examples may be given: 'The first photograph is that of a fourteen pound pike taken in a back yard from the top of

[1] *Second Part,* Act i., sc. 4.

a step-ladder'; 'A lady (through circumstances) wishes to let part of her well-furnished house'; 'The Territorial band played the hymns as well as the church organ'; 'A gentleman shortly returning to Australia wishes to meet a lady and marry her before doing so'; 'The birds were somewhat wild, but all thoroughly enjoyed the sport'; 'Long leave will be granted to Parents or Guardians of all boys who apply for it'; 'It would seem possible that almost any woman, no matter what the extent or depth of her wrinkles, might have been removed entirely and for ever by means of this lucky discovery.'[1]

A subtle form of amphiboly, and one not uncommon in controversial writings is instanced by De Morgan: "Equivocation may be used in the form of a proposition; as for instance, in throwing what ought to be an affirmative into the form of a qualified negative, with the view of making the negative form produce an impression. Thus a controversial writer will assert that his opponent has not attempted to touch a certain point, except by the absurd assertion, etc., etc., etc. To which the other party might justly reply: 'Your own words show that I have made the attempt, though your phrase has a tendency, perhaps intended, to make your reader think that there is none, or at least to blind him to the difference between *none* and *none that you approve of*.'"[2]

(ii) **Accentus.**—The *Fallacia Accentus* or *Prosodiae* was, with Aristotle, due to the fact that in Greek the same word, differently accentuated, had a different meaning. In the earlier times Greek writing was devoid of accents and breathings, and hence words when written were sometimes ambiguous which when spoken were not so. Aristotle consequently remarks that this ambiguity can scarcely occur in speech. But with us the fallacy may be committed by means of false stress either in speech or in writing. Thus, as De Morgan points out, the commandment, 'Thou shalt not bear false witness against thy

[1] The above are actual examples from the periodical literature of the day and are borrowed from *Mr. Punch*, who collects a few such gems every week.

[2] De Morgan, *ibid.*, p. 247.

neighbour,' may be read "either so as to convey the opposite of a prohibition, or to suggest that subornation is not forbidden, or that anything false except evidence is permitted, or that it may be given *for* him, or that it is only against *neighbours* that false witness may not be borne."[1] Other ways in which this fallacy may be fallen into are thus summed up by the same writer: "A statement of what was said with the suppression of such tone as was meant to accompany it, is the *fallacia accentus.* Gesture and manner often make the difference between irony and sarcasm, and ordinary assertion. A person who quotes another, omitting anything which serves to show the *animus* of the meaning; or one who without notice puts any word of the author he cites in italics, so as to alter its emphasis; or one who attempts to heighten his own assertions, so as to make them imply more than he would openly say, by italics, or notes of exclamation, or otherwise, is guilty of the *fallacia accentus.* . . . I may here observe that irony . . . is generally accompanied by the *fallacia accentus*; perhaps cannot be assumed without it. A writer disclaims attempting a certain task as above his powers, or doubts about deciding a proposition as beyond his knowledge. A self-sufficient opponent is very effective in assuring him that his diffidence is highly commendable, and fully justified by the circumstances."[2]

3. **Misinterpretation of Hypothetical Propositions.** —The characteristic fallacy of misinterpretation of a hypothetical proposition is to assume that the protasis states the cause of the apodosis, and not simply the reason for affirming it. As has been already pointed out, the latter may be a mere sign of the former or may actually be its effect.[3] Thus to misread a hypothetical proposition is, therefore, to conceive a relation as holding in reality which does not hold. Of course, the protasis may actually enunciate the cause, and in many cases does so; that, indeed, is the character of the ideal judgment of science. But it is also true that in many cases it is not so, and the identity of form in the two cases opens the way to error.

[1] *Ibid.*, p. 249. [2] *Ibid.*, pp. 249-50. [3] See p. 18.

4. **Misinterpretation of Disjunctive Propositions.**—Fallacy may arise in dealing with a disjunctive judgment from assuming that it fulfils the conditions of ideal perfection, that is, that the alternative predicates are an exhaustive enumeration of co-ordinate and mutually exclusive species falling under the subject-genus. Many disjunctive propositions are far from fulfilling these conditions. Particularly must we avoid interpreting the alternatives as formally exclusive of each other. To argue that because all individual progress implies either moral or intellectual advance, therefore, if one advances morally one cannot advance intellectually, would be an example of fallacy traceable to this source. The characteristic fault of vicious dilemmas is, on the other hand, the undue assumption that the alternatives given exhaust all possible cases. This is really at bottom a case of false opposition, and will be further considered in a later chapter.[1]

[1] See p. 156.

CHAPTER XI.

GENERAL REMARKS ON IMMEDIATE INFERENCES.

1. Nature of Immediate Inferences.

Inference, or Reasoning, is the deriving of one truth from others. By this is meant that the new judgment is accepted as true because, and in so far as, the validity of the judgments from which it is derived is accepted. Hence, every inference has a formal and necessary character, and this is not affected by the truth or falsity of the premises. The premises may be false and yet the inference may be formally valid, that is, valid in the sense of avoiding contradiction within itself. But in the wider sense of validity, in which the result of the inference must also be consistent with the whole system of knowledge, the truth of the premises is, of course, an essential element.[1] This aspect of inference will be dealt with when we consider the doctrine of Induction; we are now primarily concerned with an analysis of the formal aspects of the process.

Inference is not a mental process absolutely distinct in its character from judgment. The essence of the latter is the explanation of some element of reality by reference of it to some concept already familiar to the mind. In inference there is the same essential feature, but with this difference, that the reference is not made immediately, but indirectly through the medium of some previously accepted truth or truths. In inference, therefore, we pass beyond the judgment, or judgments, from which we start, and attain a new point of view; though, at the same time, the new judgment thus reached must be a necessary conse-

[1] Cf. pp. 9-6.

quence of the data from which we set out. Inference thus involves both a process and its result; and to each of these the name is sometimes given. But strictly speaking an inference is the whole mental construction, and sets forth the connexion between the judgment proved and the evidence which proves it. The judgments which express the data or evidence are called *Premises*; the judgment derived from them is termed the *Conclusion*.

Immediate Inference is the process by which the implications of a single judgment are unfolded. By its immediateness is not meant that no activity of thought is required to reach the new judgment, but simply that no datum is necessary besides the one given judgment. Such inferences may be styled *interpretative inferences*, as distinguished from mediate inferences obtained from a combination of judgments in which thought makes a substantial advance to a new truth.

It is often questioned whether immediate inferences are really inferences at all, as no new truth is reached by them. This is mainly a matter of words. Whether we call the processes inferences or find some other name for them, it is certain that they are not all obvious at first sight. In other words active thought is demanded before we can express explicitly all that is implicitly asserted in any single judgment, and until we can do so we are not in a position to see its exact relations to other judgments. This being so, there seems no strong objection to naming such explicative processes of thought 'Inferences.'

An examination of the forms of reasoning should begin with these Immediate Inferences; for we should know what is involved in a single judgment before we go on to enquire what results will follow from a union of several judgments.

2. **Kinds of Immediate Inferences.**—There are two main classes of Immediate Inferences—

(i) *The Opposition of Propositions*, when, from the given truth or falsity of one proposition we infer the truth or falsity of other propositions relating to the same matter —that is, having the same subject and predicate. In

other words, an examination of the opposition of propositions means a consideration of the relations as to truth or falsity which hold between the four forms of propositions, $S\ a\ P$, $S\ e\ P$, $S\ i\ P$, $S\ o\ P$, when S and P have the same signification in every proposition.

(ii) *Eductions*,[1] in which, from a given judgment regarded as true, we derive other judgments which are implied by it; or, in other words, when we look at the same truth from another point of view, and express the same matter in a different verbal form.

We shall consider these two kinds of Immediate Inferences in the next two chapters.

[1] This name is adopted from Miss Jones' *Elements of Logic*.

CHAPTER XII.

OPPOSITION OF PROPOSITIONS.

1. Opposition of Categorical Propositions.

By the Opposition of Propositions is meant the relation which holds between any two propositions which have identically the same subject and predicate. Opposed propositions thus differ in form, but refer to exactly the same matter; that is, to the same things, at the same time, and under the same circumstances. The logical doctrine of Opposition, therefore, sets forth what implications as to the truth or falsehood of each of the other forms of categorical propositions are involved in *positing* (*i.e.* affirming as true), or *sublating* (*i.e.* denying the truth of), any one proposition.

This is, evidently, an entirely technical and arbitrary use of the word 'opposition.' The natural meaning of the word would be that two opposed propositions could not both be true together; that is, that opposition could exist only between the pairs of incompatible propositions, **A** and **O**, **E** and **I**, **A** and **E**. In this sense the word was *originally used. It was, however, found convenient to* include under the same head the relations between propositions which are not incompatible, that is, those between **A** and **I**, **E** and **O**, **I** and **O**. 'Opposition' thus came to include the relation between *any* pair of propositions of different form referring to the same matter, whether that relation were one of incompatibility or of compatibility. When once this technical use of the word 'opposition' is clearly understood, it is unlikely to cause any confusion.

As we have universal and particular, affirmative and negative, propositions, the relations between them will all

be included under those subsisting between the following pairs—

(1) A universal and the particular of the same quality; **A** and **I**; **E** and **O**.

(2) A universal and the particular of opposite quality; **A** and **O**; **E** and **I**.

(3) A universal and the universal of opposite quality; **A** and **E**.

(4) A particular and the particular of opposite quality; **I** and **O**.

This gives us four kinds of opposition, to which the names (1) *Subalternation*, (2) *Contradiction*, (3) *Contrariety* and (4) *Sub-contrariety* are respectively given. We will now examine these in order.

(i) **Subalternation.**—*Subaltern Opposition exists between a universal and the particular of the same quality;* that is, between **A** and **I**, **E** and **O**. Thus, the propositions differ in quantity but not in quality. This is one of the technical kinds of opposition; for, not only are the two propositions in subaltern opposition not inconsistent with each other, but the truth of the universal necessitates that of the particular. This follows from the Principle of Identity; for, by that principle any assertion which is true of every member of a class must hold of any number of those individual members, since they must be identical with some of those included under the distributed term. The assertion, when made of an indefinite part, simply repeats an assertion which was contained in the universal proposition.

In such a pair of opposites, the universal proposition is called the *Subalternant* or *Subalternans*, and the particular the *Subalternate* or *Subaltern*. Inference from the former to the latter is styled *Consequentia* or *Conclusio ad subalternatam propositionem;* that from the latter to the former, *Conclusio ad subalternantem.*

Hence, the inference of the truth of **I** from that of **A**, and of the truth of **O** from that of **E**, are *ad subalternatam.* The assertion of 'All metals are fusible' involves that of

'Some metals are fusible'; and if we posit 'No horses are carnivorous' we equally posit 'Some horses are not carnivorous.' But, if $S\ a\ P$ is denied, then this denial holds equally if P belongs to some only of the S's, or to none of them. Hence, from the falsity of **A** we cannot say whether **I** is true or false. For example, if I deny that 'All metals are malleable' I do not thereby deny that 'Some metals are malleable.' Neither do I affirm the latter proposition (though it happens to be true in fact); for, if I did, then the denial of 'All horses are carnivorous' would involve the assertion of 'Some horses are carnivorous.' Similarly, from the denial of **E** we can neither affirm nor deny **O**. The sublating of the universal leaves us quite in the dark as to the truth or falsity of its sub-alternate.

If we now examine the inferences *ad subalternantem* (or from particular to universal), we find that the denial of the particular involves the denial of the universal. For what is not true even in some cases cannot be true in all. The denial of $S\ i\ P$ means 'There are no such things as some S's which are P,' and this, evidently, negates the assertion that *All S's are P*. Again, if **A** were true, **I** must be true by inference *ad subalternatam;* and hence, if the falsity of **I** did not involve that of **A**, it would follow that **I** could be both true and false at the same time; which is absurd. The same results hold with **E** and **O**. Thus, if we deny the truth of 'Some horses are carnivorous' we thereby deny that of 'All horses are carnivorous'; and if we assert the falsity of 'Some men are not mortal' we equally assert that of 'No men are mortal.' But, to posit the particular cannot justify us in positing the universal; for we can never justify an assertion about *Every S* by asserting that it holds good with regard to *Some S's*. For instance, though it may be true that 'Some men are red-haired,' it does not follow that all men possess that attribute; nor does the truth of 'Some men are not six feet high' imply that no men attain that height.

Hence, we reach this general result: The truth of the particular follows from that of the universal, but not *vice versâ;* and the falsity of the universal is an inference from that of the particular, but not *vice versâ.*

(ii) **Contradiction.**—*Propositions are contradictory to each other when they differ both in quality and in quantity.* Hence, there are two pairs of contradictories—**A** and **O**; **E** and **I**. By the Principle of Contradiction both the members of such a pair cannot be true together, and by the Principle of Excluded Middle both cannot be false. If 'All metals are fusible' is true, it cannot be true that 'Some metals are not fusible'; and, similarly, if 'No lions are herbivorous' is a true proposition, then it cannot be true that 'Some lions are herbivorous.' And, generally, if we make an assertion about every member of a class, the Principle of Contradiction forbids us to deny that assertion about any member of the same class. Therefore, one of the contradictories in each pair must be *false*. But, by the Principle of Excluded Middle, they cannot both be false. For, by that principle, any given attribute, *P*, must either belong, or not belong, to every individual *S*. It cannot, therefore, be false both to make an assertion of Every *S* and to deny that same assertion of Some of those *S*'s. Such propositions as 'All metals are fusible' and 'Some metals are not fusible' cannot both be false together.

We see, then, that contradictories are incompatible with respect both to truth and to falsity. It follows that when two contradictory propositions are given us we infer, by the Principle of Contradiction, that one of them is false, and, by the Principle of Excluded Middle, that one of them is true. Hence, we can deduce the falsity of one from the truth of the other, and the truth of one from the falsity of the other. The relation of contradiction is thus seen to be reciprocal; the positing of one proposition and the sublating of its contradictory are assertions of one and the same fact. It will be seen, as we examine the other forms of Opposition, that in none of them are the propositions thus mutually inferrible, and in none of them is there incompatibility with regard to both truth and falsehood.

Contradiction is, therefore, the most perfect form of logical opposition.

Whatever we affirm denies something else. The mere

asserting of every *S* that it is *P* is, in itself, a denial of any *S* whatever that it is not *P*. To assert, therefore, that *Some S's are not P*, in opposition to *Every S is P*, is the minimum of denial. It is sufficient to destroy the proposition which it contradicts, but it does not affirm the falsity of every part of it. Thus, two contradictory propositions leave no room for an intermediate supposition; one or the other must be accepted as true, as together they exhaust all possible alternatives.

Every proposition has a contradictory; if the proposition is simple, so is the contradictory, but if the proposition is compound it can be contradicted in more than one way, and its full contradictory is, therefore, compound.[1]

Contradiction is the only kind of opposition which can subsist between Singular Propositions[2]; for these can differ only in quality, and, therefore, to posit the one is to sublate the other, and *vice versâ*. This opposition of singular propositions is frequently called *Secondary Contradiction.*

(iii) **Contrariety.**—*Contrary Opposition exists between a pair of universal propositions of opposite quality;* that is, between **A** and **E**. Thus, contrary propositions differ in quality only, and not in quantity. By the Principle of Contradiction both cannot be *true* together. For, if two contraries were both true, then contradictories would also be true together. For, by subalternation, the truth of **A** would necessitate that of **I**, and the truth of **E** would secure that of **O**. Hence, **A** and **O** would be true together, and so would **E** and **I**. But this is impossible; and, therefore, **A** and **E** cannot be true together. But as a contrary proposition does not simply deny the truth of the opposed universal as a whole, but that of every part of it, and thus asserts its entire falsity, there is a possibility of an intermediate alternative. Hence, the Principle of Excluded Middle does not apply, and the propositions may both be *false*. For, while the negation of a universal allows inference by Contradiction to the truth of

[1] See pp. 97-98. [2] See p. 100.

the particular of opposed quality, this latter does not warrant us in deducing the truth of the universal to which it is subaltern. Though by sublating **A** we posit **O**, this will not enable us to posit **E**. Hence, contrary propositions are incompatible with regard to truth, but not with regard to falsity. If one is true, the other must be false, but the falsity of the one does not involve the truth of the other. It may be equally false that 'All men are red-haired' and that 'No men are red-haired'; for the one proposition does not simply negate the other, but makes the opposite assertion with an equal degree of generality. It follows that contrary propositions are not mutually inferrible, and their formal opposition is, therefore, less perfect than is that of contradictories, although, of course, they express a greater degree of material divergence.

From this lesser formal perfection, as well as from the much greater difficulty of establishing the contrary compared with that of merely disproving a given universal proposition, it follows that contrariety is of much less formal importance than contradiction. The bringing forward of one single instance which does not agree with a general proposition is sufficient to disprove it, and the contradiction is secure, as it rests on observed fact. But to establish, not merely that one *S*, or a few *S's*, but that *every S* disagrees with the general proposition we wish to disprove is a task of much greater difficulty, and the result is much less secure against being itself proved false than is the contradictory. For we can scarcely ever be sure that we have really examined every instance, and one exception is fatal to our general proposition; while the simple contradictory, being a particular, can only be overthrown by establishing the opposed general proposition. Thus we see that contradiction is sufficient for disproof, and is, obviously, a more secure position to take up than is the assertion of the contrary. One would deny that 'All men are liars' with much greater strength of conviction than one would assert that 'No men are liars.'

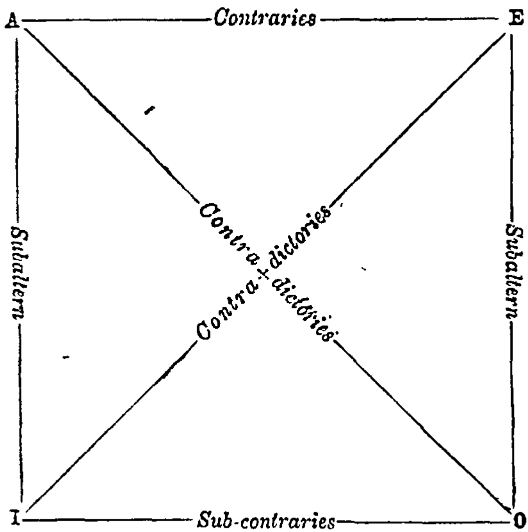
(iv) **Sub-contrariety.**—*Particular propositions stand in sub-contrary opposition to each other;* that is, **I** and **O** are

sub-contraries. This opposition depends on the Principle of Excluded Middle; for there can be no judgment intermediate between 'Some are' and 'Some are not.' Moreover, to deny the truth of one particular is to assert that of the universal of opposite quality (by Contradiction), and from this follows the truth of the particular which is subaltern to it. Hence, both these propositions cannot be *false*. But the Principle of Contradiction does not apply; for the 'some' in the one case is different in its reference from the 'some' in the other. Both propositions may, therefore, be *true*. The truth, for example, of 'Some men are red-haired' does not involve the falsity of 'Some men are not red-haired'; for it is not the same 'some men' who are referred to in both cases. But the form of the propositions does not show this, since the interpretation of 'some' must be purely indefinite. Thus, there is no real contrariety between **I** and **O**, and the name 'Sub-contrary' is entirely arbitrary. This is another instance of the technical use of the word 'opposition,' as the two propositions are perfectly compatible with each other; both may be, and often are, true, though both cannot be false. It follows, therefore, that to sublate the one is to posit the other, but not *vice versâ*. Hence sub-contrary propositions are inconsistent with regard to falsity but not with regard to truth.

2. **The Square of Opposition.**—It has long been traditional in Logic to give, as an aid to remembering the doctrine of opposition, the accompanying diagram, called the Square of Opposition.

If this diagram, with the proper positions of the letters which symbolise the four kinds of propositions, be once firmly stamped on the mind, but little difficulty will be found in retaining in the memory the whole theory of opposition. The universals are placed at the top, the particulars at the bottom, the affirmatives on the left, and the negatives on the right. The diagonals, as the longest lines, mark Contradiction, which is the most perfect and thoroughgoing form of logical opposition. The top line indicates Contrariety, and the bottom line, parallel to it,

Sub-contrariety. The fact that both are horizontal naturally suggests that each connects propositions of the same



quantity. The perpendicular lines appropriately represent Subalternation. As the diagonals run from the one top corner to the opposite bottom corner they indicate that contradictory propositions differ both in quality and quantity. Similarly, the top and bottom lines suggest a difference in quality only, and the side lines a difference in quantity only.

The following Table exhibits at a glance all the inferences which the doctrine of opposition enables us to draw. The kind of opposition through which they are reached is given by the letter, or letters, in brackets under each result. C means by contradiction; S, by subalternation; Cy, by Contrariety; Scy, by sub-contrariety. When any of these letters is printed in italics it means that the process it represents is indirect; that is, the result is

obtained, not immediately from the given proposition, but indirectly through its contradictory.

	Given	A	O	E	I
1	A true		false (C)	false (Cy, *S*)	true (S, *Scy*)
2	A false		true (C)	doubtful	doubtful
3	E true	false (Cy, *S*)	true (S, *Scy*)		false (C)
4	E false	doubtful	doubtful		true (C)
5	I true	doubtful	doubtful	false (C)	
6	I false	false (S, *Cy*)	true (Scy, *S*)	true (C)	
7	O true	false (C)		doubtful	doubtful
8	O false	true (C)		false (S, *Cy*)	true (Scy, *S*)

3. **Opposition of Hypothetical Propositions.**—Opposition applies equally well to those more definite judgments of connexion of content which are expressed in hypothetical form. The true hypotheticals: *If S is M it is P*, and *If S is M it is not P*—or expressed in the more

general but less definite symbolism *If* **A** *then* **X**, and *If* **A** *then not* **X**—are universals, and correspond to the **A** and **E** categorical forms respectively; while the particulars: *If S is M it may be P*, and *If S is M it need not be P*—or in the wider symbolic form, *If* **A** *then perhaps* **X**, and *If* **A** *then not necessarily* **X**—correspond to the **I** and **O** categorical forms. Having thus all the four necessary forms, the whole doctrine of opposition is applicable.

Similarly with the denotative forms—or conditionals as we have ventured to call them—which give more concrete expression to the content of these abstract judgments.[1] Here the four forms are—

If any S is M that S is always P—corresponding to **A**.
If any S is M that S is never P— „ „ **E**.
If an S is M that S is sometimes P— „ „ **I**.
If an S is M that S is sometimes not P „ „ **O**.

As the last two forms do not imply more than that *S* being *P* is a possible consequence of its being *M*, but not that *S* actually is *P* in any one case in which it is *M*, they may be often better expressed by—

If an S is M that S may be P—corresponding to **I**.
If an S is M that S need not be P— „ „ **O**.

4. **Opposition of Disjunctive Propositions.**—The most general symbolic form of the disjunctive proposition —*Either* **X** *or* **Y**—is most suitable to those cases in which the alternative judgments have not the same subject. A disjunctive in this form must be regarded as singular, and as, consequently, only capable of contradiction. The contradictory proposition is *Neither* **X** *nor* **Y**, and this is not itself a disjunctive judgment.

But the more perfectly stated disjunctive judgments, in which several predicates are alternatively affirmed of the same subject, admit of distinctions of quantity, and propositions of opposite quality can be found which stand to them in the relations of contradiction and contrariety.

[1] See p. 107.

Thus, with the judgment of content, *S is either P or Q*, the square of opposition can be completed by the propositions *S is neither P nor Q* (contrary); *S may be either P or Q* (subaltern); *S need not be either P or Q* (contradictory). These distinctions—as in the case of categorical judgments—stand out yet more clearly in the denotative forms of the propositions. Here we have the universal affirmative *Every S is either P or Q*; the universal negative *No S is either P or Q*; the particular affirmative *Some S's are either P or Q*; and the particular negative *Some S's are neither P nor Q*. But it will be noticed that none of the negative forms are disjunctive propositions. *S is neither P nor Q* is equally well expressed in the copulative categorical form *S is both non-P and non-Q* and similar propositions express the negative denotative forms. Hence, the full doctrine of opposition cannot be said to be applicable to disjunctive propositions.

CHAPTER XIII.

EDUCTIONS.

1 Chief Eductions of Categorical Propositions.

Eductions are those forms of Immediate Inference by which, from a given proposition, accepted as true, we educe other propositions, differing from it in subject, in predicate, or in both, whose truth is implied by it. Every Categorical Proposition gives us information of a certain subject, in terms of a certain predicate. But each of these terms has a conceivable negative; and every categorical proposition, therefore, suggests to our minds, directly or indirectly, four terms—*S*, *P*, *non-S*, *non-P*. The problem before us is to enquire what predications about each, or any, of these possible terms are implied when *S* and *P* are connected in any given categorical judgment. In other words, whether, if we take each of these terms in turn as subject, the given proposition justifies us in predicating of it any of the other terms. We need not, of course, consider any forms of proposition in which the predicate is either the same term as the subject, or its negative—as *S is S*, *S is non-S*, *P is not P*, etc.—which are either mere tautology, or are self-contradictory and, therefore, self-destructive. Our enquiry is limited to those propositions in which one term is *S* or *non-S*, and the other *P* or *non-P*.

Now, when any one of these four terms is taken as subject, we have two possible predicates offered to us; thus, we can predicate either *P* or *non-P* of *S*, and either *S* or *non-S* of *P*. This leads us to the kind of Eduction called *Obversion*, in which we retain the same subject but negative the predicate of the original proposition. Again, if *S* is the subject, and *P* the predicate, of the given propo-

sition, we can form other propositions whose subjects are respectively, *P*, *non-P*, and *non-S*, and each of these propositions can take two forms, one of which is derived from the other by obversion. Thus we get the following possible modes of inference, most of which involve a change, not only in the verbal expression, but in the form of the judgment as thought—

(1) *Obversion*—when the subject of the original proposition is unchanged, but the predicate is negatived.

(2) *Conversion*—when the subject of the inferred proposition is *P*, and its predicate *S* or *non-S*.

(3) *Contraposition*—when the subject of the inferred proposition is *non-P* and its predicate *S* or *non-S*.

(4) *Inversion*—when the subject of the inferred proposition is *non-S*, and its predicate *P* or *non-P*.

None of these can be valid inferences from any given proposition, unless the inferred proposition is involved in, and expresses the same truth as, that proposition itself expresses. We must, therefore, by careful examination, see which of them are justified by propositions of each of the four forms, **A**, **E**, **I**, **O**.

Each of the inferences (2), (3) and (4) in the above list can take two forms, one with a positive, and the other with a negative predicate. Each of these forms is obtainable from the other by the process of obversion. As, however, the simplest forms are those which have the positive predicates, the simple names, *Conversion*, *Contraposition*, and *Inversion*, are applied to the processes by which they are arrived at. Those propositions themselves are called the *Converse*, *Contrapositive*, and *Inverse*, of the original proposition; while the corresponding forms with negative predicates are termed the *Obverted Converse*, the *Obverted Contrapositive*, and the *Obverted Inverse*, respectively, of that proposition. Thus, each of these names expresses the relation in which that derived proposition stands to the given one.

If we use $\bar{S}$ and $\bar{P}$ to denote *non-S* and *non-P* respectively, we have the following empty schema of possible Eductions from categorical propositions—

i.		1	Original Proposition - - -	S—P
		2	Obverse of (1) - - - - - -	S—$\bar{P}$
ii.	Converses of (1) -	3	Converse of (1) - - - - - -	P—S
		4	Obverted Converse of (1) - -	P—$\bar{S}$
iii.	Contrapositives of (1)	5	Contrapositive of (1) - - -	$\bar{P}$—S
		6	Obverted Contrapositive of (1)	$\bar{P}$—$\bar{S}$
iv.	Inverses of (1) - -	7	Inverse of (1) - - - - - - -	$\bar{S}$—P
		8	Obverted Inverse of (1) - -	$\bar{S}$—$\bar{P}$

We have to enquire to what extent this empty schema can be filled out by either of the four kinds of categorical predication—**A**, **E**, **I**, **O**,—when the original proposition itself is of either of those forms.

Many of the inferred forms are unusual and unnatural modes of expressing the truth which is stated most simply in the original proposition. Those of them, too, which contain negative terms are open, as primary modes of statement, to the objections already made to propositions containing those terms.[1] But, when they are regarded simply as secondary modes of expressing the content of the original judgment, they are useful; as they make prominent a fresh side of the truth there enunciated.

[1] See p. 34.

And the whole of them together, by placing that assertion in every possible light, make its implications much clearer and more definite than does a mere consideration of the proposition by itself.

As Obversion and Conversion are the primary modes by which these eductions are made—for all the other inferences are obtainable by combinations of these—a detailed consideration of them should precede that of the other forms.

(i) **Obversion is a change in the quality of a predication made of any given subject, while the import of the judgment remains unchanged.** The original proposition is called the *Obvertend*, and that which is inferred from it is termed the *Obverse*.

Whenever we assert anything we, by implication, deny the opposite. That is, the affirmation of any predicate of a certain subject implies the denial of its negative; and the denial of any predicate implies the affirmation of its negative. The former of these follows from the Principle of Contradiction—for, if any *S* is *P* it cannot be *non-P*; and the latter from that of Excluded Middle—for, if any *S* is not *P* it must be *non-P*. All obversions of affirmative propositions, therefore, depend on the former of these two principles; and all obversions of negative propositions on the latter. But, to deny a negative is to affirm, for two negatives destroy each other; and to affirm a negative is to deny; and, thus, obversion involves no change of meaning. The matter, therefore, which is expressed by an affirmative proposition can always be re-expressed by a negative, and *vice versâ*. This is, however, a mere change in the mode of expression; it involves no process of thought, and consequently is not a real inference. It is, however, useful as a first step in contraposition.

From this it follows that the subject of the obverse is the same as the subject of the obvertend in every respect, as, otherwise, we should not have a true denial of the opposite of that obvertend. The *quantity* of the two propositions is, therefore, the same. The predicate of the obverse is the negative of that of the obvertend, and this, to avoid alteration in meaning, necessitates a change in

the *quality* of the proposition. This gives us the one simple rule for obverting any proposition—

Negative the predicate and change the quality, but leave the quantity unaltered.

Applying this rule to the four forms of categorical propositions, we find that

A *obverts to* **E**, **E** *to* **A**, **I** *to* **O**, *and* **O** *to* **I**;

or, expressed symbolically,

Original Proposition - -	$S\ a\ P$	$S\ e\ P$	$S\ i\ P$	$S\ o\ P$
Obverse - - - - - - - -	$S\ e\ \bar{P}$	$S\ a\ \bar{P}$	$S\ o\ \bar{P}$	$S\ i\ \bar{P}$

It must be remembered that obversion is a reciprocal process, and, thus, that $S\ a\ P$ is as much the obverse of $S\ e\ \bar{P}$ as the latter is the obverse of the former.

As material examples we may give the following pairs of propositions, each member of every pair being the obverse of the other member—

A. *All men are mortal.*
E. *No men are not-mortal.*

E. *No thoughtful men are superstitious.*
A. *All thoughtful men are non-superstitious.*

I. *Some men are happy.*
O. *Some men are not not-happy.*

O. *Some men are not rich.*
I. *Some men are not-rich.*

We may often write the obverse in a form more in accordance with the usages of ordinary speech by using a material contradictory, or a privative term, instead of the formal negative, for the new predicate. But, unless this term is exactly equivalent in meaning to the formal negative, we do not make a true obversion by its use. For instance, in obverting **A** as given above, we could say, 'No men are immortal,' for 'immortal' and 'not-mortal' exactly correspond. But we could not give 'Some men are not

unhappy' as the obverse of 'Some men are happy'; for 'happy' and 'unhappy' do not exhaust all possibilities, and, thus, the principle of contradiction does not apply to them. It is true that this proposition is justified by the given one, for 'not-happy' includes unhappy, as well as all other shades of departure from 'happy.' But it is not the obverse; for we cannot get back from it, by obversion, to our original proposition. The same holds in the case of all affirmative propositions; the obverse justifies the denial of all terms which can be brought under the formal negative. But even this is not justifiable in the case of the obversion of negative propositions. From 'Some men are not happy' we cannot conclude that 'Some men are unhappy,' for this latter proposition asserts, not merely the absence of happiness, but the presence of a certain amount of positive misery. Still less can we infer from 'Some men are not rich' that 'Some men are poor'; for 'rich' and 'poor' are contraries, and there are many intermediate stages between them. Obversion is, in short, a formal process; and, therefore, if we do not use a formal negative term for our new predicate, we must make sure that the term we do use is the exact equivalent of that formal negative.

(ii) **Conversion is the eduction of one proposition from another by transposing the terms.** The original proposition is called the *Convertend*, and that which is derived from it is named the *Converse*.

We have, evidently, here a complete alteration of standpoint, as we have changed the subject or nucleus of our proposition. The predication is now made of P in terms of S, whereas the original proposition contained an assertion about S in terms of P. Moreover, the truth of the converse follows directly from that of the convertend. Hence, the process is a real interpretative inference. Every proposition before being converted—or, indeed, used in any kind of formal inference—must be reduced to the strict logical form, *S is P* or *S is not P*, and the *whole* predicate must change places with the *whole* subject. For instance, the converse of 'Every old man has been a boy' is not 'Every boy has been an old man,' but 'Some who have been boys are old men'; for the original proposition, in its

logical form, is 'Every old man is a person who has been a boy.' As the converse simply makes the same assertion as the convertend, looked at, as it were, from the other side, it is clear that the *quality* of both propositions will be the same.

Every act of conversion involves reading the original predicate in its denotation, in order that it may be made a subject-term. That we really do make this change from a connotative to a denotative view is shown by the fact that, if the predicate of the convertend is an adjective—as in 'No crows are white'—a substantive must be supplied before we can use that term as the subject of the converse —as 'No white things are crows.' This involves a consideration of the distribution of the predicate in order that the converse may not assert more than is justified by the convertend; and may necessitate a change of *quantity*. In other words, a mere transposition of terms is not always permissible; we cannot go from 'All cats are animals' to 'All animals are cats.' The only conversion we are concerned with is *Illative Conversion*; that is, conversion which is a valid inference, and in which either both convertend and converse are true, or both are false. Such conversion must obey these two rules:

1. *The quality of the proposition must remain unchanged.*

2. *No term may be distributed in the converse which is not distributed in the convertend.*

We must now apply these rules to the conversion of each kind of categorical proposition.

(*a*) *Conversion of* **A**. In the proposition $S\ a\ P$, while S is distributed, P is not. We cannot, therefore, convert to $P\ a\ S$—for that would break Rule 2—but we must retain P in its undistributed condition, and write the converse $P\ i\ S$. Hence **A** converts to **I**, and the conversion involves a change of quantity from universal to particular. Such conversion was called by Aristotle κατὰ μέρος or *partitive* conversion. This name has, however, given place to the less descriptive one of *conversio per accidens* or *conversion by limitation.*

Though the necessity for this mode of converting **A**

propositions is obvious enough when the rules for conversion are kept in mind, yet the improper conversion of **A** propositions is one of the most frequent causes of fallacy. Because it is a fairly well established fact that lazy persons are often out of work, people jump to the conclusion that if a man is often out of work he is necessarily lazy. Since the wages of unskilled labour in England are low, it is frequently assumed that all badly paid persons are unskilful. Because all pious people go regularly to church, regular church-going is commonly regarded as a sure sign of piety. Such mistakes are continually made, yet they are on a par with arguing that every animal is a monkey because every monkey is an animal. No doubt, in some cases—as tautologous propositions and definitions, or when both subject and predicate are singular names—the simple converse, that is, converse without change of quantity, of **A** would give a true proposition. 'Every equiangular triangle is equilateral' is as true a proposition as is 'Every equilateral triangle is equiangular.' But its truth has to be established by a separate and independent demonstration; it cannot be inferred from the latter proposition by conversion. For conversion is a formal process of inference and, therefore, must be applicable to every proposition of the same kind; there cannot be two modes of formally converting **A** propositions. When the simple converse would be true in fact, it is because of special circumstances which do not appear in the statement of the convertend. Hence, as $P\ i\ S$ is the only converse which is materially true in all cases, and is formally true in any, that is the logical converse of $S\ a\ P$. For, while $S\ a\ P$ asserts positively that the attribute which P denotes is found in every S, it is not stated whether, or not, it is found in other cases.

This is further obvious from the fact that we require two diagrams to express an **A** proposition.[1]

(*b*) *Conversion of* **E**. An **E** proposition can be converted *simply*; that is, without change of quantity. For, $S\ e\ P$ asserts that the attributes connoted by P are found

[1] See p. 118.

in none of the objects which S denotes, but only in other objects. Hence, none of the objects in which P is found, and which are all denoted by P used as a substantive name, possesses the attributes which are connoted by S. The separation between the things which are S and those which possess the attribute P is total and absolute; and is, therefore, reciprocal. Whether we regard it from the side of S or of P, each individual S differs from each individual P. Thus, we can convert $S\ e\ P$ to $P\ e\ S$. If 'No horses are carnivorous,' it follows that 'No carnivorous animals are horses.'

So, if we refer to the diagrams, we see that only one is required.[1]

(c) *Conversion of* **I**. As neither term in an **I** proposition is distributed, it is clear that, by converting it simply, we shall break neither of the rules of conversion. Thus, $S\ i\ P$ converts to $P\ i\ S$, and the proposition remains particular. 'Some herbs are poisonous' gives as a converse 'Some poisonous things are herbs.' The 'some' remains, of course, purely indefinite; and when we speak of the simple conversion of **I** we do not mean that 'some' denotes the same proportion of the total denotation of the subjects of both convertend and converse. When the subject of the convertend is a genus of which the predicate is a species, the simple converse reads somewhat awkwardly. Thus, 'Some human beings are boys' converts to 'Some boys are human beings,' which, we feel, is not so definite an assertion as our knowledge of the matter would warrant us in making. This is particularly noticeable when we reconvert the converse of an **A** proposition. The converse of $S\ a\ P$ is $P\ i\ S$, and we can only convert this again to $S\ i\ P$, where the double logical process has led to a loss of fullness in the statement. For example, 'All monkeys are animals' converts to 'Some animals are monkeys,' and the simple converse of this is 'Some monkeys are animals.' This shows that conversion *per accidens* is not a reciprocal process, as simple conversion is. But, no matter what the **I** proposition is, or whence it is derived, it can, by itself,

[1] See p. 118.

only justify us in deducing another **I** proposition as its converse.

This indefiniteness is illustrated by the fact that **I** requires four diagrams to express the possible relations between P and S which it covers; from examination of these it is evident that the relation between the terms is always positive, but so far as quantity is concerned absolutely indefinite.[1]

(*d*) *Conversion of* **O**. As the predicate of an **O** proposition is distributed, but the subject undistributed, we cannot convert a proposition of that form at all. For, by Rule 1, S o P must convert to a negative proposition with S for its predicate. This would distribute S; but Rule 2 forbids this distribution, as S is not distributed in the convertend. S o P asserts that Some S's have not the attribute P, but it says nothing about the other possible S's. Hence, though the *Some* S's which form the subject are entirely separated from all those things which possess the attribute P, it does not follow that these latter are excluded from *all* the S's. It is possible that every P is S, though there are other instances of S as well which are not P. For example, 'Some men are not honest' will not justify us in inferring that 'Some honest beings are not men'; nor can we say that some who pass an examination do not sit for it, because it is true that some who sit for an examination do not pass. In many cases, no doubt, the simple converse of an **O** proposition would be materially true; thus 'Some men are not black' and 'Some black things are not men' are both true propositions, but neither can be inferred by formal conversion from the other, for neither statement is justified by the other.

If we examine the three diagrams required to represent **O** we see that no relation between P and S is common to them all.[2] Therefore we cannot convert the proposition S o P.

To sum up the results we have obtained—

A *converts per accidens;* **E** *and* **I**, *simply;* **O**, *not at all.*

[1] See p. 118. [2] See p. 118.

(*c*) *Obverted Conversion.* As any categorical proposition whatever can be obverted, we can get a new inference from the original proposition by obverting the converse, according to the rules given in sub-section (i). Thus, expressed symbolically, we get—

1	Original Proposition ...	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
2	Converse of (1)	$P\,i\,S$	$P\,e\,S$	$P\,i\,S$	(None)
3	Obverted Converse of (1)	$P\,o\,\bar{S}$	$P\,a\,\bar{S}$	$P\,o\,\bar{S}$	(None)

As material examples we may give—

Original Proposition - **A** - *Every truthful man is trusted.*
Converse - - - **I** - *Some trusted men are truthful.*
Obverted Converse - **O** - *Some trusted men are not untruthful.*

Original Proposition - **E** - *No cultivated district is uninhabited.*
Converse - - - **E** - *No uninhabited district is cultivated.*
Obverted Converse - **A** - *All uninhabited districts are uncultivated.*

Original Proposition - **I** - *Some British subjects are dishonest.*
Converse - - - **I** - *Some dishonest people are British subjects.*
Obverted Converse - **O** - *Some dishonest people are not aliens.*

(iii) **Contraposition is the inferring, from a given proposition, another proposition whose subject is the contradictory of the predicate of the original proposition.** The derived proposition is called the *Contrapositive*; there is no corresponding distinctive name for the original proposition.

The contrapositive of any given proposition is most easily arrived at indirectly. It makes a predication about the contradictory of the predicate of the given proposition. Now, this contradictory appears as the predicate of the obverse of that proposition. If, then, this obverse can be converted it gives a proposition of the form required, in which the negative of the original predicate is the subject, and the subject of the original proposition is the predicate. Hence, the simple rule for contraposition is:

First obvert, then convert.

This will give, in every case, a proposition differing in *quality* from the original one; for obversion changes the quality, and conversion does not change it back again. But the *quantity* remains unchanged, except in the case of the contraposition of **E**; for, obversion does not change quantity, and, therefore, any change in quantity must be due to the subsequent conversion. Now, as **A** and **O** obvert to **E** and **I** respectively, and both of these convert simply, the quantity will remain unaltered. But **E** obverts to **A**, which can only be converted *per accidens*, and hence the contrapositive of the universal negative is a particular affirmative. Thus, comparing the contraposition with the conversion of universal propositions in respect to quantity, it is seen that when the one inference causes a change in quantity, the other does not, and *vice versâ*.

As **I** obverts to **O**, which cannot be converted, there can be no contrapositive of **I**.

Contraposition is sometimes called *Conversion by Negation*, and, as we see, it can be applied to **O** propositions, and is the only form of 'conversion' which can be so applied. But, it is better not to use 'conversion' in this sense, as the contrapositive has not the same subject as the converse, and also differs from it in quality.

Obverted Contraposition. Having obtained the contrapositive of any proposition we can obvert it, and thus get a proposition of the same quality as the original one. This *Obverted Contrapositive* has for each of its terms the contradictory of a term in the given proposition—its subject is the negative of the original predicate, and its predicate the negative of the original subject. Some writers have confined the name Contrapositive to this form. The older logicians all did this, as they held that contraposition, being a kind of conversion, should not change the quality of the given proposition. There seems, however, to be no reason for thus restricting the application of the name. Both forms are contrapositives, and when we wish to distinguish them, we call the simpler—that is, the one which retains one of the original terms—the contrapositive, while the proposition derived from that by obversion is fitly named the obverted contrapositive.

We get, then, the following results, expressed symbolically—

1	Original Proposition - -	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
2	[Obverse of (1)] - - -	$[S\,e\,\bar{P}]$	$[S\,a\,\bar{P}]$	$[S\,o\,\bar{P}]$	$[S\,i\,\bar{P}]$
3	Contrapositive of (1) - -	$\bar{P}\,e\,S$	$\bar{P}\,i\,S$	(None)	$\bar{P}\,i\,S$
4	Obverted Contrapositive of (1)	$\bar{P}\,a\,\bar{S}$	$\bar{P}\,o\,\bar{S}$	(None)	$\bar{P}\,o\,\bar{S}$

As material examples we may give—

Original Proposition - **A** - *Every poison is capable of destroying life.*
[Obverse] - - -[**E**]- [*No poison is incapable of destroying life.*]
Contrapositive - - **E** - *Nothing incapable of destroying life is poisonous.*
Obvd. Contrapositive - **A** - *Everything incapable of destroying life is non-poisonous.*

Original Proposition - **E** - *No lazy person is deserving of success.*
[Obverse] - - -[**A**]- [*Every lazy person is undeserving of success.*]
Contrapositive - - **I** - *Some people undeserving of success are lazy.*
Obvd. Contrapositive - **O** - *Some people undeserving of success are not not-lazy.*

Original Proposition - **O** - *Some unjust laws are not repealed.*
[Obverse] - - -[**I**] - [*Some unjust laws are unrepealed.*]
Contrapositive - - **I** - *Some unrepealed laws are unjust.*
Obvd. Contrapositive - **O** - *Some unrepealed laws are not just.*

The great value of contraposition is this. The aim of science is to reach propositions which are in fact reciprocal. In such propositions the predicate is stated so definitely that it is strictly characteristic of the subject, that is, it belongs in exactly that form to nothing else, and the knowledge expressed by the proposition is, therefore, of the most precise form attainable. When then $S\ a\ P$ is established, we want to know if $P\ a\ S$ is also true; and the readiest way to establish this is generally to examine cases

of $\bar{S}$ and endeavour to establish the proposition $\bar{S}\ e\ P$ which is the contrapositive of $P\ a\ S$. The importance of this will appear more clearly in the discussion of Induction.

(iv) **Inversion is the inferring, from a given proposition, another proposition whose subject is the contradictory of the subject of the original proposition.** The given proposition is called the *Invertend*, that which is inferred from it is termed the *Inverse.*

The inverse of any given proposition is most easily arrived at indirectly, through some of the forms of eduction we have already considered. We can only obtain the contradictory of a term by obverting the proposition of which that term forms the predicate. S must, therefore, have been made the predicate of a proposition, and then that proposition must have been obverted for us to get *non*-S. Two eductions—the obverted converse and the obverted contrapositive—satisfy these conditions. If, then, we can convert either of these we have an Inverse. Hence the rule for Inversion is —

Convert either the Obverted Converse or the Obverted Contrapositive.

In the case of **A** the obverted converse is $P\ o\ \bar{S}$, and this is inconvertible. But the obverted contrapositive is $\bar{P}\ a\ \bar{S}$, which can be converted to $\bar{S}\ i\ \bar{P}$. As both the terms of this proposition are contradictories of those which appear in the original proposition, it is not the simple, but the obverted, inverse. As, however, obversion is a reciprocal process, we can obvert this to $\bar{S}\ o\ P$, which gives the simple inverse.

In the case of **E** the obverted converse is $P\ a\ \bar{S}$, which, by conversion, gives the inverse $\bar{S}\ i\ P$; this we can obvert to $\bar{S}\ o\ \bar{P}$, which is the obverted inverse.

In the case of **I** the obverted converse is $P\ o\ \bar{S}$, which cannot be converted; and it has no obverted contrapositive; therefore, it can have no inverse.

In the case of **O** there is no obverted converse, and the obverted contrapositive is $\bar{P}\ o\ \bar{S}$, which cannot be converted; **O** has, therefore, no inverse.

So the possible inverses expressed symbolically are—

1	Original Proposition	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
2	[Obverted Converse of (1)]		$[P\,a\,\bar{S}]$		
3	[Obverted Contrapositive of (1)]	$[\bar{P}\,a\,\bar{S}]$			
4	Inverse of (1)	$\bar{S}\,o\,P$	$\bar{S}\,i\,P$	(None)	(None)
5	Obverted Inverse of (1)	$\bar{S}\,i\,\bar{P}$	$\bar{S}\,o\,\bar{P}$	(None)	(None)

As material examples we may give:—

Original Proposition - **A** - *Every truthful man is trusted.*
[Obvd. Contrapositive] - [**A**] - [*Every not-trusted man is untruthful.*]
Inverse - **O** - *Some untruthful men are not trusted.*
Obvd. Inverse - **I** - *Some untruthful men are not-trusted.*

Original Proposition - **E** - *No unjust act is worthy of praise.*
[Obvd. Converse] - [**A**] - [*Every act worthy of praise is just.*]
Inverse - **I** - *Some just acts are worthy of praise.*
Obvd. Inverse - **O** - *Some just acts are not unworthy of praise.*

All the forms of Eduction may be thus tabulated—

			A	E	I	O
i.	1	Original Proposition	$S\,a\,P$	$S\,e\,P$	$S\,i\,P$	$S\,o\,P$
	2	Obverse of (1)	$S\,e\,\bar{P}$	$S\,a\,\bar{P}$	$S\,o\,\bar{P}$	$S\,i\,\bar{P}$
ii.	3	Converse of (1)	$P\,i\,S$	$P\,e\,S$	$P\,i\,S$	
	4	Obverted Converse of (1)	$P\,o\,\bar{S}$	$P\,a\,\bar{S}$	$P\,o\,\bar{S}$	
iii.	5	Contrapositive of (1)	$\bar{P}\,e\,S$	$\bar{P}\,i\,S$		$\bar{P}\,i\,S$
	6	Obverted Contrapositive of (1)	$\bar{P}\,a\,\bar{S}$	$\bar{P}\,o\,\bar{S}$		$\bar{P}\,o\,\bar{S}$
iv.	7	Inverse of (1)	$\bar{S}\,o\,P$	$\bar{S}\,i\,P$		
	8	Obverted Inverse of (1)	$\bar{S}\,i\,\bar{P}$	$\bar{S}\,o\,\bar{P}$		

2. **Eductions of Hypothetical Propositions.**—Though true hypotheticals are universal, yet we have seen that some particulars take the same general form, and may be regarded as imperfectly developed hypotheticals.[1] Embracing these propositions we have forms corresponding to each of the four forms of categorical propositions, and the full table of eductions is applicable to them. These inferences are seen, perhaps, more clearly when the propositions are not written in the abstract form directly expressive of connexion of content, but in the following more concrete and denotative forms which are justified by and correspond to them, and which we have called conditional[2]—

A. *If any S is M, then always, that S is P.*

E. *If any S is M, then never, that S is P.*

I. *If an S is M, then sometimes, that S is P.*

O. *If an S is M, then sometimes not, that S is P.*

It must be remembered that 'sometimes' is purely indefinite, like 'some,' and moreover it does not necessarily imply the actual occurrence of the consequent in any one instance; its force is really 'it may be,' while 'sometimes not' simply means, 'it need not be.'

As examples we will give the eductions from **A** expressed symbolically—

Orig. Prop.	**A**	*If any S is M, then always, that S is P.*
Obverse	**E**	*If any S is M, then never, that S is not P.*
Converse	**I**	*If an S is P, then sometimes, that S is M.*
Ob. Conv.	**O**	*If an S is P, then sometimes not, that S is not M.*
Contrap.	**E**	*If any S is not P, then never, that S is M.*
Ob. Contr.	**A**	*If any S is not P, then always, that S is not M.*
Inverse	**O**	*If an S is not M, then sometimes not, that S is P.*
Ob. Inv.	**I**	*If an S is not M, then sometimes, that S is not P.*

[1] See p. 108. [2] See p. 107.

3. **Eductions of Disjunctive Propositions.**—Eductions can only be drawn from disjunctive propositions in which alternative predicates are affirmed of one subject. They are more clearly seen if we take the denotative forms of proposition, corresponding to the categorical **A** and **I**, and the same eductions can be drawn from the former as from the latter. The derived propositions, however, are not themselves disjunctive.

The symbolic expressions of the eductions from a universal disjunctive are—

Orig. Prop.	*Every S is either P or Q.*
Obverse.	*No S is both* $\bar{P}$ *and* $\bar{Q}$.
Converse.	*Some things that are either P or Q are S.*
Ob. Conv.	*Some things that are either P or Q are not* $\bar{S}$.
Contrap.	*Nothing that is both* $\bar{P}$ *and* $\bar{Q}$ *is S.*
Obv. Contr.	*Everything that is both* $\bar{P}$ *and* $\bar{Q}$ *is* $\bar{S}$.
Inverse.	*Some* $\bar{S}$*'s are neither P nor Q.*
Obv. Inv.	*Some* $\bar{S}$*'s are both* $\bar{P}$ *and* $\bar{Q}$.

CHAPTER XIV.

FALLACIES INCIDENT TO IMMEDIATE INFERENCE.

1. **False Opposition.**—Any of the inferences based on the opposition of propositions may be wrongly drawn, and we then have a fallacy of opposition. Thus, for example, to infer the falsity of the subaltern from that of its subalternans, or the truth of the subalternans from that of its subaltern, would be such a fallacy. But the most dangerous and frequently committed fallacies of opposition are those connected with contradiction. The contrary may be confused with the contradictory. It is easy to state a number of alternatives and assume that they are all that are possible. As we have already noted, one form of misinterpretation of a disjunctive proposition is due to thus assuming that the alternatives given exhaust the possibilities. Under this head may be brought Aristotle's fallacy of Many Questions.

Plures Interrogationes.—The *Fallacia Plurium Interrogationum* or *Fallacy of Many Questions*, is the attempt to get a single answer to several questions asked in one, as in the old example: 'Have you left off beating your father?' Other examples would be 'Where did you hide the goods you stole last night?' 'Have you cast your horns?' This last is a traditional example, and from it the fallacy is sometimes called the *Cornutus*. The whole is to-day quite frivolous, and the only justification for noting it as a separate class of sophism was that the common method of disputation amongst the Greeks was a procedure by question and answer. It lingers among ourselves as a common device of a cross-examining counsel. The essential nature of the fallacy is false opposition, due to the wrong application of the principle of Excluded Middle. The usual

examples take the form 'Is it x or y?' and assume that it must be either x or y; that there is no other alternative. Thus the first example omits the alternative 'You never did beat your father,' and assumes that the only possible alternatives are 'You still beat your father' and 'You used to beat him but do so no longer.'

2. **Illicit Conversion.**

(i) **Abstract.**—To convert a proposition so that a term is used universally in the converse which was only used particularly in the convertend is a fallacy not infrequently committed. This is only possible with **A** and **O** categorical propositions, and with the hypothetical forms which correspond to them. When the fallacy is committed openly it may be called *abstract*; when it is hidden by the language it may be said to be *concrete*. The former class of fallacies has been already discussed with sufficient fullness.[1] Under the latter head come two of Aristotle's fallacies, viz. *Accidens*, where the illicit conversion is that of an **A** categorical proposition, and *Consequens*, where it is of a universal hypothetical.

(ii) **Accidens.**—The *Fallacia Accidentis* arises when a predication which can be correctly made of any subject is made of all the 'accidents' of that subject. But by 'accident' is here meant, not what is denoted by that name in Porphyry's Scheme of Predicables,[2] but, any subordinate part of a general notion. Thus, every species and individual is an accident of its genus, in this sense of the term. The fundamental invalidity in such inferences is the simple conversion of an **A** proposition, whence the correct conversion of that form of proposition is said to be *per accidens*. Thus, to take an example given by Aristotle: "Every triangle has its three angles equal to two right angles; every triangle is a figure; therefore, every figure has its three angles equal to two right angles." Here we have a spurious syllogism with illicit process of the minor term; that is, 'figure' is distributed in the conclusion though not in the premise. But if the minor premise were simply con-

[1] See p. 146. [2] See pp. 40 42-43

verted the formal argument would be a correct syllogism, though with a false minor premise. The error is obvious. "Yet," says Grote, "Aristotle intimates that a scientific geometer of his day, in argument with an unscientific opponent, would admit the conclusion to be well proved, not knowing how to point out where the fallacy lay: he would, if asked, grant the premises necessary for constructing such a syllogism; and, even if not asked, would suppose that he had already granted them, or that they ought to be granted," which, as Grote remarks, "affords us a curious insight into the intellectual grasp of the scientific men contemporary with Aristotle."[1] Under this head should come a form of fallacy which is classed by De Morgan under the head of *æquivocatio*. "To call you an animal is to speak truth, to call you an ass is to call you an animal; therefore, to call you an ass is to speak truth."

Many logicians have failed to understand the nature of this fallacy, and have regarded it as but another name for the *Fallacia a dicto simpliciter ad dictum secundum quid*.

(iii) **Consequens.**—The *Fallacia consequentis* was intended by Aristotle to denote simply the formal error of inferring the truth of the antecedent from that of the consequent, or the falsity of the consequent from that of the antecedent. In the former case the error involved is that of invalid conversion; in the latter case we have an instance of illicit inversion. Like other fallacies, the invalidity is often hidden by the length and complexity of the argument in which it occurs. The most dangerous form of it is, undoubtedly, the assumption that a conclusion is necessarily wrong because it is supported by invalid arguments, or, conversely, that the arguments urged in support of a proposition accepted as true must necessarily be cogent. In both these cases personal bias and prejudice have abundant scope to come into operation. When a conclusion is deduced from an invalid argument, we are only justified in saying that such a conclusion is not proven, not that it is disproved. In any case, it must be

[1] *Aristotle*, p. 391.

remembered, a proposition can only be disproved by a cogent argument establishing its contradictory.

3. **Illicit Contraposition.**—Contraposition is illicit when the conclusion contains a distributed term which was undistributed in the premise. As contraposition involves both obversion and conversion, it will be found that the invalidity is always in the latter process. Thus, a fallacy of contraposition is only possible when the given premise is an **E** or **I** proposition or a hypothetical proposition of corresponding form.

4. **Illicit Inversion.**—To infer from *Every S is P* that *No* $\bar{S}$ *is P* or from *No S is P* that *Every* $\bar{S}$ *is P* would be to commit the fallacy of illicit inversion. Thus to argue that 'Thought is existence, therefore, what contains no element of thought is non-existent' or that 'A bad man must be miserable because happiness is the result of well-doing' is to commit this fallacy. Similarly to assume that because the proposition *If S is M it is P* is established, therefore, we are justified in assuming that *If S is not M it is not P* would be to commit this fallacy. Inductive enquiry is somewhat liable to this error. The sufficiency of a positive condition to secure a given result must not, however, be held to demonstrate that the result could be attained in no other way. Because a certain flower is fertilised when it is visited by insects, it does not follow that it could not be fertilised if no insects came near it. Whether it is so or not must be settled by a special enquiry conducted under conditions which exclude the positive agent already known to be sufficient to secure the result.

CHAPTER XV.

GENERAL METHOD OF KNOWLEDGE.

1. **Nature of Logical Method.**—The aim of thought is to reach exact knowledge. Such knowledge is a branch of science, no matter with what material it is concerned, and the thought which deals with it is scientific thought. The popular restriction of the words 'science' and 'scientific' to knowledge of the material universe has no logical meaning. It is true that in some such departments of knowledge greater precision and certainty have been reached than in those which deal with human affairs, either in the present or in the past. That is because the material is more amenable to manipulation and more uniform in its nature than are the motives and, consequently, the actions of men. But whether we deal with external nature or with the deeds of men in history, politics, economics, ethics, or any other aspect of human activity, the aim is always to reach real exact knowledge. That is, the aim is the establishment of science, and the means adopted are scientific in intention and in conception, even though they cannot be employed with as much care, certainty, and exactness as in physics or in chemistry.

In whatever department of knowledge the scientific investigator is working, much sad experience proves that he is liable to error. His conclusions may be true or may be false, or they may be a mixture of truth and falsity. What standard is there by which to test whether the results reached are trustworthy? As the very ultimate postulate of thought is that the truth cannot contradict itself, it is evident that consistency both with what is already known and with what may in the future be established is the only conceivable test. If as knowledge

in that sphere advances such agreement continues to hold, the conclusions reached may be regarded as practically established. If it fails to hold, those conclusions must be either rejected or modified.

In every form of human activity success depends on skill acquired by long and patient practice. But skill means that the activity, be it bodily or be it mental, has gradually discovered the best way—or method—of setting to work. That there is a skill in discovery and in thinking, and that that skill is the outcome of practice becoming more and more effective, rejecting the needless and discerning with increasing certainty the pertinent, is evidenced by the notorious fact that it takes a long apprenticeship to make a successful worker in any department of knowledge. The question for logic is whether this skill has features common to all its applications to this or that sphere of thought. Here logic has evidently to wait on the sciences. In the actual work of the study and the laboratory some methods have proved successful and have consequently been refined and extended: others have issued in failure and have needed modification if not rejection. As science advances it improves its methods, and, it may be, modifies the conceptions which guide them. Such adaptation, in so far as it touches the common features of which we have spoken, must be recognised and accepted by logic. In other words, the establishment of the logic of method is itself a scientific work which uses as its material the special methods of the various sciences which deal directly with the facts of the world. But with the special features of these methods logic has no direct concern. Most certainly it makes no claim to dictate them, and most certainly also it did not—and could not—originate them.

2. **Analysis and Synthesis.**—Now scientific thought has two main objects—to reach truth and to expound it. The method suitable to the one of necessity differs from that appropriate to the other. For in the second case the whole system pertinent to the matter in hand is known, and the task is to set it forth as clearly and accurately as

possible. Thought arranges the parts in their ascertained relations to each other in a known and understood system. The whole arrangement can be made orderly and correctly proportioned, just because the whole is known through and through. This, for example, is what is done in a good 'summing-up' by the judge of a case in the law-courts. Such a method has long been known as *Synthetic*, because it puts together the elements of a whole in an orderly manner determined by the nature of that whole.

So we see why it has been called the method of instruction, and all scholastic history shows that for long centuries it was practically so employed. That friend of our childhood, 'Euclid's Elements of Geometry,' is essentially synthetic in its arrangement. So, too, was the traditional method of teaching Latin grammar in abstract rules which had to be deductively applied to interpret Latin sentences.

Nowadays we see that this method of exposition is only suitable when the object is to put a reasoned result before one who has sufficient knowledge to appreciate the force of our premises. In other words, when a speaker or writer adopts this method he is—at any rate presumably—assuming that his hearers or readers have knowledge of like character with his own, if not of equal extent.

When, however, the object is to teach those who have not such knowledge, the emptiness of the mind of the learner should be recognised. Whatever it may be to the teacher, the advance on the part of the learner is a discovery of knowledge, and the object to be accomplished is not merely the gathering of information but yet more the acquirement of skill in dealing with reality in thought and in act. In the essentials of method—*i.e.* of the progress of thought—the learner is always a learner, no matter at what part of the ideally complete system of knowledge he is working. He may be a beginner, or he may see his system nearing completion. But in each case he has to start with fragments of knowledge and to try to connect those fragments together and to relate them to yet other fragments, as yet unknown, till a systematic whole stands before his mind.

But his thought deals with reality, and in reality alone can it find its material. Yet that material cannot be gathered straight away. The relations which bind thing to thing do not usually lie on the surface. On the contrary they have to be sought with much labour and pain and frequent disappointment. After ages of effort many of the secrets of nature are secrets still. Such seeking is an analytic process of thought, that is a process which separates and holds apart what is combined in actual fact, often in so complex a way that the combination appears to the investigator as confusion. So the method of discovery is essentially *Analytic*.

When the two methods are compared it is evident that the former presents a much more complete appearance than does the latter. As far as it goes it is a finished whole, but the latter always shows that its end is not reached. Hence it follows that the path of science is through analysis to synthesis, and that the more the contents of any department of knowledge can be exhibited synthetically the nearer has that branch of human effort approached its goal.

Abstractly, then, analysis and synthesis stand contrasted. Yet as the one leads to the other they are not antagonistic. Nor are they in practice held apart. The progress of knowledge is gradual, and at every step a synthesis of the results attained is the one sure basis of further advance. In no science is the one method used to the exclusion of the other, though in some sciences, as has been said, synthesis is more possible than in others. This is so in proportion as the relations considered are few and simple, as is emphatically the case with mathematics, where the relations dealt with are ideal and hypothetical and are only approximately discoverable in the real things of the world. They are relations which exist in their purity only for thought; consequently thought can deal with their results. But these results also are only hypothetically true —that is, they hold only under conditions which are never fully realised in fact. Still, to whatever extent mathematics can be applied to any branch of knowledge that science becomes synthetic. For example, astronomy and physics

are much more synthetic than chemistry or geology, and these more so than history or politics.

Broadly speaking, the physical sciences present us with particular objects and phenomena which demand from the enquiring mind an explanation. They suggest problems for solution, but the principles on which the solution depends are not apparent on the surface. Indeed "between these principles and the facts under consideration the distance is often immense. To pretend to traverse it at a stride, as Descartes says on the subject of method in general, is to wish to rise to the summit of a tower while neglecting the ladder intended for the purpose."[1] That ladder is analysis. We seek by careful observation, and where possible experiment, to detach what is permanent and essential from what is variable and accidental. The connexions so discovered are formulated as a law or principle, and further investigation is directed to establishing the dependence of this law on others expressing relations of wider and yet wider generality. Such is the method most appropriate to the sciences which deal with concrete reality. Of these we may mention chemistry, biology and geology as being largely analytical.

But the process is one of great practical difficulty, and rarely is it immediately evident what principle expresses the relations which an examination of the facts suggests. Various possible principles may seem to offer a solution, and frequently the only way to decide between them is to assume one of them to be true, to draw out the consequences which follow from it, and to compare the results with the facts. But to argue from principles to consequences is a process of synthesis, and thus one of the most important uses of synthesis is to eke out the deficiencies of analysis. The two together have made many discoveries possible which neither could have achieved alone.

At the same time there are some departments of knowledge rich in accepted general principles, in which the method is mainly synthetic. Such is mathematics. Start-

[1] Rabier, *Logique*, p. 298.

ing from certain axioms, definitions and postulates with regard to number and space, a closely connected system of truth is developed which in its most complex particulars is ultimately derived from those fundamental assumptions. Indeed the greatest step in transforming the general method of science is when considerations of exact quantity are first made applicable. The science is then brought into touch with mathematics, which has of all the sciences the most complete, because the most synthetic, method. Thus, for example, the science of astronomy has been since the time of Newton largely a mathematical application to the movements of the heavenly bodies of the laws of motion which he first formulated; so, too, physics is becoming increasingly quantitative and therefore synthetic in its methods.

But even mathematics is not entirely synthetic. When, for example, a theorem is proposed to a geometrician, he may assume the theorem to be true and work backwards to the conditions which must be fulfilled if the truth of the theorem is to be established. If in the course of this regress he arrives at a condition which is realised in the particular case under consideration, he regards the theorem as demonstrated. If it be a problem instead of a theorem, he seeks for conditions which can be realised and then effects the construction desired.

For example, let it be required to describe a circle through two given points M, N. If P be the centre of the given circle obviously it must satisfy the conditions of equidistance from M and N: the middle point Q of MN is such a point $\therefore$ Q is a centre. Can more circles than one be described? Let R be the centre of any other circle through M and N. Join QR. Again R must be equidistant from M and N. That would be so if the $\triangle$s RMQ, NRQ were equal: they would be equal if $\angle RQM = \angle RQN$. These will be so if they are right angles. Realise this condition and the problem is solved. The reasoning is analytic throughout and may be symbolised: A depends upon B, B upon C, C upon D. D is a realised or realisable condition $\therefore$ A at once follows. Indeed in mathematics, though synthesis is of such im-

portance, progress is frequently made by a careful analysis of suggested theorems or problems leading back to those which have been previously established or solved. Yet in directly communicating the results of research to others the synthetic method is adopted.

Whenever a great and wide-reaching general principle is suggested the work in that science becomes at once synthetic and analytic. The truth of the principle being hypothetically assumed, its results are worked out deductively, and so is reached a synthetic exposition of the system as it would be were the assumption true. But at the same time, and at every step, analytic enquiry into pertinent facts is pursued, guided, of course, by the hypothesis, yet not blinded by partiality for it so as to misread the facts. The aim of the analysis is not to establish the hypothesis, but first to test it. Its result may be modification or even rejection. In any case the synthesis, after it has taken up into itself the results of the analytic work, must be a very different thing—fuller, more clearly conceived, more exactly constructed—than was the preliminary synthesis. Moreover, it has passed from being merely a construction of possibilities to being a representation of reality. Such combined analytic and synthetic work goes on for years—even for centuries—and at every stage those two crutches of human thought in its slow march towards the truth are indispensable. In our own day we see one of the most striking examples of this which the history of thought has shown in the gradual establishment of the hypothesis of evolution in the biological sciences.

We see, also, how the possibility of synthesis in one branch of knowledge acts not only as an incentive but also as a clue to possible synthesis in others. Evolution is now the guiding thought in all that concerns human life—in art, in religion, in language, in law, in politics, in economics, in ethics, as well as in what concerns man's animal nature. It has lifted all these sciences to a level where thoroughgoing synthesis for the first time appears possible. It has done the same kindly service to geology, and even the more purely physical sciences have felt its influence. Thus

this great principle bids fair to do as much for the ultimate synthesis of the whole of knowledge into a systematic conception of the universe as the epoch-making theory of gravitation has done.

Though analysis and synthesis thus co-operate, there is, nevertheless, a profound difference between their points of view. In the one case thought advances to an understanding of the whole through consecutive mastery of its parts and of their relations; in the other it progresses to a more thorough understanding of the parts through their relation to the whole it is making more definite. Yet analysis retains its hold on the whole it analyses, and synthesis its grasp of the parts it is synthesising. As has often been remarked since Condillac, it is impossible to take a watch to pieces without at the same time perceiving, more or less clearly, how the parts we detach were related to the mechanism as a whole. Yet to take a watch to pieces is not the same thing as to construct a watch from its constituent parts. So, in reasoning we deal with wholes and with parts, and we may proceed from either to the other. But the parts are meaningless except as constituents of the whole, and the whole is only secured when the parts are fully known and related.

The illustration of the watch leads us to remark that it is necessary to distinguish carefully between analysis and synthesis as methods by which thought grasps reality, and physical processes which are sometimes called by the same names. Doubtless the former may be aided by the latter. The task of unravelling the sensible complex of nature includes the separation of the simple elements which compose it; and the success of such separation is evidenced by the possibility of reconstructing the original complex by a combination of the elements. It would be better to name such physical operations—of which chemistry offers the most obvious examples—Decomposition and Composition, and to restrict Analysis and Synthesis to the processes of thought. When decomposition and composition are possible they simply express in matter what has already been planned in thought. And in many cases thought deals with elements which cannot be physically separated,

and combines them in relations which are never perfectly realised in fact, as, for instance, those stated by the primary laws of physical motion.

Again, a physical whole may be actually divided into parts related in space to each other, as, for instance, England into counties. Such physical partition has no relation to the analysis of thought. The division is arbitrary, and a knowledge of the parts as related does not increase our understanding of the whole. To enumerate the counties of England is not an act of thought, but merely a series of acts of perception. It is the analytic and synthetic processes of thought with which logic is concerned, and if any physical processes are initiated, they are initiated as auxiliary to the processes of thought.

Analysis and synthesis increase understanding, that is they are methods of explanation. This proves that the composition and decomposition of substances in chemistry are not instances of them. These do not give the *reasons* for the combinations and separations of elements which are effected. Why should the mixture in a certain proportion of two gases with such diverse properties as oxygen and hydrogen produce, when an electric spark is passed through them, a substance so different from each of them as water? A true synthesis would enable us to pass from the properties of the elements to those of the compound as their necessary resultant. "We do not suppose a mysterious force entering on the scene and taking possession of the oxide of hydrogen as soon as it is formed. . . . We live, on the contrary, with the hope and confidence that some day, thanks to the progress of molecular physics, we shall be able to pass from the constituents of water to the properties of water itself as easily as we to-day deduce the workings of a watch from the form of the parts and the manner in which they are arranged."[1] This brings out the point. It is not the actual physical arrangement of the parts of a watch, but the understanding of that arrangement which enables one to deduce the working. In

[1] Huxley, *cit.* Rabier, *Logique*, p. 313, *footnote*. Cf. the whole chapter, pp. 293-316.

other words it is the analytic thought which makes possible the synthetic thought which sees the watch *as a watch* and not merely as a collection of queer-shaped pieces of metal.

We may distinguish, further, between full scientific analysis and a preliminary division of the matter under consideration into manageable portions. Yet this division will mislead us unless it is based on something more than convenience. To be fruitful it must express a true theory of the relations of the parts. Take as an example the broad and apparent distinction between body and mind, and consider how much both the biological and the mental sciences have suffered from their too rigid separation.

Analysis, then, is the method by which discovery most naturally starts when it is concerned with concrete reality. But to say that all discovery is made by analysis would be to ignore the advances made in knowledge by the aid of mathematics, for instance in astronomy, and to forget that once a principle has been demonstrated it is ever receiving fresh applications which lead again and again to a fuller apprehension of the system within which it applies. Such, for instance, is the application of the doctrine of evolution to many different branches of knowledge. Nor must we forget the important work of synthesis in testing principles provisionally taken for granted during the course of an enquiry. This procedure is in many cases an indispensable part of the process of discovery.

Synthetic also is the higher work of the scientific imagination. Men of powerful and original views in science have an exceptional facility in drawing out the implications of principles already established in directions hitherto undreamed of, or in combining principle with principle to open up new fields of thought. Of such a nature are the hypotheses by which it is sought to explain a thought as at once a mental and physiological phenomenon, or by which the atom is conceived as a centre of force with a difference of electric potential. The construction of such theories differs only in degree, of course, from the testing of principles to which we have referred, but they are of wider scope and much less obviously suggested by the facts under consideration.

Moreover there are some cases where synthesis is the only method of investigation open. No body, for example, is ever found to be acted upon by one force alone, and of the various forces which keep any given body at rest or in motion, the effects are not observable separably and therefore cannot be isolated. Consequently the investigator is bound to work out the results of forces known or conjectured to be in operation in order to show how their combination can give the phenomena which he is examining.

On the other hand synthesis sets out in orderly form the results reached by thought and thus enables us to get a kind of bird's-eye view of some department of knowledge. But synthesis is not always possible as it is in mathematics. There, wherever an analysis has been effected, the corresponding synthesis consists simply in reversing the order of thought. But it is not always possible to do this in other sciences. The psychologist has made considerable progress in analysing the functions of consciousness, in describing, for instance, the nature of attention, feeling, and so forth, but the synthesis which shows these functions as the necessary correlatives of the nature of consciousness is yet to seek. So also the biologist has much to tell of the functions of life as revealed to careful analytical enquiry, but cannot explain how they are synthesised in the different forms of life in the organic world. And the reason is that the analysis is not yet complete: the greater the imperfection of knowledge, the more partial will be the syntheses which are feasible. Just as in the whole system of knowledge only a perfect apprehension of the whole would enable man to pass from point to point by synthesis or analysis at will, so it is in all branches of knowledge in which for the purposes of study the whole is divided. What is aimed at is not an actual synthesis as a chemist makes a compound out of the elements. That would be to ask for nothing else than the creation of the world. But a conceptual synthesis is sought in which the world will find its explanation.[1] Scientific investigation is most

[1] Cf. Ch. 33.

complete when the synthesis has been so far accomplished that it is possible within a more or less limited range to apply the principles which in the main have been revealed by analysis. Thus synthesis is used to verify the results of analysis.

3. **Rules of Method.**—Whether method as an orderly process of thought be analytic or synthetic it is always controlled throughout by purpose. The general purpose of all science is to reach truth. But this is, of course, too vague to be a practical guide in actual thought. There are first the wide aims which separate one 'science' from another, and which lay down broadly the limits within which thought in any one process confines itself. This division into departments of investigation and of thought is itself a considerable contribution to method, and the increasing sub-divisions are signs of its progress. It is only in this way that considerations which are not relevant to the particular aspect of reality under examination can be safely ignored. The unity of the whole is assured by the unity of purpose which binds the separate sciences together. Further, in each separate investigation in a single science there is some distinct purpose which guides and controls it. Unless this is clearly apprehended thought is likely to stray upon the way—to include what is not pertinent, to neglect that which is, to fail to see the relative importance of the elements considered.

Thus the possibility both of orderly arrangement and of correct estimation of the relative importance of the elements involved depends upon a clear conception of the end the enquiry is meant to attain. Of course when the aim of the investigation is to reach new knowledge and not simply to convey knowledge new to another though familiar to the expounder, this conception of the end is not definite and precise knowledge: if it were there would be no room for the enquiry. But the nature and general character of the end to be attained must be clearly apprehended at the outset, and kept in view throughout the process. Every step must be a progressive filling out and determination of this end till at length the problem is solved. No learner

or enquirer can make true and regular progress by groping in the dark. Attention must be concentrated and kept to the point, ready to see the relevance or irrelevance of each new consideration, and to dwell on it or reject it accordingly. And relevance to the end can only be estimated when the nature of the end is with more or less exactness known.

Thus the first rule of method is—

Make the purpose clear.

The questions and problems which give the purpose always grow out of what is already known. Every advance in knowledge throws a search-light on ignorance still ahead. Unless we are clear as to what we *do* know, as well as to what we want to discover or to prove, we again cannot proceed in confidence or in the most advantageous way. Thus a second rule of method is—

Make sure of the starting-point.

If we begin, as in the case of mathematics, with principles and proceed to develop their consequences it will be incumbent on us to accept only those principles which have been already demonstrated, or which are too fundamental to need demonstration, or, indeed, to admit of it. But if our investigation lies within the sphere of the sciences which are concerned with the interpretation of material facts, the first thing to do will be to secure that the facts have been rightly and completely apprehended. Unless this is done there will be no guarantee that the superstructure of knowledge built on them as foundation will have any stability. Now the method used in ascertaining the facts is primarily that of observation. Nothing is more common, as we shall see, than to find observation at fault. It is therefore important to determine its exact nature and to indicate how it must be conducted in order to avoid error. The importance of observation is evident when we remember that it is the means of acquiring that sense experience on which all our knowledge is based.

But the direct range of observation is small, and even

where conditions can be repeated at will, as in experiment, the facts are so multitudinous that no one man can verify them all. Moreover, in many cases, as for example in history, direct contact with the facts is impossible. Therefore, no matter what the department of knowledge may be, the scientific investigator is dependent to a very considerable extent on the testimony of others. This testimony will reach him in many different ways and will be of diverse kinds. Not only will its transmission often enhance the possibility of error, but its value as a record of actual observation will vary from absolute untrustworthiness to the highest credence. Logical method must therefore indicate on what grounds testimony is worthy of acceptance or rejection.

When the facts have been determined the more intricate task remains of interpretation and explanation. For only in this way can the facts be shown to constitute in their universal relations an orderly system for thought. The constitution of such a system is effected by a process of reasoning which passes beyond the immediate data of sense, and traces out the laws which the phenomena exemplify. The process is carried further by showing the connexion of the laws amongst themselves, their application in new spheres, and where possible their dependence on laws more universal still. The facts from which a start is made are expressed in judgments, and so far as the facts are correctly expressed these judgments are true. From the truths thus given the mental construction of an organised body of knowledge proceeds by the derivation of other truths. In this process it is essential to advance cautiously and step by step, making sure of each before passing on to the next. As Locke put it: "General observations drawn from particulars are the jewels of knowledge, comprehending great store in a little room; but they are therefore to be made with the greater care and caution, lest, if we take counterfeit for true, our loss and shame be the greater when our stock comes to a severe scrutiny. One or two particulars may suggest hints of enquiry, and they do well who take those hints; but if they turn them into conclusions, and make them presently general rules, they

are forward indeed, but it is only to impose on themselves, by propositions assumed without sufficient warrant."[1]

These cautions were summarised by Descartes in the Second Part of his *Discourse on Method* in the following rules which may be added to the two fundamental ones already given, which are, indeed, preliminary precautions without which no methodical process of thought can be even entered upon—

Rule I. Never to accept anything as true which we do not clearly know to be so.

Rule II. To divide each of the difficulties under examination into as many parts as possible, and as may be necessary for its adequate solution.

Rule III. To conduct our thoughts in such order that by commencing with objects the simplest and easiest to know, we may ascend by little and little, and, as it were, step by step, to the more complex; assigning in thought a certain order even to those objects which in their own nature do not stand in a relation of antecedence and sequence.

Rule IV. In every case to make enumerations so complete, and reviews so general, that we may be assured nothing is omitted.

These rules, from their very generality, are no doubt difficult of application, but as the writers of the *Port Royal Logic* observe, "it is always advantageous to have them in the mind, and to observe them as much as possible when we try to discover the truth by means of reason, and as far as our mind is capable of knowing it."[2]

Speaking generally, we may say that the essence of all the rules may be summed up in the directions to make sure of our starting-point, to know the end we wish to attain, and to go from the starting-point to that end by orderly and consecutive steps, each of which is seen in its true relation to all the rest of the enquiry.

[1] *Of the Conduct of the Understanding*, Sect. 25.
[2] Eng. Trans., p. 316.

4. **Inferential Nature of Method.**—Whether our thought be analytic or synthetic it proceeds by deriving new truths from truths already accepted. To this derivation of truth from truth the name of *Inference* properly belongs. Consequently inference is a constant and essential characteristic of the operations of thought by which systematic knowledge is developed.

The distinction between synthesis and analysis corresponds broadly to that between deductive and inductive inference. So as analysis and synthesis mutually co-operate, and to some extent each implies the other, we must take care not to oppose deduction and induction to each other as though they were independent modes of thought. Both help to interpret the one system of reality. They differ in their starting-points, and consequently in their modes of advance.

All thought takes place within some system which it seeks to explain, whether it be the system of reality as a whole or some more restricted province of thought, as one of the particular sciences. The process of thought will aim at defining more clearly those relations holding between the parts of the system in virtue of which it is an organic unity. At any given moment the organisation of the system will be more or less complete. So far as it is complete we shall have a number of propositions expressing relations holding universally within the system; and so far as it is incomplete a number of particulars awaiting explanation through the universals which they express. To exhibit these particulars as inter-related parts of the whole we may proceed to develop the imperfect organisation in either of two ways. Suppose the universal to serve as the starting-point of explanation. In that case it is applied to the particulars which come under it, and thus we arrive at a fuller knowledge of the system. For example, our universal may be the law of gravitation, and we may apply it to the case of a bullet shot from a gun in a given direction with a given velocity, and thus determine the path which the projectile must necessarily take. Such a process is deductive and synthetic and on its formal side is examined in the theory of the syllogism.

On the other hand we may start with the unexplained particulars and try to read them off as members of the system. This can only be accomplished by finding the universal relations which bind them together. Thus the conjunction of the south-west monsoon with heavy rain is shown after a careful examination of the particulars involved to depend mainly on the general laws which govern the action of heat on the sea and atmosphere. Such a procedure is called inductive, and the examination of the facts involved is that which we have already described as analysis. "It is an insight into the nature of the whole or system based upon a careful examination of the parts."[1]

But Induction is commonly used to cover the whole of scientific method by which we seek to go from particular facts to general law, and it is, therefore, a wider term than analysis. For, as we shall see more fully later, the inductive method is based on deductive principles throughout. All methods of experimental enquiry involve deduction, and whenever in the course of the enquiry observation suggests a hypothesis to explain the facts, the results of the hypothesis are unfolded in syllogistic form. In short, we may repeat all that has been said on synthesis as supplementing analysis as applying to the relation between induction and deduction.

Nevertheless, there is a distinction of aspect between induction and deduction. In induction reality presents itself in concrete and particular isolated instances, and the task of inference is to discern the universal which is more or less hidden in those instances. To this universal a clue is sought, while in deduction on the other hand the universal is given. In the one case we are seeking some bond of connexion; in the other, we are given the connexion and seek to know what is bound together by it. Thus in deduction reality first presents itself in its universal aspect, and inference traces that universal through the differing and complex instances in which it manifests itself, while in induction the direction of the whole process is reversed. The distinction is, therefore, solely one

[1] Creighton, *An Introductory Logic*, p. 385.

point of view whether the judgment so demonstrated is, or is not, new. Obviously we may seek reasons for a fact which is familiar, though we may never have been conscious of the grounds which constrain us to accept it as it is. For example, we may seek the meteorological conditions on which the coldness of the east wind depends. But when we reach the conclusion, '*therefore*, the east wind is cold,' though we know the fact now in a new way, and see its relation to facts hitherto not known or disregarded in this connexion, it is not the novelty of the relation but its cogency which constitutes the inference. Indeed, the whole series of premises might be as well known as the conclusion to start with, and the logical character of the process would remain the same. For it consists essentially in the exhibition of the necessary dependence of the facts on one another.

Nor is it different in those cases where a conclusion is reached which is new in the sense that Newton's laws of motion were new when he first enunciated them. Here the same test has to be satisfied as in the first example we considered. Are the propositions so combined that the conclusion issues from them inevitably, and are they of such a nature that they have a like necessity immediate or derived? Then it does not matter whether the final judgment is familiar or unfamiliar: the conditions of correct inference will have been fulfilled.

It is not correct, therefore, but is to confuse logic with psychology, to describe logical inference as a passage from the known to the unknown, even if we define the unknown as that which is seen for the first time to be dependent on given truth. The mathematical demonstration is as much inference at its last repetition as on its first discovery. The essential point is that the conclusion of our reasoning be necessarily conditioned by the reasons advanced in its favour. What is meant by the description is that these reasons must be known to be true if there is to be any valid inference in the wider sense of the term.

The problem then arises, how can there be, from known truths taken as premises, the assertion of another truth distinct from them, and yet owing its assertion to them.

If we affirm that "Flowers are transient" and that "Sables are expensive," and try to link the two facts together our thought is baffled. As we say, we cannot see the connexion between them. But suppose that instead of the first fact we have the statement that "All expensive things have a limited sale" our thought at once goes forward to the assertion that "Sables have a limited sale," because of the common quality of expensiveness. Now this quality is a universal: that is, it is common to both propositions. And, unless there is to be the abrupt arrest of thought experienced, for instance, in the attempt to draw an inference from the transiency of flowers and the expensiveness of sables, we must find some universal through which the facts are connected. Only through the existence of such universals is inference possible at all. If one fact had nothing in common with another we could never seek a reason for it outside itself. Each would be presented to the mind in complete isolation, and organised knowledge would not exist. Hence science resolves itself into a search for universals, and in proportion as these are discovered, the inter-relation of fact with fact becomes more evident, and a firm basis for inference is secured.

In all inference there is both a formal and a material factor. The first is concerned solely with the justification of the transition itself, and it is more prominent in deduction than in induction. To formal considerations we owe the whole doctrine of the syllogism, and though all syllogistic inferences are abstract in that they do not take account of all the complex conditions which are found in every piece of reality, yet no inference is valid which violates the formal principles of syllogistic reasoning. The practical danger is in extending them beyond their proper scope.

The material factor is concerned mainly with the grounds for the judgments between which the transition from truth to truth is made. Of the adequacy of these in any particular case logic cannot judge. It can only lay down general principles of adequacy: whether those principles are satisfied in any one instance demands a sufficient knowledge not only of logic but of the branch of knowledge in question.

6. **System.**—Inference would be impossible apart from a system. Now the conception of a system is one that we have frequently employed. By it we mean a whole in which each part has a definite and necessary relation to every other if the whole is to be constituted at all. A machine may be taken as an example. Displace a lever, unscrew a bolt, re-distribute the parts, and the machine as a machine ceases to exist. The nature of the whole determines the nature of the parts, and the nature of the parts determines the nature of the whole: there is an essential thread of relation binding all together and each to each. Contrast a heap of firewood. It forms a whole in which the sticks are the separate parts, but the position of each in reference to the rest is a matter of indifference, and there is no necessary relation between them.

If we suppose, then, some knowledge of a system, the possibility of inference is at once apparent. For given the truth about any part we can pass to the truth of some other part through the universal relations in which it stands to the whole it helps to constitute. These relations express its nature, and in them the nature of the whole is potentially revealed. Consider the case of a workman who in the course of excavations turns up a human jaw-bone. He may not recognise it as such, or if he does, he regards it curiously, and casts it away. He has no useful system into which the fact will fit, and so receive a fuller meaning. But the geologist with the same preliminary data can commence a process of thought by which he constructs mentally the conditions of life that must have prevailed during the time when the human being of whom this sole relic remains was still alive. He determines perhaps that it must have been preserved from pre-historic times. The anatomist, too, from the size and shape of the bone can tell a great deal about the physical frame with which alone it would have been consistent. He may, perhaps, show that only muscles of much greater strength than is common to-day could have moved it.

In some such way a number of inferences impossible to the workman might be drawn by the man of science because he possesses a knowledge of the systems appropriate to the

fact to be interpreted. But he too would have failed to make an inference if he had not had a system of inter-related truths enabling him to pass from what was given and recognised as a part of the system, to its meaning in the light of the whole.

The extent of possible inference will coincide with our knowledge of the system within which our thought is working for the time being. The citizen knows enough of the tramway system of his city to be able to decide what car he must catch, and where, in order to get to any place within a specified time. But beyond that, and indeed often within those limits, his thought is apt to be vague. Ask him what power is needed to move the cars, how it is generated and distributed, what inclines can be safely attempted and why, and he is soon at the end of his tether. Not so with the electrical engineer, whose knowledge of the system goes deeper, and whose capacity, therefore, to answer questions that arise is greater. If it is proposed to introduce a new type of car, he can tell in his study what conditions it must fulfil to be safe; or if any extension of the service is contemplated, he can work out from the data which a preliminary survey supplies, what alteration of his source of supply will meet the new demand. Both citizen and engineer infer; but the wider scope of the one is the result of a wider acquaintance with the relations subsisting between the various parts of the system.

It is evident, then, that within the one ideal system of perfect knowledge there are many actual smaller systems which are known with more or less completeness. Of these the most important are those which serve a scientific purpose. A minor system which satisfies a practical need, such as that represented in a railway time-table, may be known through and through, but that is never the case where the governing aim is a knowledge of reality. The power to infer breaks down at some point or other; the engineer, for example, cannot answer every question. Where that is so a problem arises for solution. Now, however the solution is reached, whether by a process of analysis or synthesis, or by both, it presents itself as

a claim to form part of a system. The claim is established when the system as it stands, or if need be as reorganised, incorporates it in such a way as to make a chain of inference more complete; it is disallowed when it is found to be repugnant to existing systems and to suggest no modification of them in which it would find a consistent and necessary place. All advance in knowledge is made by the perfecting of systems through inference, whether in the way of completing their internal organisation or in establishing connexions between them which bring us nearer to the ideal system of perfect knowledge.

But knowledge, though essential, will not alone give to the individual the power of inference in any department of thought. Especially is this the case in the labour of discovery. Men differ in the sagacity they display in seizing on the relevance of facts and in discerning the right kind of relation. The search for the universal which explains is easier to the genius of one man than to that of another. "Darwin was noted for his keenness in detecting connexions which escape the ordinary eye, as well as for his skill in giving explanations of them. On one occasion, he observed that in that part of the country where he lived, clover was abundant in the fields which were situated near villages, while the outlying fields were almost destitute of it. What now, he asked himself, is the connecting link between these facts? Some investigation of the matter convinced him that the three agencies which produced this result were humble-bees, mice, and cats. The bees fertilise the clover flowers, and thus make the plant abundant, the field mice destroy the bees' nests, but the cats go out from the villages into the fields near by, and destroy the mice."[1] Thus the existence of the personal factor precludes the laying down of detailed precepts for the conduct of processes of inference. At the same time it is possible to frame general rules of method which should be observed in the conduct of scientific investigations.

[1] Creighton, *An Introductory Logic*, pp. 381-382.

CHAPTER XVI.

SPECIAL METHODS.

1. **Applications of General Method.**—Thought in its attempt to attain systematic and exact knowledge always proceeds by analysis of some part of what is given in experience and synthesis of the results of that analysis. These are always combined, but the relative shares of each and the mode of their interaction are determined by the nature of the material with which thought deals. Thus arise three great departments of thought—the mathematical, the physical, and the historical. Not that these are isolated: on the contrary they not only overlap but intertwine.

2. **Mathematics.**

(i) **Analysis of Data.**—The most general, because the most abstract, is mathematics, which began in investigations of relations of quantity. Now quantity is at once discrete and continuous, and these aspects may be dealt with apart. So have developed the algebraic and the geometric mathematical doctrines.

In the algebraic sciences of number the element of continuity involved in the need for constant and uniform units in all enumeration is assumed. In the geometric sciences of continuous quantity—or occupation of space—the possibility of indefinite division is similarly taken for granted. For example, any number of points—or positions—may be conceived in a line. But mutual implication is not identity. A line is not made up of points, nor a surface of lines; in neither case can the multiplication of zero give aught but zero. We can so analyse a surface as to

think of its length, but no synthesis of mere lengths can give the idea of anything but length. That is to say, the unit reached by analysis of a line is one of length only.

It is certain, however, that surface is apprehended by the senses, that lines can be traced by the eye as boundaries of surfaces, and, indeed, that all mathematical ideas have their birth in sense-experience. From this it has been maintained—as by Hume and J. S. Mill—that mathematics is an empirical science in the same sense as physics. But this implies that the mind passively receives impressions. On the contrary, it deals actively with them by putting on one side all it does not need for its present purpose. A surface is known in sense-experience not simply as a surface, but as the surface of some solid; a line is not merely a line, but a boundary of some surface; a point is not mere position, but a small surface. Not the senses, but the analytic power of mind, seizes on one aspect and ignores the rest. This aspect can be made an object of thought but cannot be presented in sense-imagination. We can *think* of a point or a line, but we cannot *picture* one. Nor can ideal notions be reached from comparison of imperfect objects. Mill argued that the idea of a straight line results from eliminating divergences from straightness. This begs the question hopelessly, for, without the idea of straightness, nothing could be apprehended as divergence.

Though then it is probable that mathematical ideas were first suggested by reflexion on sense-experience, yet those ideas are not pictures—or pale ghosts—of such experience, but are tools forged by the analytic power of the mind for dealing for its own purposes with all those relations given in experience into which space or number enters. This, too, makes it easy to see why mathematical relations are never exactly exemplified in physical relations. It is because the latter always contain more than the former, so that the former have to be dug out from the matrix in which they are embedded.

It is, then, the peculiarity of mathematical ideas that they are constructions of thought dealing with relations that can be defined exactly, and permitting of deductions which

need no confirmation of their validity by any further reference to experience.[1] This does not mean that they are entirely arbitrary. "We want our numbers not merely to verify mathematical formulae, but to apply in the right way to common objects."[2] But when number has been so conceived that it allows this application, the science of number can be developed without further consideration of the things to which it may be applied. It is the same with geometry. "Experience plays an indispensable rôle in the genesis of geometry: but it would be an error to conclude from this fact that geometry is an experimental science, even in part."[3] Yet, in spite of the fact that its conceptions are abstract and can be logically elaborated without necessary reference to external reality, geometry like number has its practical applications.

Since the whole structure of mathematical knowledge depends on the accuracy of the notions which are its foundation, it is necessary that these should be clear and precise. Thus definition is of first importance. On the other hand, as these notions are the results of analysis, it is inevitable that their distinctness should be progressively attained. As analysis pierces more deeply it is apt to lay bare uncertainty and absence of rigorous definition even in the most fundamental notions. Much work of this kind was done by the leading mathematicians of the last century. "The labours of the great analysts—Gauss, Cauchy, Riemann, and Weierstrass—all tended to increase our knowledge of the higher mathematical relations, but also to reveal the uncertainty and absence of rigorous definition of the foundations of arithmetic and of geometry. Accordingly we find these great thinkers continually interrupting their more advanced researches by examinations of these principles."[4] As a result of this movement older proofs have been remodelled or abandoned. Assumptions tacitly made have been justified or shown to be untenable.

[1] Cf. pp. 469-473.

[2] Russell, *Introduction to Mathematical Philosophy*, p. 9.

[3] Poincaré, *La Science et l'Hypothèse*, p. 90.

[4] Merz, *A History of European Thought in the Nineteenth Century*, Vol. II., pp. 708-9.

A clearer insight into the nature of the foundations of mathematics has been gained. Such notions as continuity, the infinite, and form, have been examined more exactly and defined more precisely, and, as a consequence, a more adequate conception of their significance has been reached. As Mr. Whitehead says: "The whole difference between the older and the newer mathematics lies in the fact that vague half-metaphorical terms like 'gradually' are no longer tolerated in its exact statements. Modern mathematics will only admit statements and definitions and arguments which exclusively employ the few simple ideas about number and magnitude and variables on which the science is founded." [1]

But, further, the ideas derived by the first act of analysis and expressed in such definitions as those in geometry of point, line, etc., are themselves subjected to analytic enquiry. "Instead of asking what can be defined and deduced from what is assumed to begin with, we ask instead what more general ideas and principles can be found, in terms of which what was our starting-point can be defined or deduced." [2]

(ii) **Synthetic Use of Data.**—There is, then, always analytic work to be done in mathematics, but it is within the power of those few minds which have mastered all that has been achieved already. The ordinary worker is concerned rather with applying knowledge already established. Either in pure mathematics theoretical consequences are inferred by a continuous train of reasoning, or in mixed mathematics relations of quantity are applied to the explanation of physical phenomena or to the construction of physical instruments. In both cases the procedure is essentially synthetic, the derivation of conclusions from premises. The first premises are such axioms and definitions as have stood the test of analysis. These are closely inter-related. For an axiom is a statement of which the truth is manifest when the terms in which it is

[1] Whitehead, *An Introduction to Mathematics*, pp. 155-6.
[2] Russell, *Introduction to Mathematical Philosophy*, p. 1.

expressed are precisely apprehended. Such a statement is called 'self-evident,' which means, not that it is so plainly a truism that nobody can fail to accept it, but that no evidence outside itself can be brought forward to support it. From these descend interminable chains of inferences, in which the conclusions of each step form the premises of other steps. Throughout, the material dealt with is abstract—relations of magnitude simply as magnitude, and wholly indifferent to questions as to what things the magnitudes represent. The reasoning is syllogistic when a more general relation is applied to a less general case, but it may be an immediate synthesis of the general relations combined in the specific case; as in the forms of relation specified in Chapter XXIV., § 3.

(iii) **Symbols.**—Attention is more easily confined to the aspect under consideration, and the abstract character of mathematical reasoning thus more emphasised, when ideas or things are represented by symbols. This very emphasis of the abstract brings out its general nature, so that its steps are more immediately apparent. Comparison of a numerical operation expressed arithmetically in figures with the same relation translated into general algebraic literal symbols gives a familiar illustration.

So in geometry the relations dealt with may be embodied in figures. But here the tendency to lean on the figure is apt to become operative. And this is a danger. For the figure used is a specific material thing, and, as such, has many qualities, even of magnitude, other than those with which the problem under consideration is concerned. Nor is the danger one only to the schoolboy who is misled by the accidental shape of the triangles he has drawn to deduce general conclusions from these accidental specific relations, but, as complexity of geometrical construction increases, the tendency to appeal to the figure as evidence is greater. But much as a figure may aid in making the conditions of the problem clear it can never furnish proof. As Mr. Whitehead puts it: "the space-intuition which is so essential an aid to the study of geometry is logically irrelevant: it does not enter into the

premises when they are properly stated, nor into any step of the reasoning."[1]

In its search after exactness and precision, then, mathematics has freed itself ever more completely from the material given by sense-perception with which it started. It has learnt that "intuition cannot give us exactness, nor even certainty, and this has been recognised more and more. It teaches us, for instance, that every curve has a tangent—that is to say, that every continuous function has a derivative—and that is untrue. As certainty was required, it has been necessary to give less and less place to intuition."[2] So the ideal of mathematical method may be summed up in the words of Mr. B. Russell: "No appeal to common sense, or 'intuition,' or anything except strict deductive logic, ought to be needed in mathematics after the premises have been laid down."[3]

This by no means implies that mathematics has no relation to the real world of things. On the contrary, of that world, mathematical relations are constituents, and mathematical calculations are capable of deducing physical truths, as, for example, when by a wonderful exercise of pure mathematics Clerk-Maxwell predicted in 1865 that the speed of the electro-magnetic impulse is a constant of 300,000,000 metres (roughly 186,000 miles) a second, identical with that of light.

(iv) **System.**—Method in mathematics thus shows the elements of analysis and synthesis in, as it were, a skeleton form. All extraneous matter is stripped from relations of quantity, all that would interfere with their perfect operation in the world of things is ignored. The result aimed at is a system of relations self-contained, and, within those limits, ideally perfect. This system is not yet complete. Not only is the synthesis of results already achieved not perfect, but there is no reason to suppose that no further mathematical truths remain to be discovered. But the process is further advanced than in any other department

[1] Whitehead, *op. cit.*, p. 242.
[2] Poincaré, *Science and Method*, Eng. trans., p. 123.
[3] *Op. cit.*, p. 145.

of knowledge. "Mathematical sciences as they develop dovetail into each other, and share the same ideas in common. It is not too much to say that the various branches of mathematics undergo a perpetual process of generalisation, and that as they become generalised, they coalesce."[1]

3. **Physical Sciences.**—Next in order of decreasing simplicity to mathematics comes the group of sciences which deal with inorganic nature. Yet, even here, the complexity of phenomena is so great that it can be attacked only in detail. So arise the mechanical and chemical sciences. Further, each of these is a general branch of knowledge including divisions and sub-divisions, each dealing with its own sets of facts and problems. And as insight into detail increases, the sub-division of investigation becomes ever more minute. Analysis is prominent at every stage.

Yet this division of labour is only a matter of convenience, or at most of necessity, imposed by human limitations of time and capacity. Not only does it not affect the unity of the material world, but no true scientific investigator ignores that unity. Each science lends its aid to the others. Physics and chemistry, for example, are mutually helpful, and each owes many of its greatest achievements to the aid of mathematics. Synthesis is operative throughout.

The aim of the physical sciences is to understand the inorganic world by bringing its phenomena under one far-reaching generalisation. It must, therefore, begin with facts of experience, and it must be justified by the power of its generalisations to explain those facts in all their complexity as due to specific combinations of general forces. Its method is, therefore, that process of induction which is examined in detail in Chapters XXV.-XXXIV., and which need not, therefore, be further discussed here. Suffice it to say that—whereas in mathematics validity of conclusion depends wholly on accuracy of reasoning from

[1] Whitehead, *op. cit.*, p. 116.

self-evident statements of clearly apprehended relations, so that the task is predominantly one of inference—in physics and chemistry the difficulty of the task is essentially in ascertaining the general relations involved in actual occurrences which are always extremely complex. Hence, every such relation is apprehended tentatively. It is, in short, at first a guess—or hypothesis—the justification of which must be sought in ever-deepening analysis of the concrete facts with which alone experience furnishes us.

Hypothetical synthesis, therefore, interacts with analysis ever pushed more deeply into experience. In that analysis physical objects and processes have to be manipulated. But, as was pointed out in the general discussion of method, this manipulation is not logical analysis and synthesis.[1] Those are modes in which thought works in view of the disentanglement of physical facts which is facilitated by mechanical means.

The further the work advances the more the interconnexion of the abstract relations discovered by thought becomes apparent, and the possibility is naturally suggested of bringing all under one conception. Such a conception is the mechanical theory of the inorganic universe—the finding the explanation of all the pertinent facts in combinations of one mechanical force. This claim of dynamics to be ultimate "amounts to this: namely, that the various qualities of things perceptible to the senses are merely our peculiar mode of appreciating changes in position on the part of things existing in space," thus making the laws of motion "the ultimate laws of physical science."[2] On the adequacy of this theory, even in the realm of the inorganic, the discovery of radium and the Einstein theory of relativity have thrown increasing doubt, so that at present the claim cannot be regarded as substantiated.

4. **Natural Sciences.**—However this may be, the extension of the mechanical theory to the organic world, including man, is wholly unjustified. For there the new fact of vital energy is dominant. The doctrine of evolu-

[1] Cf. pp. 167-169. [2] Whitehead, *op. cit.*, pp. 47, 48.

tion has profoundly modified the point of view from which the phenomena of living beings must be regarded. These are now seen to be products not of mechanical forces impinging on inert matter, but of the reaction of a non-mechanical force on the inorganic environment. Analysis of the existent is inadequate by itself to lay bare the course of events. Hypothetical assumptions of laws of vital force must be made and tested. And in the testing constant appeal must be made to all the evidences of the past which bear on the subject as well as to observation of changes in the forms of life which may be observed or brought about in the present. The method is still inductive, but the scope of the inductive inquiry is much widened.

At present no certainty has been attained. Darwin's theory of natural selection is not generally accepted as more than a preliminary and imperfect hypothesis, and the great question of the inheritance of acquired characteristics is still unsettled. The historical systematisation of the facts of organic life is, therefore, still in the making. What is certain is that no mechanical theory is adequate to explain the phenomena of life.

5. Historical Science.

(i) **Gathering of Facts.**—In its reference to evolution in time natural science is intermediate between physical science and history, which, in the usual sense of the term, is the study of the thoughts and actions of men. Here a further element is introduced, in that man's acts are determined by purposes consciously conceived. Not merely life, but a life guided and determined by will, is to be explained. 'As the matter of the physical sciences is more complex than that of mathematics, and the matter of the natural sciences more complex than that of the physical sciences, so the matter of history is more complex than that of the natural sciences. We have here reached the highest end of the scale. It is evident, then, that mechanism is even more inadequate as an explanation of the evolution of civilisation than as explanatory of the forms of vegetable and animal life. Yet, during the dominance of the

mechanical theory in the last century, such attempts were confidently made, as in Buckle's *History of Civilisation*.

History not only deals with the most complex material but it is largely unable to help itself by experiment and can get but little help from direct observation. It is true that the consequences which follow some definite social change—as *e.g.* a law of prohibition of all forms of alcoholic liquor—may be observed and statistics gathered. But there is no certainty that the results are what at first sight they appear to be, for never does the assumed cause operate alone and unmodified by other forces of human will. Moreover, the range of such observation is obviously very limited: it gives no direct help in gaining knowledge of the past, and it is with the past that history is mainly concerned.

The facts of history include every form of record of human agency. Written documents, material remains, oral traditions, systems of law, religions, philosophies, languages—all must be taken into account. And the number of such records is indefinitely great, though, of necessity, they embrace but an infinitesimal part of the facts of the past. Specialisation is consequently as imperative in historical research as in other forms of investigation. So, here as elsewhere, the method is guided by definite purpose which defines the scope of its application. The purpose may be to study a period limited in time and in geographical range; or it may be to take a more general survey of a wider range; or it may be to study some one aspect of human activity—such as the political, economic, social, or religious. Whatever the purpose, it determines the range and kind of facts which will be sought.

An indispensable *first step* in historical *investigation* is the discovery, collection, and classification of sources. This work, though far from complete, has been carried out on a large scale. Archives, libraries, museums, general and special bibliographies, by the material they furnish and by an orderly arrangement of it, where such exists, may all be regarded as contributions to method in history.

It is assumed that the historian has the knowledge necessary to understand his sources; that, for example, he

knows the language in which a document is written, can decipher a manuscript or an inscription, is familiar with the varying forms of official documents, in short, that he has studied the 'auxiliary sciences' needed to equip him for the special researches which he may undertake. Thus equipped he is prepared to begin the process of making sure of the facts by criticism of the sources themselves.

External criticism seeks to determine the date of a document, the author, and place of origin. Each of these particulars may be quickly and surely determined, or may demand careful, perhaps prolonged inquiry. The text will be examined for any indications of time, place, or persons from which inferences to the date, authorship, or place of origin may be drawn; the style may give a clue to the period in which it was written; references in other works, if they exist, must be scrutinised and assessed. The result of this process of reasoning may vary through all degrees of probability to moral certainty. Whatever use is made of the document for the establishment of facts will be conditioned by the degree of certitude attained. Before reliance can be placed on a document which has been shown to be of a certain authorship and date, it must be ascertained that it is not a copy, in whole or part, of an earlier document, and the accuracy of the text must be established. Printed documents are not immune from error, and prior to the days of printing, when manuscripts were multiplied by hand, corruptions of the text crept in in numerous ways. Mistakes in copying, additions, falsifications, omissions, are common; they cannot be detected by a comparison with the original manuscript, since, as a rule, this has disappeared. Copies that remain are of varying date, value, and completeness. The earliest manuscript of Plato, for example, belongs to the ninth century; of Thucydides, to the tenth century. To obtain as accurate a text as possible manuscripts are compared with one another, and with quotations or paraphrases of passages that may be found in earlier writers, and, whenever the text is still obscure, emendations may be conjecturally made.

When external criticism has done its work it is followed

by internal criticism, the object of which is to find exactly the meaning of the author, to sift out the facts he records, and to determine the value of his testimony to them. This raises questions as to the trustworthiness of testimony which are discussed in Chapter XXVIII.

The facts recorded in documents refer to persons, events, ideas, and objects, that cannot be known directly. They must be reconstructed in imagination. This process makes the chances of error greater than in direct observation. Often the facts themselves are not complete, and it is necessary to fill in the gaps by inference from the data already established. But the mental construction of the past is conditioned throughout by reality, and the objective reference is as essential as in the physical and natural sciences: the difference lies in the method by which the facts are ascertained and verified. Many historical works are devoted mainly to the establishment of facts, and in them *interpretation* takes a subordinate place. Since the facts are disparate in character, they must be classified. This is a preliminary step in methodical thought about them. The most obvious bases for classification are time and place, with reference to individuals or groups. The determination of periods of time, the limitations of place, and the decision on what constitutes a group will depend partly on the facts, and partly on the object in view. Even so, the classification will contain heterogeneous material, and this can be arranged under headings derived from the nature of the facts themselves—such headings, for example, as war, finance, language, science, religion, industry, commerce, education. The facts so grouped may be thoughts and actions shared and repeated by a number of individuals, or peculiar to one individual.

Less liable to misinterpretation than documents are such records of man's activities as buildings, codes of laws, the languages in which he has expressed his needs desires and thoughts, which have grown with his growth, and at every stage embody the views of life and of the universe which he has held.

In all this dealing with records in order to ascertain facts the whole process of induction is essential. There is no

greater error than to confuse record with fact, and to assume that whatever has been recorded is true. Records must be examined critically, and guesses—or hypotheses—formed as to what the facts which they reveal—or disguise—really were; and these hypotheses tested by relating them to other facts similarly suggested or more surely witnessed by language, laws, buildings, artistic products.

(ii) **Explanation.**—Historical explanation seeks to pass from the facts to the conditions which have determined them. Ultimately these conditions are to be found in the motives and actions of men in relation to their physical environment and to one another. The ideal would be to exhibit the facts in a causal series which would show the successive changes that have taken place as conditioned by what preceded them, and conditioning what followed. Such a construction can never be more than an imperfect approximation. All the facts are not known, and many that must be used are the result of a more or less uncertain conjecture made with a view to understand the changes that a comparison of different periods of time shows have somehow taken place. The historian may not only be embarrassed by a paucity of facts; he may be overwhelmed by their superabundance. It is then necessary for him to make a selection. Inevitably he will choose what in his judgment has had a determining influence on later events and ideas; the personal factor in historical construction is a greater disturbing element than the personal equation in the investigations of physical and natural science.

In his reasoning the historian is guided by his knowledge of human nature, which is assumed to have remained the same in its essential features throughout the historic period. By means of this knowledge, however acquired, he endeavours to relate fact to fact, in order to show the causal connexion between them. The facts have been established by analysis: the reasoning from them is deductive or synthetic. An appropriate general principle is sought under which the historical judgment of fact can be subsumed, and from these premises the conclusion is drawn. It is true that in exposition the transition often seems to

be directly from fact to fact, but on examination it will be found that a general principle or 'law' must be assumed if the connexion is to be established.

Sequence in time does not in itself prove causal relation Here great caution is necessary. As Dr. J. Holland Rose remarks on the course of events in the French Revolution: "We, who know how one event led on to another, find it difficult to escape from the attractive but fallacious conclusion that the sequence was inevitable. The mind loves to forge connecting links, and then to conclude that the chain could not have been made otherwise—a quite gratuitous assumption."[1]

The certainty of the conclusion cannot go beyond the certainty of the premises. There may be an element of doubt about the facts themselves, and the general principles of human nature rest on a precarious foundation ranging from a high degree of probability to pure conjecture. If historical generalisations form part of the reasoning, they are infected with a like uncertainty. Nevertheless, there are many facts that admit of no doubt and principles which it would be pedantic to reject; and a conclusion weak in itself may be strengthened by convergent lines of reasoning.

Ideally, historical truth requires the determination of the whole of the conditions operative in bringing about a given change; but the field of history is too vast for this requirement to be fulfilled, so far as it can be under the limitations already outlined, without the cooperation of many workers. The historian is obliged to confine his original investigations to a definite period, or to some definite phase of the life of the past. Where a synthesis is attempted over a broad sweep of time, he is necessarily dependent for his material on the facts, and, to some extent, on the conclusions of his predecessors. Reasoning in history is, in the first instance, inductive—from effect to cause; and to reach the true cause, the whole of the influences at work must be taken into account. The specialist, who confines himself to tracing out one line of

[1] *William Pitt and National Revival*, p. 541.

development, for example, political, economic, social, or ecclesiastical history, abstracts from the fullness of events that phase in which he is interested. But in the concrete life of the past each phase is connected more or less closely with every other, and unless this connexion is recognised, the causes assigned for any given development may be inadequate. To ensure accuracy, either the specialist himself, or another using the results of the work of specialists, *must bring out this reciprocal influence between the separate* phases of experience.

History does not repeat itself: that conditions never recur exactly is what is meant in speaking of the uniqueness of historical facts and causes. Conclusions valid in one set of circumstances cannot be applied in other circumstances, which, though they may be analogous, will never be exactly the same; still less can conclusions drawn from the past be used to predict the course of events or ideas in the future. In this respect history differs from the physical sciences. Yet the historian, no less than the natural scientist, generalises. His statements of fact are often generalisations, as in the passage:—"The long-continued war raised the taxes, checked the growth of manufactures, and drained the country of its best peasant blood, only to return to it a body of brigand soldiers ruined in morals and incapable of honest industry."[1] Beliefs, customs, institutions, language, and other phenomena common to members of a community or group, may persist relatively unchanged, and the historian may seek to determine their general features during the period of their existence. Such generalisations are valid only within the field from which they are drawn.

Although *the* complex of conditions which account for the concrete case which it is sought to explain is never exactly reproduced, and the historian may consider his task finished when he has determined what the conditions are, it is possible to raise other questions which can only be answered when this preliminary work has been done. To recur to the quotation given above, we may enquire

[1] Wakeman, *The Ascendancy of France*, p. 168.

whether there are any general effects which war tends to produce. This will lead to a comparison of wars in different ages and in different countries, and to an endeavour to formulate points in which they agree. As a matter of fact we know that there are general propositions about war believed to be true and to be supported by the experience of the past, although in themselves they do not suffice to explain the course or results of any particular war. By means of the comparative method the historian seeks to discern the general conditions of historical development, to detect its general tendencies, and to reveal whatever purposes may be immanent in the facts.[1] Generalisations reached in this way belong to the 'philosophy' rather than to the 'science' of history; unless they arise from facts, and are continually verified by them, they pass outside the domain of history altogether.

[1] Cf. Wundt, *Logik*, 3 Aufl., Bd. 3, p. 394.

CHAPTER XVII.

FALLACIES INCIDENT TO METHOD.

§ 1. **Petitio Principii.**—By a *principium* Aristotle means a truth which can be known of itself. *Petitio principii* is, therefore, committed when a proposition which requires proof is assumed without proof. In other words, in this sense, the *petitio principii* is the undue assumption of axioms. Now an axiom is a self-evident truth, such as are the fundamental assumptions of mathematics and of logic. But it is very easy and very common to confuse both the desired and the familiar with the obviously true. So that what requires proof is assumed as not at all open to question, and from it all sorts of consequences are deduced and regarded as proved. The Middle Ages, with their reverence for authority in matters of thought and knowledge as well as in those of faith and conduct, were prolific in such spurious axioms. That everything found in Aristotle's works was true, and everything not found there false—as when Scheiner's monastic superior rejected his discovery of spots on the sun in the words: "I have searched through Aristotle, and can find nothing of the kind mentioned: be assured, therefore, that it is a deception of your senses or of your glasses"[1]—was, perhaps, the assumption that did most to prevent men from seeking fresh knowledge. It was long regarded as axiomatic that the colours in the rainbow were arranged in a different order from that in which they actually appear—so far can pre-conception go in determining what is actually perceived. Other assumptions were due to a kind of common-sense estimate of experience—as

[1] See Baden Powell, *History of Natural Philosophy*, p. 171.

that bodies fall with a rapidity proportioned to their weight; that some substances, such as flame, are positively light. Others were mere figures of speech—as that 'nature abhors a vacuum.' One of the greatest advances in human thought is the recognition that nothing is axiomatic which admits of verification.

In our own days the fallacy of assuming axioms is most frequently found in arguments relating to human affairs. That all change is for the better is regarded by many earnest social and political reformers as axiomatic, and the nature of the assumption is hidden by the question-begging term 'progress.' This—which properly means nothing more than advance in *some* direction—is assumed to be advance in a *desirable* direction. The common fallacy of party politics of assuming the party proposals to be necessarily wise and good, and those of the opposed party to be inevitably foolish and evil, and the related assumption that "the will of the people" is expressed by the party cries, common though they be, are almost too foolish to merit notice in a serious work on Logic.

By logicians generally the term is, however, used more widely to denote *petitio quæsiti*, or the assumption in some form of the very proposition to be proved, as a premise from which to deduce it. This may be done, Aristotle tells us, in five different ways:—by assuming the very proposition to be proved; by assuming, when the conclusion is particular, a universal which involves it; by assuming, when the conclusion is universal, a particular involved in it; by assuming piece by piece the proposition to be proved; by assuming a proposition which necessarily implicates the proposition to be proved. The assumption, it will be seen, is in every case of the same character. The two first modes of committing the fallacy are, however, of the greatest importance and of the most frequent occurrence.

The direct assumption of the proposition to be proved would seem to be hardly possible when it is expressed on both occasions in the same terms. Still, people can be found who commit it. Commenting on an attempt to square the circle published by a Mr. James Smith in a

work entitled *Nut to Crack*, De Morgan says: "Mr. Smith's method of proving that every circle is $3\frac{1}{8}$ diameters is to assume that it is so,—'if you dislike the term datum, then, by hypothesis, let 8 circumferences of a circle be exactly equal to 25 diameters,'—and then to show that every other supposition is thereby made absurd. The right to this assumption is enforced in the 'Nut' by the following analogy:—'I think you (!) will not dare (!) to dispute my right to this hypothesis, when I can prove by means of it that every other value of π will lead to the grossest absurdities; unless, indeed, you are prepared to dispute the right of Euclid to adopt a false line hypothetically for the purpose of a *reductio ad absurdum* demonstration in pure geometry.' Euclid assumes what he wants to *disprove*, and shows that his *assumption* leads to absurdity, and so *upsets itself*. Mr. Smith assumes what he wants to prove, and shows that *his* assumption makes *other propositions* lead to absurdity. This is enough for all who can reason."[1] Such direct assumption is, however, by no means unusual when synonyms are used. Put symbolically, it being agreed that S is identical with A, and P with B, the fallacy is committed when the proposition A is B is assumed as a premise from which to prove S is P. This is, in the strictest sense, "begging the question." According to the directness of the invalid process two sub-forms may be distinguished—the *hysteron proteron* and the *circulus in demonstrando*.

In the *hysteron proteron* the conclusion and the premise are really the same. Here the fallacy is committed in a single step of inference, as 'The volume of a body diminishes when it is cooled, because the molecules then become closer.' 'Opium induces sleep because it has a soporific quality.' It is obvious that the richer a language is in synonyms the more likely is a *hysteron proteron* to appear in arguments expressed in that language. Often a proposition expressed in abstract terms is given in proof of the same proposition expressed in concrete terms, as when we are told that the loadstone attracts iron because of its

[1] *Budget of Paradoxes*, p. 327.

magnetic power, or that oxygen combines with hydrogen because it has an affinity for it. A good instance of this form of fallacy was unconsciously supplied by the member of Parliament who argued that 'the bill before the House is well calculated to elevate the character of education in the country, for the general standard of instruction in all the schools will be raised by it.'

The fallacy may even be committed in single words, by the use of what Bentham terms "question-begging appellatives," that is, those which beg the question under the pretence of stating it. "These," says Dr. Davis, "are potent when laudatory, but even more so when vituperative; as Radicals, Rebels, and most political catch-words. The word 'innovation' having acquired a bad sense, the admission, which is unavoidable, that a new measure is an innovation is always construed to its disadvantage."[1] So, as has been noted, 'progress' is applied to change as a form of approval.

It must not be supposed that it is difficult to fall into this fallacy unintentionally, or that only persons of untrained minds can be guilty of it. Galileo accuses Aristotle himself of having committed it in the following argument: —'The nature of heavy things is to tend towards the centre of the universe, and of light things to fly from it; experience proves that heavy things tend towards the centre of the earth and that light things fly from it; therefore, the centre of the earth is the centre of the universe.' But Aristotle could only say that heavy things tend towards the centre of the universe by assuming that centre to be identical with the centre of the earth, which is the very proposition he undertakes to prove.[2]

Put symbolically, the *hysteron proteron* takes either the form S *is* P, S *is* P, $\therefore$ S *is* P; or S *is* P, S *is* S, $\therefore$ S *is* P. An examination of the latter shows that whenever a definition is used as a premise in an argument meant to be demonstrative, the question is begged; and this Aristotle distinctly points out. For if we argue—'Every rectilinear three-sided figure has its angles equal to two right angles, Every tri-

[1] *Theory of Thought*, p. 284. [2] Cf. *Port Royal Logic*, p. 249.

angle is a rectilinear three-sided figure, Therefore, every triangle has its angles equal to two right angles,' there is no proof. The major premise assumes the very point to be proved, for the minor premise does nothing but tell us how such a figure is named. Of course, such a use of a definition is permissible in popular explanations, though the form may be the same as the above; for when no demonstration is offered, no begging of the question is possible.

The *circulus in demonstrando* or *reasoning in a circle* is when the assumption of the conclusion as a premise is separated by a greater or less interval from its statement as a conclusion. It may be symbolically expressed thus—*M is P, S is M, ∴ S is P; S is P, M is S, ∴ M is P.* Of this Whately gives a good example: "Some mechanicians attempt to prove, (what they ought to lay down as a probable but doubtful hypothesis), that every particle of matter gravitates equally; 'Why?' 'Because those bodies which contain more particles ever gravitate more strongly, *i.e.* are heavier: 'But (it may be urged) those which are heaviest are not always more bulky'; 'No, but still they contain more particles, though more closely condensed'; 'How do you know that?' 'Because they are heavier'; 'How does that prove it?' 'Because all particles of matter gravitating equally, that mass which is specifically the heavier must needs have the more of them in the same space.'"[1]

The second mode of committing *petitio principii* is when a universal which involves the particular proposition to be proved is assumed. Here there is no formal syllogistic fallacy. The argument takes the form of a valid syllogism in *Barbara.*[2] But it is an offence against the principles of Method, as the proposition assumed is one which needs proof quite as much as that which is inferred from it. This is the most accurate sense in which the term *petitio principii* is applied. A *principium*, as was said above, was a self-evident truth; and the true *petitio principii* was the assumption of some proposition of inferior rank as such a principle. Some such truths must be accepted as not

[1] *Logic*, p. 225. [2] See p. 224.

requiring proof, or all proof would be impossible, as there would be no starting point for the process. The fallacy in this form is, then, not committed whenever a proposition is assumed without proof, but when the proposition thus assumed is one which needs just the same kind of proof, bearing on just the same point, as the proposition which is to be deduced from it. "Sound probation," says Dr. Davis, "must depart from such principles as are either immediately given as ultimate, or mediately admit of proof from other sources than the proposition itself in question."[1] As examples of this form of the fallacy we may give: 'His cowardice may be inferred from his cruelty, for all cruel men are cowards'; 'A table of logarithms must be entertaining, for all books are so.' A striking example is found in the First Chapter of Mr. Spencer's *Education*. After stating that "acquirement of every kind has two values—value as *knowledge* and value as *discipline*," Mr. Spencer goes on to discuss the value of different subjects from the point of view of knowledge. He then turns to the disciplinary value of studies, and commences his disquisition with the following flagrant *petitio*:—"Having found what is best for the one end, we have by implication found what is best for the other. We may be quite sure that the acquirement of those classes of facts which are most useful for regulating conduct, involves a mental exercise best fitted for strengthening the faculties. It would be utterly contrary to the beautiful economy of Nature, if one kind of culture were needed for the gaining of information and another kind were needed as a mental gymnastic."

It is this mode of the fallacy which is referred to when it is said that the syllogism is a *petitio principii*.[2]

The other modes of *petitio principii* are not of much importance. The third mode—of assuming the particular to prove the universal which involves it—is of the nature of a generalisation from simple enumeration. "Aristotle himself seems to be guilty of this when he maintains that slavery is in accord with natural law, on the ground that

[1] *Op. cit.*, p. 288. [2] See pp. 287-291.

the neighbouring barbarians, being inferior in intellect, are the born bondsmen of the Greeks."[1]

The fourth mode is only a variety of the first. Thus, to take Aristotle's example, when, in trying to show that the healing art is knowledge of what is wholesome and unwholesome, it is successively assumed to be the knowledge of each.

The fifth mode is when a proposition which is in reciprocal relation to another proposition is assumed as a means of proving the latter. Aristotle's example is the assumption that the side of a square is incommensurable with the diagonal when the proposition to be proved is that the diagonal is incommensurable with the side. Other examples are: London is north of Brighton, therefore, Brighton is south of London; Philip was the father of Alexander, therefore, Alexander was the son of Philip; 'Everywhere the light of life and truth was lacking, for darkness covered the earth, and gross darkness the people.' In all these cases we have obviously no passage of thought; it is the same judgment which is expressed in different words; nor does the new expression unfold any meaning which was previously implicit.

2. **Ignoratio Elenchi.**—By an *elenchus*—*i.e.* a refutation—Aristotle meant a syllogism with a conclusion contradictory of the thesis to be refuted. The *ignoratio elenchi* was then applicable only to disputation, and consisted in arguing beside the mark, in answering to the wrong point, in establishing a proposition which did not overthrow the original thesis. But the scope of the fallacy may well be extended—as it usually is by modern logicians—to include all cases in which instead of the required conclusion, a proposition which may be mistaken for it is established. This might appropriately be called *ignoratio* or *mutatio conclusionis*. In every case the error consists in proving the wrong point. It is thus a violation of the most general rule of method—to be clear as to the purpose. As an example of *ignoratio elenchi* we may take

[1] Davis, *op. cit.*, p. 289. *See* Aristotle, *Politics*, i. 2.

the common argument against a classical education that "throughout his after career, a boy, in nine cases out of ten, applies his Latin and Greek to no practical purposes."[1] This is to ignore the fact that the advocates of a classical education do not claim that Latin and Greek are of direct use in practical life. What they do urge is that the study of the classics furnishes an unrivalled mental training; and it is to the disproof of this proposition that a true *elenchus* must be addressed.

No fallacy is more common or more easily committed than *ignoratio*. Anyone who has had experience of disputations and debates knows how constantly recurrent is the tendency to wander from the real point at issue, especially when the subject under discussion is a wide one, and how necessary it is for a speaker occasionally to begin his remarks by reminding the disputants what the question under discussion really is. This tendency must be guarded against especially when any practically important results flow from the conclusion reached. Thus, as De Morgan tells us: "The *pleadings* in our courts of law, previous to trial, are intended to produce, out of the varieties of statement made by parties, the real points at issue, so that the defence may not be *ignoratio elenchi*, nor the case the counter fallacy . . . *ignoratio conclusionis*. If a man were to sue another for debt, for goods sold and delivered, and if the defendant were to reply that he had paid for the goods furnished, and plaintiff were to rejoin that he could find no record of that payment in his books, the fallacy would be palpably committed. The rejoinder, supposed true, shows that either defendant has not paid, or plaintiff keeps negligent accounts; and is a dilemma, one horn of which only contradicts the defence. It is plaintiff's business to prove the sale from what is in his books, not the absence of payment from what *is not*, and it is then defendant's business to prove the payment by his vouchers."[2]

This leads on to that form of the fallacy which consists in throwing the burden of proof on the wrong side. Proof of an assertion should generally be given by the person

[1] Spencer, *Education*, Ch. I.

[2] *Op. cit.*, p. 260.

who makes that assertion, and to endeavour to transfer to an opponent the task of proving the negative of that assertion is an *ignoratio elenchi*. It is often said that it is difficult, if not impossible, to prove a negative. And this is true so long as the negative is a bare denial. But the establishment of every positive proposition proves a number of negatives. If, then, the number of possible alternatives are few, the proof of any one of them negates all the others. This principle is adopted in law. "For instance, a homicide, as such, is considered by the law as a murderer unless, failing justification, he can prove he had no malice. . . . The case stands thus:—the alternatives are few, so that proving the negative of one, which the accused is called on to do, can be done by proving the affirmative one out of a small number. There is but malice, heat of blood, misadventure, insanity, etc., to which the action can be referred. Of these few things it is easier for the accused to establish some one out of several, above all when motive is in question (of which only himself can be in possession of the most perfect knowledge), than it is for the prosecutor to establish a particular one. Another principle on which he is called on to establish a negative (or rather another positive) is that the burden of proof fairly lies on the one to whom it will be by much the easiest."[1]

One form of the fallacy is to confuse objections against the thesis proposed with its disproof. Especially is this likely to be committed when the question at issue is some proposed change, say in the law. Against most reforms some objections can be urged, but to treat these as necessarily fatal is an *ignoratio elenchi*; the point to be established is that those objections outweigh the reasons for the proposed change, and simply to point out their existence is entirely beside the mark. One common specimen of this form of the fallacy is to object to a certain conclusion as tending to establish a position deemed undesirable. Here we have what De Morgan calls "the great fallacy of all, the determination to have a particular conclusion, and to find

[1] De Morgan, *ibid.*, p. 261.

arguments for it,"[1] coming into play. The conclusion being fore-ordained, all arguments which make against it are refused a hearing.

Another form of the fallacy is to prove, or disprove, part of what is required, and to dwell on that to the exclusion of the remainder. Thus, if one disputant supports a conclusion by weak arguments, his opponent may confine himself to showing the weakness of his arguments, and leave his readers or hearers to infer that, consequently, the conclusion drawn from those arguments is false. The handle thus given to the enemy should be a warning against the practice of urging bad arguments in support of a good cause.

A more extreme case of the same mode of committing the fallacy is the taking exception to a mere illustration or part of an illustration, which has no essential connexion with the point in dispute. The very use of illustration at all is liable to be an *ignoratio elenchi*. For an illustration is intended to make some point of difficulty clearer and easier of comprehension to the hearer or reader. But the user of the illustration may mistake the point which will need elucidation, and may illustrate the wrong point. And there may equally be an *ignoratio* on the part of the pupil. As De Morgan says: "The greatest difficulty in the way of learners is not knowing exactly in what their difficulty consists; and they are apt to think that when *something* is made clear, it must be *the* something."[2] And he rightly goes on to point out the danger incident to the use of concrete examples in the study of the rules of formal inference. "If the student receives help from an example stated both in matter and form, the odds are that the help is derived from the plainness of the matter, and from his conviction of the matter of the conclusion. . . . The right perception may, no doubt, be acquired by reflection on instances; but the minds which are best satisfied by material instances are also those which give themselves no further trouble."[3] The use of illustrations is also liable to the fallacy in another way—the person

[1] *Ibid.*, p. 264. [2] *Ibid.*, p. 266. [3] *Ibid.*, pp. 266-267.

to whom an illustration is addressed may not see the analogy of the matter.

One common and most objectionable form of *ignoratio elenchi* is summed up in the advice to counsel: 'No case; abuse the plaintiff's attorney.' Discussions, especially on subjects of real practical importance, such as politics and religion, are not usually conducted entirely in cold blood. Prejudice and even bad faith lead members of one party to attribute all kinds of base motives to their opponents, and to discount all the statements they make. "The testimony is, in the receiver's mind, of a low order; the proposer is a radical, and the receiver is of opinion that a radical would pick a pocket: or else, perhaps, the proposer is a tory, and the receiver is of the belief that a tory must have picked a pocket."[1] But it must be remembered that abuse is not argument, and that to prove any amount of ill conduct against the proposers of a certain measure or the maintainers of a certain proposition will not prove the measure unwise or the proposition untrue.

Closely allied to this form of the fallacy is the *argumentum ad hominem*, or the *tu quoque*—you're another—style of argument. All recrimination, all charges of inconsistency, are of this character, unless indeed the very point in dispute is personal character or consistency. But when a statesman brings forward a certain measure it is no objection to that measure to point out that he has hitherto opposed it. Very often, indeed, when a disputant tries to turn his opponent's former arguments against himself, it is not, as De Morgan says, "absolutely the same argument which is turned against the proposer but one which is asserted to be like it, or parallel to it. But *parallel cases* are dangerous things, liable to be parallel in immaterial points, and diverge in material ones."[2]

A sub-form of this species of the fallacy is the *argumentum ad baculum*. As Mr. Stock remarks: "To knock a man down when he differs from you in opinion may prove your strength, but hardly your logic."[3]

[1] De Morgan, *ibid.*, p. 263. [2] *Ibid.*, p. 265.
[3] *Deductive Logic*, p. 313.

Another form of *ignoratio* is the *argumentum ad populum,* or appeal to popular passion or prejudice. The following 'argument' against a literary education appears to us to be a striking instance: "When a mother is mourning over a first-born that has sunk under the sequelæ of scarlet-fever—when perhaps a candid medical man has confirmed her suspicion that her child would have recovered had not its system been enfeebled by over-study—when she is prostrate under the pangs of combined grief and remorse; it is but small consolation that she can read Dante in the original."[1]

Closely allied to this is the *argumentum ad ignorantiam,* which consists in trusting that the ignorance of the hearer will lead to the acceptance as proved of statements which are by no means proved. And this is frequently allied with the *argumentum ad verecundiam* or appeal to a respected authority. 'You should accept this conclusion, or advocate this measure, because so-and-so supports it' is an *ignoratio elenchi* not infrequently heard in political life. This also involves a fallacy of illicit generalisation.[2] Undue respect for authority leading to a neglect to examine the evidence for and against a given proposition is not a state of mind favourable to the detection of fallacy.

3. **Non sequitur or Non propter hoc.**—This fallacy is committed whenever the conclusion is not a necessary consequence of the premises. Both the premises and the conclusion may be granted, and yet the derivation of the one from the other denied. Or the premises may be accepted, and the conclusion ostensibly drawn from them rejected. If one wishes to refute a proposition by showing that it leads to absurd results, and one includes it as a premise among others, then though the conclusion drawn from their combination may be absurd or impossible it does not follow that the absurdity or impossibility is due to that one particular premise. If it be argued that the increase of schools has been evil, and for proof it be pointed out that official returns show a continuous increase

[1] Spencer, *Education,* Ch. I. [2] See pp. 494-498.

of crime we have an obvious *non-sequitur.* For it may be retorted that there are many antecedent conditions besides the increase of schooling, and that increase of crime may with more probability be traced to them: or that the increase is only apparent and is really proof of better administration of the law rather than of its more frequent violation.

In the first case distinguished there is not a true syllogism but really four terms, for were the syllogism a good one the conclusion must be granted if the premises be accepted. This is the position of Aristotle. In the *Analytica Priora* he says: "The most obvious case of the irrelevance of the thesis to the conclusion is when the thesis is not connected by any middle term with the conclusion."[1] It thus appears that this fallacy involves, at any rate in some instances, the formal syllogistic fallacy of four terms. But when it is considered generally, its essential connexion with proof by *reductio ad impossibile* classes it more appropriately as a fallacy incident to method.

In the second case the presence of the fallacy may be detected by trying whether the premise in question can be omitted without interrupting the sequence of the argument. But it must be remembered that the superfluous premise may have been tacitly assumed, as in the example Aristotle gives: "We assume that the opposite of destruction is generation; therefore, the opposite of a particular destruction is a particular generation; but death is a particular destruction and its opposite is life; therefore life is generation, and to live is to be generated. This is absurd. Therefore life and soul are not identical." Here the premise assumed is that 'Life and soul are identical.' The premise tacitly assumed must be made explicit before it can be seen whether the absurdity of the conclusion is really due to it and to it alone.

A common example of a *non-sequitur* is when an opponent of some suggested reform objects that "it may be good in theory but is bad in practice." If we grant the conclusion as true yet the badness in practice does not follow from the

[1] ii. 19.

goodness in theory. It is assumed that theory and practice are incompatible—which is absurd. The badness of the practice results from the inadequacy—and therefore, badness—of the theory, from the incompetence of the practitioner, or from interferences from without which prevent the practice being a real example of the theory. Political speeches afford a happy hunting-ground for the seeker after this, and every other, kind of fallacy. When an oration is self-destructive we may be said to have a very real *non-sequitur*, for as the premises annihilate each other there is nothing left from which a conclusion can be drawn.

The reader may be interested in comparing the following propositions from one and the same speech recently made in the House of Commons by Mr. Asquith[1]:

"The House of Lords has long ceased to have any real control over policy and administration. . . .

"Then followed the four years, 1906-1909. . . . During those years . . . the House of Lords resolutely opposed and successfully defeated the principal controversial measures passed by the largest majorities in the annals of the House of Commons. The climax was reached in the autumn of 1909 when the House of Lords rejected the finances of the year. . . .

"If that precedent had been allowed to stand the Lords could always, as they did then, by destroying the finance of the year, compel the Government of the day to resign office or to appeal by dissolution to the electorate."

The remarkable thing is not that such self-contradictory positions should be taken in the course of a speech of some length, but that neither friends nor foes seemed aware of the logical character of the argument addressed to them.

[1] Reported in '*The Times*,' February 22, 1911.

CHAPTER XVIII.

GENERAL NATURE OF SYLLOGISM.

1. Definition of Syllogism.

A Syllogism is an inference in which, from two propositions, which contain a common element, and one, at least, of which is universal, a new proposition is derived, which is not merely the sum of the two first, and whose truth follows from theirs as a necessary consequence.

The word Syllogism may be considered as retaining its strict etymological meaning—'a collecting together'—and as implying that the elements of a syllogism are thought together. The word thus emphasises the fact that a syllogistic inference is one indivisible act of thought.

As one of the propositions given as data must be universal, every syllogism is an inference from the general; in many cases it is an argument from the general to the particular or individual. Syllogism is the one means by which a general principle can be applied to specific instances; and in no case can the derived proposition be more general than those from which it is drawn.

The whole force of a syllogism depends upon the necessity with which the inferred proposition follows from those given as data, and this necessity must be evident from the mere form of the argument.

The *matter* of a syllogism is given in its terms, which vary according to the subject to which the argument refers. Its *form* consists in that relation of the terms by which they are united in two propositions necessitating a certain conclusion. Syllogistic inference is, thus, purely formal,

and can, consequently, be entirely represented by symbols. We are concerned in a syllogism, not with the truth or falsity of either of the individual propositions which compose it but, simply with the dependence of one of them upon the other two, so that, if we grant the latter, we, of necessity, accept the former. The derived proposition, therefore, propounds no truth which was not contained in the data. But this is no objection to the syllogism as a process of inference; it is, indeed, a necessity if that process is to be wholly regulated—as we shall show in the next chapter that it is—by the Laws of Thought.

If the given data are objectively true, the proposition inferred from them must also be true; but, if the given data are objectively false, it may accidentally happen that the derived proposition is true in fact. This is, however, a mere coincidence; its truth is known from other sources, and is not established by the syllogism. For example, from the data

Lions are herbivorous
Cows are lions

we derive the proposition *Cows are herbivorous*, which is true, but whose truth cannot be held to be a consequence of the given data, which are both false.

It is essential that the propositions which form the data should have a common element, as, otherwise, they would have no bond of connexion with each other, and, consequently, no third proposition could be drawn from their conjunction. But this common element does not appear in the derived judgment, which is an assertion connecting the remaining elements of the syllogism.

The *Elements of a Syllogism* are the propositions and terms which compose it. 'Terms' is here used widely to cover, not only the true terms of categorical propositions, but also the propositions which form the antecedents and consequents of hypothetical propositions. The three propositions which compose a syllogism are called its *Proximate Matter*, and the terms (in the wide sense just noted) which are united in those propositions are styled its *Remote Matter*. The derived proposition is the *Conclusion* of the

syllogism, and the two propositions from which it is derived are the *Premises*.

These names are applicable when the syllogism is stated in the ordinary and strictly logical form, in which the premises precede the conclusion—as when we say 'Everything which tends to reduce the supply of any article tends to raise its price; Protective Duties tend to reduce the supply of those articles on which they are imposed; therefore, Protective Duties tend to raise the price of those articles on which they are imposed.' But, when the conclusion is put forward first, as a thesis to be proved, it was called by the old logicians the *Question*, and the propositions which establish it, and which are then introduced by 'because,' or some other causal conjunction, were termed the *Reason*. In this form, the syllogism given above would read—'Protective Duties tend to raise the price of those articles on which they are imposed, because they tend to reduce the supply of those articles; and everything which tends to reduce the supply of an article tends to raise its price.' These latter terms are, however, but little used by modern writers.

The element common to the two premises is called the *Middle Term*, and is most conveniently symbolised by M; while the other two terms are styled the *Extremes*. Distinguishing between the extremes, that which is the predicate of the conclusion is called the *Major Term*, and may be expressed by the symbol P; that which is the subject of the conclusion is named the *Minor Term*, and is appropriately represented by S. The premise in which the major and middle terms occur is known as the *Major Premise*; that in which the minor and middle terms are found is called the *Minor Premise*. The order in which the premises are stated is, of course, of no consequence so far as the validity of the argument is concerned; but, as the relation of the propositions is most clearly expressed by stating the major premise first, that order must be regarded as the legitimate logical form of a syllogism.

The use of the words Minor, Middle, and Major to denote the terms of a syllogism arose from the consideration of that form of syllogism in which the conclusion is a universal affirmative proposition, and both whose premises

are also universal affirmatives. This syllogism may be symbolised by

$$\begin{array}{r} M\ a\ P \\ S\ a\ M \\ \hline \therefore S\ a\ P \end{array}$$

Here, as the extent of the predicate of an affirmative proposition must be, at least, as great as, and is generally greater than, that of the subject, it is plain that P must be at least as wide as, and is probably wider than, M in extent, and similarly with M and S. Hence, the extent of M is, in most cases, intermediate between that of S and that of P, and, in other cases, is coincident with that of one, at least, of those terms. This relation of extent does not hold in all syllogisms and is not essential to the validity of syllogistic argument. For instance

$$\begin{array}{r} M\ a\ P \\ M\ a\ S \\ \hline \therefore S\ i\ P \end{array}$$

is a perfectly valid argument, though S is here greater than, or at least as great as, M in extent. Similarly, when one of the premises is negative, this relation of extent is not assured. For example, in

$$\begin{array}{r} M\ e\ P \\ S\ a\ M \\ \hline \therefore S\ e\ P \end{array}$$

the inference is perfectly just whether P be greater than, equal to, or less than, M in extent; we cannot tell which is the case, nor is it material, as the total exclusion of P, which does not depend on its extent relatively to that of M, is secured. The names Minor, Middle, and Major are not, therefore, appropriate in all cases, if they are regarded as referring to the extension of the terms; but they are universally accepted and recognised, and are as convenient as any others which could be invented. In another sense, moreover, the expression 'Middle Term' is quite appropriate, for that term in every syllogism *mediates* the conclusion, and is the middle bond of union connecting the premises.

This terminology of Terms and Premises is primarily applicable to syllogisms which are entirely composed of categorical propositions, but it may be broadly transferred to those which consist, wholly or in part, of hypothetical or disjunctive propositions.

2. **Kinds of Syllogisms.**—As there are different kinds of propositions—Categorical, Hypothetical, and Disjunctive—all of which can be used in syllogistic arguments, it follows that syllogisms can be of different kinds, or *relations* as it is technically called.

When both the premises in a syllogism are of the same character as regards the relation of the terms—categorical, hypothetical, or disjunctive—the syllogism is said to be *Pure*, and the conclusion is, in every case, of the same relation as the premises. Thus, two categorical premises yield a categorical conclusion, two hypothetical premises necessitate a hypothetical conclusion, and from two disjunctive premises there follows a disjunctive conclusion. There are, therefore, three kinds of pure syllogisms—the *Categorical*, the *Hypothetical*, and the *Disjunctive*.

When the premises are propositions of different relations the syllogism is called *Mixed*. In the first place, the major premise may be either hypothetical or disjunctive, and the minor categorical. A syllogism in which this order was reversed would be impossible, as the minor premise must state, in a definite manner, the special case which is to be brought under the more general statement of the major premise. This gives two kinds of Mixed Syllogisms—the *Hypothetical* and the *Disjunctive*. These Hypothetical Syllogisms are sometimes called Hypothetico-Categorical, but it is more usual to name a mixed syllogism in accordance with the relation of the major premise. To avoid confusion, we shall always call syllogisms in which all the propositions are hypothetical or disjunctive propositions *Pure Hypothetical* and *Pure Disjunctive Syllogisms*; while those with categorical minor premises and conclusions we shall style *Mixed Hypothetical* and *Mixed Disjunctive Syllogisms*, according to the character of the major premise.

In the second place, the major premise may be hypo-

thetical and the minor disjunctive. This gives that peculiar form of mixed syllogism called the *Dilemma,* in which, according to the number of terms in the major premise, the conclusion is either categorical or disjunctive.

We thus get the following table of kinds of syllogisms—

Syllogisms	1. Pure	(*a*) Categorical.
		(*b*) Hypothetical.
		(*c*) Disjunctive.
	2. Mixed	(*a*) Hypothetical.
		(*b*) Disjunctive.
		(*c*) Dilemmas.

The distinction between Pure Hypothetical and Pure Disjunctive Syllogisms on the one hand, and Categorical Syllogisms on the other is not of as great importance as is the distinction between hypothetical, disjunctive, and categorical propositions; for, in all cases the force of the syllogism depends on the necessity with which the conclusion follows from the premises, and the same rules will be found to apply to all kinds of Pure Syllogism. But the Mixed Syllogisms require somewhat different treatment.

We shall, in the next three chapters, confine our attention to Pure Syllogisms, working out the details fully with categorical syllogisms, and then showing how they can be applied to pure hypothetical and pure disjunctive syllogisms. We shall then, in the following chapter, discuss Mixed Syllogisms.

CHAPTER XIX.

POSTULATES OF DEDUCTIVE INFERENCE.

1. Basis of Pure Syllogistic Reasoning.—As syllogistic reasoning is purely formal, it rests ultimately upon the Laws of Thought. The Principle of Identity is the basis of every affirmative categorical syllogism, and that of Contradiction of every negative categorical syllogism. For pure hypothetical syllogisms an additional reference is required to the Principle of Sufficient Reason.

As both the premises of every syllogism contain the same middle term each affirmative categorical premise must state that an element of identity exists between that term and one of the extremes, and each negative categorical premise must assert a separation between the middle term and one of the extremes. If, then, both premises are affirmative categoricals, the extremes are connected with each other mediately in so far as each is identical with the middle term; identity to the same extent is established between them. Of course, if restrictions of quantity are introduced into the premises, they limit the identity, and the same limitation must appear in the conclusion.

If, out of two categorical premises, one is negative, then, as one extreme is excluded from *M*, it is excluded from everything which is identical with *M*, and, therefore, from the other extreme; for the other premise must be affirmative, and a term cannot at the same time agree with *M* and with a term which is incompatible with *M*. Thus, symbolically, if *S is M*, and *M is not P*, then *S is not P*.

These principles apply equally to pure syllogisms whose premises are hypothetical propositions. But here the pro-

position which forms the 'middle term' of such a syllogism gives the reason why the proposition which forms the 'minor term' is the antecedent, whose affirmation is the ground for the assertion of the proposition which forms the 'major term,' and is, therefore, the consequent of the conclusion. Thus, symbolically, from *If* **A**, *then* **B**; and *if* **B**, *then* **C**, it follows that *If* **A**, *then* **C**; the 'Sufficient Reason' being found in the relation of both these extremes to **B**.

2. **The Dictum de omni et nullo.**—Instead of appealing directly to the simple statements of the Laws of Thought, logicians have been accustomed to give various axioms—which are more or less expansions of those statements—as the bases of syllogistic reasoning from categorical propositions.

The most important of these is the *Dictum de omni et nullo.* The scholastic logicians regarded as the perfect type of categorical syllogism that in which the middle term is the subject of the major premise and the predicate of the minor premise—that is, in which the empty schema is

$$\begin{array}{c} M\text{——}P \\ S\text{——}M \\ \hline \therefore\ S\text{——}P. \end{array}$$

All other forms of syllogism can be reduced to this by applying the various modes of eduction to the premises. The validity of such other forms can, therefore, be tested, by first reducing them to this standard form, and then enquiring whether or not they conform to the general axiom which applies directly to this form only. These logicians, therefore, gave one axiom as the fundamental principle of syllogistic reasoning. This is the time-honoured *Dictum de omni et nullo,* which is, perhaps, most satisfactorily expressed by saying:

Whatever is distributively predicated, whether affirmatively or negatively, of any class may be predicated in like manner of anything which can be asserted to belong to that class.

This axiom is simply an expanded statement of the Principles of Identity and Contradiction; for, to predicate anything of a term used distributively is to make the same predication of each of the constituents of the denotation of that term.

3. General Rules or Canons of Categorical Syllogisms.

(i) Derivation of Rules from the 'Dictum.'—The *Dictum de omni et nullo*, as has been said, is directly applicable to syllogisms in whose premises the middle term is the subject of the major, and the predicate of the minor, premise. To all other forms of syllogism it applies indirectly through this form. The *Dictum* may, therefore, be taken as the special axiom of all syllogistic inference; and, consequently, all rules which govern such inferences must be deducible from it. An examination of the *Dictum* will give these in a specific form, corresponding to its own direct reference to one form only of syllogism; but by slight generalisations they can be made directly applicable to all forms of syllogism. Such an examination shows that—

1. The *Dictum* speaks of three, and of only three, terms. There is the 'Whatever is predicated'—which is the major term; the 'class' of which it is predicated—the middle term; and the 'anything asserted to belong to that class' —the minor term. This gives the rule that *a syllogism must have three, and only three, terms.*

2. Similarly, there are three, but only three, propositions contemplated by the *Dictum*. There is that in which the original predication is made of the 'class'—the major premise; that which declares something 'to belong to that class'—the minor premise; and that in which the original predication is made of that included something—the conclusion. Hence, the rule that *a syllogism must consist of three, and only three, propositions.*

3 The *Dictum* says the original predication is made of some 'class.' Now this 'class' is, as has just been said, the middle term, which is directly regarded by the *Dictum*

as the subject of the major premise. Thus, the *Dictum* tells us that in this form of syllogism the middle term must be distributed in the major premise. Generalising this, we get the rule that *the middle term must be distributed in one, at least, of the premises.*

4. The *Dictum* says the original predication may be made of 'anything' which can be asserted to belong to the class; therefore, that predication must not be made of a term *more definite than* this 'anything.' Hence, if the 'anything' is undistributed in the premise it must be undistributed in the conclusion. Similarly, the *same* predication which is made of the 'class' in the major premise can be made of this 'anything' in the conclusion; we are, therefore, not justified in making a *more definite* predication, and hence, if this predication is made by means of an undistributed term in the predicate of the major premise, it must be made by a similarly undistributed term in the predicate of the conclusion. Generalising this, we get the rule that *no term may be distributed in the conclusion which is not distributed in one of the premises.*

5. According to the *Dictum* the minor premise, in the form of syllogism to which it directly refers, must be affirmative, for it must declare that something *can be* included in the 'class' (*i.e.* in the middle term). This, when generalised, gives the rule that *one, at least, of the premises must be affirmative.*

6. The *Dictum* recognises the possibility of the original predication—that is, the major premise in such a syllogism as it directly applies to—being either affirmative or negative, and declares that the predication in the conclusion must be made 'in like manner.' As, according to 5, the minor premise in such a syllogism is always affirmative, it follows that when both premises are affirmative the conclusion is affirmative, and when the major premise is negative, then, and only then, the conclusion is negative as well. By generalising this, we get the rule that *a negative premise necessitates a negative conclusion, and there cannot be a negative conclusion without a negative premise.*

(ii) **Examination of the Rules of the Syllogism.**—We thus, get the traditional six general rules, or canons, of the syllogism. Each of these is directly applicable to every form of syllogism, and no syllogism is a valid inference in which any one of these rules is violated. An examination of them shows that the first two relate to the nature of a syllogism, the second two to quantity—or distribution of terms, and the last two to quality. They may, therefore, be summarised thus:

A. Relating to Nature of Syllogism:

I. *A syllogism must contain three, and only three, terms.*

II. *A syllogism must consist of three, and only three, propositions.*

B. Relating to Quantity:

III. *The middle term must be distributed in one, at least, of the premises.*

IV. *No term may be distributed in the conclusion which is not distributed in a premise.*

C. Relating to Quality:

V. *One, at least, of the premises must be affirmative.*

VI. *A negative premise necessitates a negative conclusion, and to prove a negative conclusion requires a negative premise.*

We will now examine each of these rules in detail.

Rules I and II.—These are not rules of syllogistic inference, but rules for deciding whether or not we have a syllogism at all. Rule I forbids all ambiguity in the use of the terms employed in the syllogism; for, if any term is used ambiguously, it is really two terms,[1] and so the argument really contains four, instead of three, terms, and is not a true syllogism at all, though it may, at first sight, appear to be one. If there is ambiguity it is most likely

[1] See pp. 20; 60-64.

to occur in the case of the middle term, and hence, Rule III is frequently stated with the additional words 'and must not be ambiguous.' But this is unnecessary, for Rule I provides against that error, and also against a similar fault in connexion with either the major or the minor term, which, if not so common, is equally fatal, when it does occur, to the validity of the inference.

A good example of an ambiguous middle is given by De Morgan[1]—

" *All criminal actions ought to be punished by law:*
" *Prosecutions for theft are criminal actions;*
" ∴ *Prosecutions for theft ought to be punished by law.*

"Here the middle term is doubly ambiguous, both *criminal* and *action* having different senses in the two premises." If the middle term is not exactly the same in both premises it is evident there is no connecting link between the major and minor terms. There must be a common element, and this must be identical in the two premises. Mere resemblance, however close, is not enough; for then S and P might resemble M in different ways, and so no connexion be established between them.

Again, if S or P is used in a different sense in the conclusion from that which it bears in the premise in which it occurs, the inference is invalid; for the premises justify only the predication of that *same* P which was connected in the major premise with M, of that *same* S which was related in the minor premise to the same M.

Of Rule II but little need be said. If there are three terms, two of which are to occur in each proposition, and the same two in no two propositions, it is evident there must be three, and only three, propositions. The very definition of a syllogism secures this rule directly, and Rule I indirectly; for two premises with a common term contain evidently three, and only three terms, and the conclusion relates the two terms which are not common to the two premises.

[1] *Formal Logic*, pp. 241-242.

Rule III.—The violation of this rule is called the *Fallacy of the Undistributed Middle.* It is essential that the middle term should be distributed in one, at least, of the premises, as only thus can there be any assurance that there exists that element of identity which is necessary to constitute a bond of connexion between the extremes. Unless it is certain that the extremes are related to one and the same part of the middle term, there can be no inference as to the relation in which those extremes stand to each other. Now, if only an indefinite reference is made to the middle term in each premise, either the same, or an entirely different, part of its extent may be, in fact, involved in each case. For example, because All Englishmen are Europeans, and All Frenchmen are Europeans, it does not follow that All (or any) Frenchmen are Englishmen. In fact, every possible relation between S and P is consistent with the two propositions *All P is M* and *All S is M*, where M is an undistributed middle term. This is seen at a glance by a reference to Euler's diagrams, which give all the possible objective relations of two classes.[1] Each of those five figures may be entirely enclosed in a larger circle representing M, and in each case $P\ a\ M$ and $S\ a\ M$ will hold true. Thus, it is evident that from two such propositions no inference whatever can be drawn as to the relation of S and P. Similar ambiguity will be found to follow from every other case in which, in a pair of propositions, M is not once distributed.

As, then, we have no security when M is undistributed that there is any bond of connexion between S and P, we can draw no inference concerning the relation of those two terms. For formal inference can depend only upon that bare minimum of assertion which the premises must be held to make unconditionally; and, therefore, as it is formally possible that the same part of M is not referred to in both premises, we must not assume that it is so referred to in any particular case. Only, then, by securing the whole of the middle term in one premise can we be formally certain that there is an identical element in both

[1] Cf. p. 117.

premises. And, though the middle term may be distributed in both premises, yet a single distribution is sufficient to secure this. For, if the whole extent of M is related to one of the extremes, no matter what part of M is related to the other extreme, it must be identical with some at least of the M referred to in the former case.

Of course it is the identical part of M which is the ground of the inference. The rule of distribution is only a formal safeguard that such a common bond of reference is present.

Rule IV.—The violation of this rule is called Illicit Process. If the minor term is distributed in the conclusion and not in the minor premise, we have the *Fallacy of Illicit Process of the Minor Term*; if the same unwarranted treatment is accorded to the major term, it gives rise to the *Fallacy of Illicit Process of the Major Term.* As the conclusion must follow necessarily from the premises, we can never be justified in making a predication about a definite *All S* when the minor premise only refers to *Some S*. For, if merely an indefinite part of S is stated to be related to M, that statement can give us no right to trace, through M, a connexion between P and *all S*. Because all criminals are deserving of punishment, and *some* Englishmen are criminals, it does not follow that *all* Englishmen are deserving of punishment. The conclusion must be no more definite than the premises warrant. And the same holds of the major term. We are justified in relating the whole of P to S only when the whole of P has been previously related to M in the major premise.

It will be noticed that we can only use P universally in the conclusion when that conclusion is negative, for P is always its predicate, and the predicate of an affirmative proposition is always undistributed. In this case, therefore, one of the premises must be negative (Rule VI). If that premise be the minor, then P must be the subject of the major premise, which must be an **A** proposition; but if the negative premise be the major, P may always be its predicate, though it can be its subject only when it is universal. In every other case, if P is distributed in the conclusion, we have Illicit Process of the Major. For

example, if we argue from 'All fishes are oviparous,' and 'No birds are fishes,' to 'No birds are oviparous,' our inference is invalid. In this case, as in most such cases, the conclusion is also false in fact. But even were it true in fact it would not be a valid inference from the premises; its truth would not be a result of their truth, but would be an accidental coincidence and known only through some other source. For instance, 'All fishes are cold-blooded, No whales are fishes, therefore No whales are cold-blooded' is exactly as invalid an inference as the one just considered, though the proposition given as its 'conclusion' is objectively true. For there is nothing in the premises to deny to whales the attribute 'cold-blooded,' as will be seen by substituting the word 'snakes' for 'whales,' when the 'conclusion' becomes false in fact. Thus, no conclusion is justified in which any term is distributed which is undistributed in the premise in which it occurs. The violation of this rule may be compared with the simple conversion of an **A** proposition, a process which has been already shown to be illicit.[1]

Rule V.—From two negative premises no conclusion can be drawn; for, from mere negation of relation no statement of relation can be deduced. It is only when one of the extremes is connected with the middle term, that we can, through that connexion, infer its agreement with, or separation from, the other extreme. For, if both S and P are declared to be separated from M, there is, clearly, no bond of union to connect them with each other. They may, or may not, be related in fact; but whatever relation they hold, it is impossible to infer it from the negation of relation with the common element which is contained in the premises. Compare, for instance, the pairs of negative propositions: 'No cows are carnivorous—No sheep are carnivorous'; 'No men are immortal—No negroes are immortal.' In the first case the minor term is, in fact, wholly excluded from, and, in the second case, wholly included in, the major term; but neither the exclusion nor the inclusion can be deduced from the premises, which

[1] See pp. 145-146.

simply separate both the classes represented by the major and minor terms from the common attribute expressed by the middle term; and premises identical in form must, if they give any conclusion, always give one of the same form.

That from two negative premises no conclusion can be derived has been questioned. For example Jevons wrote: "The old rule informed us that from two negative premises no conclusion could be drawn, but it is a fact that the rule in this bare form does not hold universally true; and I am not aware that any precise explanation has been given of the conditions under which it is or is not imperative. Consider the following example—

> " *Whatever is not metallic is not capable of powerful magnetic influence;*
> " *Carbon is not metallic;*
> " ∴ *Carbon is not capable of powerful magnetic influence.*

" Here we have two distinctly negative premises, and yet they yield a perfectly valid negative conclusion. The syllogistic rule is actually falsified in its bare and general statement."[1]

Expressed symbolically this argument is

$$\begin{array}{c} \text{No } non\text{-}M \text{ is } P \\ S \text{ is not } M \\ \hline \therefore\ S \text{ is not } P; \end{array}$$

where we have, apparently, four terms, S, P, M, and $non\text{-}M$. But, if we examine the argument more closely, we shall see that what the minor premise really asserts is that S is included in those things of which P is denied, *i.e.* the $non\text{-}M$'s. Hence, the middle term is really $non\text{-}M$, and it is this which is predicated *affirmatively* of S in the minor premise. We reduce the above argument, therefore, to a syllogism by obverting the minor premise, when we get

$$\begin{array}{c} \bar{M}\, e\, P \\ S\, a\, \bar{M} \\ \hline \therefore\ S\, e\, P \end{array}$$

[1] *Principles of Science*, 2nd Ed., p. 63.

which is perfectly valid and in which Rule V is observed, as the minor premise is affirmative.

In the following example there is also an apparent inference from two negatives—

> *Jones is not over six feet tall;*
> *Jones is not under six feet tall;*
> ∴ *Jones is six feet tall.*

But the inference is really from the implied disjunctive major, *Everyone is either six feet, over six feet, or under six feet tall.* The two premises given are really the minor, *Jones is neither over nor under six feet.*

Rule VI.—If one premise is negative, the other must, by Rule V, be affirmative. Hence, the two extremes are related to the middle term in opposite ways. Now, if two terms agree with each other, they must, necessarily, stand in the same relation to any third term. If, then, the relations of S and P respectively to M are not the same, but contradictory, relations, S and P cannot agree with each other. We reach the same result in a slightly different way by considering that, in so far as anything agrees with M, it must be separated from everything from which M is separated. If, then, one premise declares the agreement of one extreme with M, and the other premise asserts the incompatibility with M of the other extreme, those extremes must be inferred to be incompatible with each other. Hence, a negative premise involves a negative conclusion.

And the converse of this is also true. The non-agreement of S and P with each other must follow from the fact that one agrees with, and the other is separated, wholly or in part, from M. For, if they both agreed with M, they would agree with each other. Therefore, a negative conclusion can only be inferred when one of the premises is negative.

(iii) **Corollaries from the Rules of the Syllogism.**—Though the four rules of syllogistic inference (Rules III-VI) are not equally fundamental, yet all are so far independent that all are necessary for the immediate detection of invalidity in syllogistic inference; and for this purpose

they are sufficient. There are, however, three corollaries from these rules which, though not absolutely requisite for the detection of syllogistic fallacy, are useful for that purpose. It was long customary to give the first two of these as independent rules of syllogism. They are—

1. *From two particular premises nothing can be inferred.*
2. *If one premise is particular the conclusion must be particular.*
3. *From a particular major and a negative minor nothing can be inferred.*

We will now show in detail how each of these can be deduced from the rules already given.

Cor. 1. This may be established by an examination of each of the possible combinations of particular premises—

In every valid syllogism the premises must contain one distributed term more than the conclusion. For if any term is distributed in the conclusion it must also be distributed in the premise in which it occurs, and, in addition to this, the middle term must be distributed once, at least, in the premises. From this it follows that no conclusion can be drawn from two particular premises. For, if they are both **O** propositions, by Rule V nothing can be inferred. If they are both **I** they contain no distributed term at all and, thus, break Rule III. If one is **I** and the other is **O** then the conclusion must be negative (Rule VI), and, consequently, it distributes the major term. But **I** and **O** distribute only one term between them, and, therefore, cannot distribute both the major and the middle terms. Hence, nothing can be inferred, for to draw a conclusion would be to break either Rule III or Rule IV.

Cor. 2. That a particular premise necessitates a particular conclusion may be thus proved—

If both premises are affirmative and one particular, they can, between them, distribute only one term, which must be the middle term (Rule III). Hence, both the extreme terms are undistributed in the premises, and, consequently,

must be undistributed in the conclusion (Rule IV)—that is, the conclusion must be *particular* affirmative.

If, however, in such a syllogism, one premise is negative, it distributes its predicate, and the premises, therefore, contain between them two, but only two, distributed terms. One of these must be the middle term (Rule III). Hence, only one distributed term can enter the conclusion (Rule IV). But the conclusion must be negative (Rule VI), and it, therefore, distributes the major term, which must, consequently, be the second distributed term in the premises (Rule IV). The minor term is, therefore, undistributed in the premises and must be undistributed in the conclusion—that is, the conclusion must be *particular* negative.

As both premises cannot be negative (Rule V), these are the only possible cases.

Cor. 3. That nothing can be inferred from a particular major and a negative minor may thus be proved—

As both premises cannot be negative the major is affirmative particular (**I**), and distributes neither of its terms. The major term, therefore, cannot be distributed in the conclusion (Rule IV). But, as one premise is negative, the conclusion must be negative. Therefore, the major term must be distributed in the conclusion. This contradiction shows that no valid inference can be made.

4. Application of the Rules to Pure Hypothetical and Pure Disjunctive Syllogisms.

(i) **Pure Hypothetical Syllogisms.**—Since hypothetical propositions admit of the same distinctions of quality and quantity as categorical propositions, there can be forms of pure hypothetical syllogisms corresponding to every form of categorical syllogism. Hence, all the rules given above apply to pure hypothetical syllogisms. The denotative, or conditional, forms bear a closer analogy to the ordinary quantified forms of the categorical syllogism than do the pure abstract hypothetical forms, and the application of the rules is more clearly seen when those quantified forms are considered. The 'terms' here are, however, propositions—the consequent of the conclusion

corresponding to the major term of a categorical syllogism, the antecedent of the conclusion to the minor term, and the element which appears only in the premises to the middle term.

In considering the distribution of these 'terms' it must be remembered that, as 'always,' 'never,' 'sometimes,' 'sometimes not,' in conditional propositions correspond to 'all,' 'no,' 'some,' 'some not,' in categorical propositions, these words indicate the quantity of the antecedent. The quantity of the consequent must be determined by the same rule which decides the quantity of the predicate of a categorical proposition. That is to say, the consequent of a negative conditional proposition is distributed, and that of an affirmative conditional is undistributed. For example, *If any S is M then always that S is P* does not distribute the proposition *that S is P*; for it neither states nor implies that the only possible condition of *S* being *P* is that it should be *M*—it is quite possible that *S* is *P* under many other conditions, as when it is *N* or *Q* or *X*. In short, the distribution of the 'terms' in a pure hypothetical syllogism must not be determined by a reference to those terms by themselves and out of connexion with their context, any more than in a categorical syllogism.

(ii) **Pure Disjunctive Syllogisms.**—Since disjunctive propositions are all affirmative, the syllogistic rules (V and VI) relating to quality do not apply. The rule for securing the distribution of the middle term (III) can only be fulfilled when one of the alternatives in the minor premise is the negative of one of those in the major premise. This will be more fully considered in the next chapter.[1]

[1] See p. 249

CHAPTER XX.

FIGURE AND MOOD.

1. Distinctions of Figure.

Figure is the form of a syllogism as determined by the position of the middle term in the two premises.

If account be taken of the premises alone, so that it is immaterial which of the extreme terms is the subject, and which the predicate, of the conclusion, only three figures are possible. For M must either be (1) subject in one premise and predicate in the other, (2) predicate in both, or (3) subject in both. If, however, it is determined which term shall be the subject, and which the predicate, of the conclusion, the distinction of major and minor is introduced into the premises. The first alternative now becomes two-fold, according as M is subject in the major and predicate in the minor premise, or predicate in the major and subject in the minor.

There are thus four possible Figures of syllogism—

First Figure: M is subject in major, and predicate in minor, premise.

Second Figure: M is predicate in each premise.

Third Figure: M is subject in each premise.

Fourth Figure: M is predicate in major, and subject in minor, premise.

The empty forms of syllogisms arranged in Figures, and with the premises written in the usual order of major first, are, therefore—

Fig. I.	Fig. II.	Fig. III.	Fig. IV.
M——P	P——M	M——P	P——M
S——M	S——M	M——S	M——S
$\therefore S$——P	$\therefore S$——P	$\therefore S$——P	$\therefore S$——P

It should be noted that the distinction between the First and Fourth Figures in no way depends upon the order in which the premises are written, but upon the distinction between major and minor premise, which is due to the predetermined order of the terms in the conclusion.

Only the first three Figures were recognised by Aristotle. The Fourth was added by the Scholastic Logicians for the sake of formal symmetry and completeness. It is, however, an artificial arrangement of terms, and arguments forced into it are usually more naturally expressed in one of the other figures, as will be seen in nearly every actual case by converting whichever of the premises admits of the simple form of that process. Of course, as absolute a formal validity can be secured in the Fourth as in any other Figure.

2. **Special Rules of the Four Figures.**—The arrangement of terms in the premises of each Figure necessitates the imposition of certain limitations of quality and quantity. We thus get special derivative rules for each Figure.

(i) *In the First Figure,* if one premise is negative it must be the major in order to secure that P is distributed; therefore the minor premise is always affirmative. This necessitates that M be distributed in the major premise which must therefore be universal. So the special rules are—

1. *The minor premise must be affirmative.*
2. *The major premise must be universal.*

(ii) *In the Second Figure,* distributed middle can be secured only if one of the premises is negative. This implies the distribution of P in the consequent negative conclusion; therefore the major premise must distribute P. So the special rules are—

1. *One premise must be negative.*
2. *The major premise must be universal.*

(iii) *In the Third Figure,* as P occupies the same position as in the First Figure, the same reason necessitates that

the minor premise be affirmative. Consequently it does not distribute its predicate *S*. So the special rules are—

1. *The minor premise must be affirmative.*
2. *The conclusion must be particular.*

(iv) *In the Fourth Figure*, the distribution of *M* can be secured either by a negative major premise or by a universal minor premise; and that of *P* by a universal major premise. Consequently no definite limitation in quality of either premise exists, and the special rules are conditioned by the possibilities of quality in the premises.

If the arrangement of terms be examined it will appear that when there is a negative premise, and consequently a negative conclusion, the major premise must be universal so as to secure the distribution of *P*; that when the major premise is affirmative the minor premise must be universal so as to distribute *M*; that when the minor premise is affirmative it does not distribute *S*. So the special rules are—

1. *If either premise is negative the major premise must be universal.*
2. *If the major premise is affirmative the minor premise must be universal.*
3. *If the minor premise is affirmative the conclusion must be particular.*

3. **Determination of Valid Moods.—Mood is the form of a syllogism as determined by the quality and quantity of the three constituent propositions,** *e.g.* **A A A, E A E, A O O** are moods.

As all moods are not valid in every figure, a syllogism is only described fully when both its Figure and its Mood are given. There thus arises a more specific use of the term 'mood of a syllogism' in which the same general mood—*e.g.* **E A E**—is regarded as including specifically different moods in the different Figures. This is the more convenient use of the term 'mood' and one sanctioned by custom.

We will now see how many such moods are justified by

the general laws of the syllogism. The simplest way of doing this is to ask what moods will give as formal valid conclusion each of the four forms of categorical proposition, **A**, **E**, **I**, **O**.

(*a*) *To prove* **A**.—Both premises must be affirmative (Rule VI); and, consequently, they distribute only their subjects. As the conclusion is universal, S must be distributed in the minor premise. This leaves M to be distributed in the major premise. Consequently only one syllogism of the general mood **A A A** is possible, and that is in the First Figure—

$$\begin{array}{c} M\,a\,P \\ S\,a\,M \\ \hline \therefore S\,a\,P \end{array}$$

(*b*) *To prove* **E**.—One premise must be negative (Rule VI) and one affirmative (Rule V). These premises must distribute between them all the three terms S, P, M (Rules III, IV). One premise must, therefore, be **E**, and in this the order of the terms is immaterial. The other must be **A** and must distribute the extreme term—S or P—which occurs in it. There are thus four specific moods according to whether the major or minor premise is affirmative and to the order of terms in the negative premise—

$$\begin{array}{cccc} (1) & (2) & (3) & (4) \\ M\,e\,P & P\,e\,M & P\,a\,M & P\,a\,M \\ S\,a\,M & S\,a\,M & S\,e\,M & M\,e\,S \\ \hline \therefore S\,e\,P & \therefore S\,e\,P & \therefore S\,e\,P & \therefore S\,e\,P \end{array}$$

Of these (1) and (2) are specific forms of the general mood **E A E** in Figures I and II; (3) and (4) are specific forms of **A E E** in Figures II and IV.

(*c*) *To prove* **I**.—Both premises must be affirmative (Rule VI). As neither S nor P is distributed in the conclusion, M is the only term which must be distributed in the premises, though P may be distributed, or M distributed twice, without making it possible to draw another conclusion. If, however, S were distributed in the minor

premise, though the conclusion $S\ i\ P$ would be valid it would be inadequate, as we could then deduce $S\ a\ P$. We may have, then, any combination of affirmative premises which secure the distribution of M and make that of S impossible. We thus get—

(1)	(2)	(3)	(4)	(5)	(6)
$M\ a\ P$	$M\ a\ P$	$M\ i\ P$	$P\ i\ M$	$M\ a\ P$	$P\ a\ M$
$S\ i\ M$	$M\ i\ S$	$M\ a\ S$	$M\ a\ S$	$M\ a\ S$	$M\ a\ S$
$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$	$\therefore S\ i\ P$

Of these (1) and (2) are in the general mood **A I I** in Figures I and III; (3) and (4) in **I A I** in Figures III and IV; (5) and (6) in **A A I** in Figures III and IV.

(*d*) *To prove* **O**.—One premise must be negative (Rule VI) and one affirmative (Rule V). The distribution of M and P must be secured in the premises (Rules III and IV) and that of S avoided for the reason given under **I**. It is immaterial whether M is distributed once or twice.

If the major premise is **E**, it secures in itself all the distribution required, no matter what the order of its terms: the minor premise may then be any affirmative proposition except $S\ a\ M$. If the major premise is **O** it must secure the distribution of P by having that term as its predicate; then the minor premise must be $M\ a\ S$ to secure the distribution of M. The minor premise cannot be **E**, as then it would distribute S. If it is **O** it must have M as its predicate to secure its distribution, for the affirmative major must distribute P, and, therefore, be of the form $P\ a\ M$. We thus get—

(1)	(2)	(3)	(4)
$M\ e\ P$	$P\ e\ M$	$M\ e\ P$	$P\ e\ M$
$S\ i\ M$	$S\ i\ M$	$M\ i\ S$	$M\ i\ S$
$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$
(5)	**(6)**	**(7)**	**(8)**
$M\ e\ P$	$P\ e\ M$	$M\ o\ P$	$P\ a\ M$
$M\ a\ S$	$M\ a\ S$	$M\ a\ S$	$S\ o\ M$
$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$	$\therefore S\ o\ P$

Of these the first four are in the general mood **E I O**, one in each figure; (5) and (6) in the general mood **E A O** in Figures III and IV; (7) is the only form of the mood **O A O** and is in Figure III; (8) is the only form of **A O O**, and is in Figure II.

Collecting our results it appears that:—

A can be proved in only one mood, and only in Figure I.

E can be proved in four moods, and in every Figure except the Third.

I can be proved in six moods, and in every Figure except the Second.

O can be proved in eight moods, and in every Figure.

Thus **O** is seen to be proved in the greatest number of moods, and **A** in the smallest. But these propositions are contradictories, and the establishment of the one disproves the other. Hence, it is often said that **A** is the most difficult proposition to establish and the easiest to disprove. At the same time, it must be remembered that "*universal affirmative* conclusions have the highest *scientific value*, because they advance our knowledge in a positive manner and admit of reliable application to the individual. The *universal negatives* come next; they guarantee only a negative but a distinctly definite view. Then come the *particular affirmatives*, which promise a positive advance, but leave us helpless in the application to individual cases. Lastly, the *particular negative* conclusions are of the lowest value. Particular propositions, however, are by no means without scientific meaning. Their special service is to ward off false generalisations. The universal negative or affirmative judgment, falsely held to be true, is proved not true by the particular affirmative or negative conclusion, which is its contradictory opposite."[1]

4. **The Mnemonic Lines.**—We have found that there are nineteen valid specific moods—four in Figure I, four in Figure II, six in Figure III, and five in Figure IV. It

[1] Ueberweg, *Logic*, Eng. trans., pp. 436-437.

has long been customary to designate these moods by the names which compose the following mnemonic lines; each of these names containing three vowels, and thus specifying a mood by indicating the quality and quantity of the constituent propositions by the usual symbols—**A, E, I, O**; thus *Cesare* denotes the mood **E A E** in Figure II—

Barbără, Cĕlārent, Dărĭi, Fĕrĭōque prioris:
Cĕsărĕ, Cāmĕstres, Festīnŏ, Bărŏcŏ, secundæ:
Tertia, *Dărapti, Dĭsămis, Dătĭsĭ, Fĕlapton,*
Bŏcardō, Fĕrīsŏn, habet: quarta insuper addit
Brāmantip. Cămĕnes, Dĭmăris, Fĕsăpŏ, Frĕsĭson.

These mnemonics are given here for the convenience of referring to the moods by their ordinary names. They will be more fully explained in the chapter on Reduction.

5. **Strengthened and Weakened Syllogisms.**—In three of the nineteen valid moods—viz. **A A I** in Figure III and **E A O** in Figures III and IV—the middle term is distributed twice, and in **A A I** in Figure IV the distribution of P in the major premise is not demanded by the conclusion. These are therefore sometimes called *Strengthened* Syllogisms because in each a premise may be reduced to its subaltern particular without affecting the conclusion. The matter is, however, of no importance.

In discussing the moods which prove **I** and **O** we required that S should be undistributed in the premises, on the ground that otherwise the conclusion would be weaker than the premises justify. There is, however, no fallacy in proving less than is warranted by the evidence. If, then, in these moods we strengthen the $S\ i\ M$ into $S\ a\ M$ and the $S\ o\ M$ into $S\ e\ M$ and still conclude only to $S\ i\ P$ and $S\ o\ P$, in every case we draw a particular conclusion when the premises justify a universal. These conclusions are the subalterns of the legitimate **A** or **E** which the premises justify. So these quite worthless and rather misleading forms—**A A I** in Figure I, **E A O** in Figures I, II, and **A E O** in Figures II and IV—are called *Subaltern Moods* and the syllogisms are said to be *Weakened*.

6. We will now give some examples of valid moods in each figure.

(i) **Barbara.**—This is the most important of all the forms of syllogistic inference, and the one most frequently employed—though often elliptically—not only in all branches of sciences but in common life; for to establish a universal connexion between subject and attribute is the constant effort of thought, and object of research. Its schema is

$$\begin{array}{r} M\,a\,P \\ S\,a\,M \\ \hline \therefore S\,a\,P \end{array}$$

In *Physics* the syllogistic form of thought is the only one by which particular phenomena can be explained; and *Barbara* is the most important mood. From the general law of the radiation of heat—that, unless some medium intervenes, a warm body radiates part of its heat through the atmosphere to a colder body surrounding it—we infer that, as the surface of the earth on a clear night is a warm body under those conditions, it will thus become cooled.

The application of *Law* is equally syllogistic. The whole aim of legal procedure is to determine whether or not a particular case does, or does not, fall under a certain general rule, and, if it does, what are the resultant consequences. Thus, in a criminal trial, the law which has been violated furnishes the major premise, the conclusion drawn from an examination of the acts of the accused person supplies the minor premise; the verdict of 'Guilty' or 'Not Guilty' gives the conclusion from those premises; a conclusion to which the sentence of the judge gives practical effect.

From such examples it stands out clearly that syllogistic reasoning is inevitable within its sphere, but that its premises are abstract, in that they single out one relation from the very complex whole in which alone that relation is found. So its conclusion is true only within the same limitations: it is the whole truth only on the assumption that the premises are not only true but complete.

It is further evident that even when—as in law—we have the required major it is a task of no small difficulty in practice to determine whether the case before us is a real minor premise. Syllogism, indeed, is a convenient synthesis of work already done. Its function in the extension of knowledge will be considered in the discussion of Induction.

(ii) **Celarent.**—This is the typical mood in which it is proved that a certain subject does not possess certain attributes. As it is neither of so much importance, nor of so much interest, to prove what a thing is not as to show what it is, this mood is not so universally used as *Barbara*. Its schema is

$$\begin{array}{r} M\,e\,P \\ S\,a\,M \\ \hline \therefore S\,e\,P \end{array}$$

As an example we may give: 'Duties on imports levied solely for the purposes of revenue are not protective; all English import duties are of this class; therefore, no English import duty is protective.'

(iii) **Baroco.**—The schema of this mood is

$$\begin{array}{r} P\,a\,M \\ S\,o\,M \\ \hline \therefore S\,o\,P \end{array}$$

As an example of *Baroco* we may give: 'All truly moral acts are done from a right motive; some acts which benefit others are not done from such a motive; therefore, some acts which benefit others are not truly moral.'[1]

(iv) **Bocardo.**—The schema of this mood is

$$\begin{array}{r} M\,o\,P \\ M\,a\,S \\ \hline \therefore S\,o\,P \end{array}$$

A good example is given by Ueberweg: "Some persons

[1] Aristotle, *Ethics*, III., 4.

accused of witchcraft have not believed themselves to be free from the guilt laid to their charge; all those accused of witchcraft were accused of a merely feigned crime: hence some who were accused of a merely feigned crime have not believed themselves free from the guilt laid to their charge."[1]

(v) **Bramantip.**—The schema of this mood is

$$\begin{array}{l} P\,a\,M \\ \underline{M\,a\,S} \\ \therefore S\,i\,P \end{array}$$

Very few arguments fall naturally into the Fourth Figure. The following is an example of *Bramantip.* From the premises 'All moderate physical exercise is beneficial to health; everything beneficial to health is inculcated by the Moral Code,' we may, if our attention is concentrated on moral precepts, most naturally conclude that 'Among the commands of the Moral Code is one which insists on moderate physical exercise.'

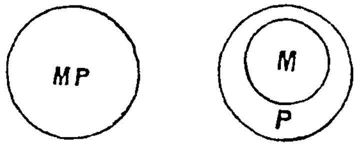
7. **The Representation of Syllogisms by Diagrams.**—The main purpose of applying diagrams to the representation of syllogisms is to make immediately obvious to the eye, by means of geometrical figures, the relation established between the extreme terms by the premises, and, thus, to render easier the apprehension of the conclusion.

This purpose is very imperfectly fulfilled by the circles of Euler. The fundamental objections to the application of these diagrams to the fourfold scheme of propositions have been already stated.[2] As every proposition—except **E**—requires a plurality of diagrams for its complete representation, it is evident that the combination of the two premises of a syllogism can only be fully set forth by a series of diagrams, which must, by its very complexity, go far to prevent that obviousness which is an essential feature of any diagrams which are to be an aid, and

[1] *Logic*, Eng. trans., pp. 425-426. [2] See pp. 118-119.

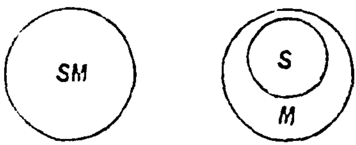
not a hindrance, in apprehending the result of an argument. Take, for example, the representation of *Barbara*. Each premise requires two diagrams to express it; thus—

M a P



(i) (ii)

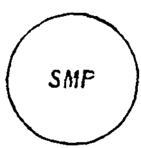
S a M



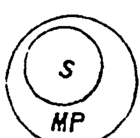
(*a*) (*b*)

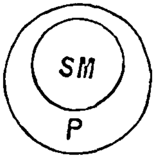
To represent the conclusion, we must combine each of the diagrams which express the major premise with each of those setting forth the minor premise. This gives a combination of four diagrams, and unless they are all considered, we cannot be sure that the result given by those we have examined will not be inconsistent with that yielded by those we have omitted. Thus—

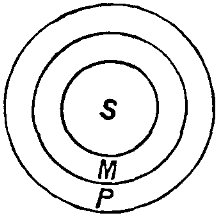
(i) and (*a*) give



(i) and (*b*) give



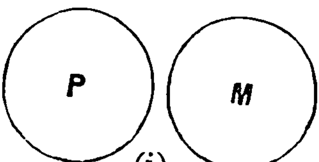
(ii) and (*a*) give 

(ii) and (*b*) give 

To draw the conclusion we must seek out the relation of *S* and *P*, and in this respect the last three of the diagrams coincide—*S* in each is contained in *P* but does not exhaust it. We then compare this result with the first diagram, and we find that in every case it is true to say that *Every S is P*.

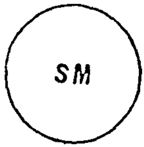
Similarly, if we combine **E** and **A** propositions as premises, we require two diagrams to represent the syllogism, for **A** can only be fully expressed by using the two diagrams given above, and **E** by diagram V. on p. 117. There are, therefore, two combinations, and these, moreover, will be lettered and interpreted differently according as the **A** proposition is the major or the minor premise.

If we take a syllogism involving a particular premise, the representation becomes still more complex. To take *Festino*, for instance, the major premise requires only one diagram, but for the minor four are needed—

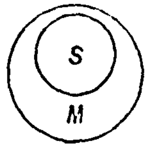
P e M 

(i)

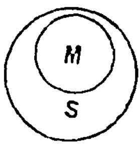
S i M



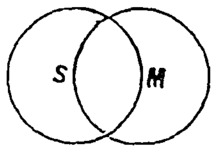
(*a*)



(*b*)



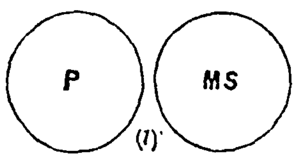
(c)



(*d*)

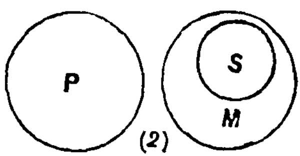
The combination of major and minor in every possible way yields no less than eight diagrams—

(i) and (*a*) give



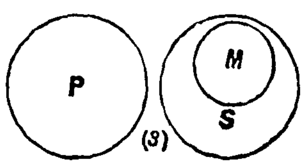
(*1*)

(i) and (*b*) give

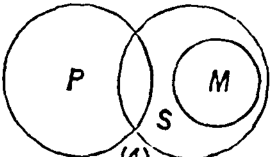


(*2*)

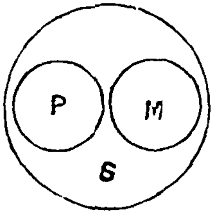
(i) and (*c*) give



(*3*)

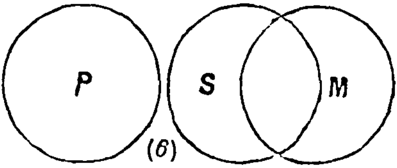


(4)

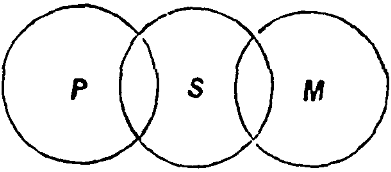


(5)

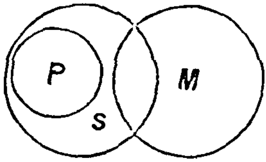
(i) and (*d*) give



(6)



(7)



(8)

From this group of figures, we have, by disregarding *M*, to find the relation of *S* and *P*. On examination we find that (1), (2), (3), (6) express the relation of entire mutual exclusion between *S* and *P*; that (4) and (7) represent the partial coincidence and partial exclusion of those terms, and (5) and (8) give the case in which *P* is entirely included in, but does not form the whole of, *S*. We reach, then, the three diagrams which express the proposition *S o P*.

It will be obvious from the above examples that to represent thus the different moods of the syllogism scarcely makes the reasoning more immediately self-evident. Indeed, the chief value of this system of diagrams is the negative one of showing what premises will *not* yield a valid conclusion; when the diagrams are compatible with every possible relation between *S* and *P*—as in the case of two negative premises—we know that no conclusion can be drawn.

8. **Pure Hypothetical Syllogisms.**—As hypothetical propositions have the same distinctions of quality and quantity as categorical propositions, it follows that they can be combined into syllogisms in exactly the same number of ways. There can, therefore, be forms of pure hypothetical syllogism corresponding to every figure and mood of categorical syllogism, and governed by the same rules. But, as the universal hypothetical propositions are the only ones of much importance, it follows that the important pure hypothetical syllogisms are those composed of such propositions; and of these, those which correspond in form to *Barbara* are the most useful, and the most frequently employed.

Moreover, as the whole force of syllogistic inference consists in the necessity with which the conclusion follows from the premises, and as this necessity is not affected by the hypothetical or categorical form in which those premises are expressed, it follows that such hypothetical premises can always be reduced to the categorical form without affecting the validity of the inference. This reduction is most conveniently made when the quantified—or conditional—forms of the hypothetical are employed, as they correspond most closely with the quantified form in which the propositions composing a categorical syllogism are usually written. Of course, when this is done, though the inference is equally necessary, the abstract and necessary character of the conclusion is more or less hidden.

It will be sufficient to give an example of a pure hypothetical syllogism in each figure expressing each of our propositions in the quantified denotative form.

Figure I. Corresponding to *Barbara* we have the form

If any S is X, that S is P,
If any S is M, that S is X,
$\therefore$ *If any S is M, that S is P.*

As a material example may be given: 'If any person is selfish, he is unhappy; if any child is spoilt, that child is selfish; therefore, if any child is spoilt, he is unhappy.'

Figure II. Corresponding to *Cesare* is the form

If any S is P, then never is it X,
If any S is M, then always it is X,
∴ If any S is M, then never is it P.

An example is: 'If any act is done from a sense of duty, it is never formally wrong; if any act is done from purely selfish motives, it is always formally wrong; therefore, if any act is done from purely selfish motives, it is not done from a sense of duty.'

Figure III. Corresponding to *Bocardo* is the form

If an S is X, then sometimes it is not P,
If any S is X, then always it is M,
∴ If an S is M, then sometimes it is not P.

We may give as an example: 'If a war is just, it is sometimes not successful; if any war is just, it is always waged in defence of some right; therefore, if a war is waged in defence of some right, it is sometimes not successful.' Here it is evident nothing is lost by transferring the syllogism to the categorical form, and saying: 'Some just wars are not successful; all just wars are waged in defence of some right; therefore, some wars waged in defence of a right are not successful.' This has exactly the same force as the conditional form, for the latter does not imply that the want of success is a necessary consequence of the character of the war. But, in the examples with universal conclusions, it is evident there is such a dependence of consequent upon antecedent, which is lost if the syllogism be transferred to the categorical form.

Figure IV. Corresponding to *Dimaris* is the form

If an S is P, it is sometimes X,
If any S is X, it is always M,
∴ If an S is M, it is sometimes P.

This may be illustrated by: 'If the currency of a country consists of inconvertible bank notes, it is sometimes depreciated; if the currency of any country is depreciated, it

causes an artificial inflation of prices; therefore, if the currency of a country causes an artificial inflation of prices, it sometimes consists of inconvertible bank notes.' Here, again, it is evident that the antecedent does not state the necessary ground or reason for the consequent, and nothing is lost by reducing the whole argument to the categorical form.

9. **Pure Disjunctive Syllogisms.**—The possibility of syllogisms consisting entirely of disjunctive propositions has not usually been considered by logicians. Indeed, it is only with certain limitations that such syllogisms are possible at all. They can, to begin with, only be syllogisms with an affirmative conclusion, as no disjunctive proposition can be negative. Only the affirmative moods are, therefore, possible, and, of these, that corresponding to *Barbara* is the only one of any importance. Further, we only secure a middle term when one of the alternatives in the minor premise negatives one of those in the major premise. From

$$\begin{array}{c} S \textit{ is either } P \textit{ or } Q \\ S \textit{ is either } P \textit{ or } R \end{array}$$

no conclusion can be drawn, except that S *is either* P *or* Q *or* R, which simply sums up the premises. But from

$$\begin{array}{c} S \textit{ is either } P \textit{ or } Q \\ S \textit{ is either } \bar{P} \textit{ or } R \end{array}$$

we can draw the conclusion S *is either* Q *or* R. This will, perhaps, be more clearly seen if each premise is expressed as a hypothetical proposition. We can write the premises in the form

$$\begin{array}{c} \textit{If } S \textit{ is } \bar{P} \textit{ it is } Q \\ \textit{If } S \textit{ is } \bar{R} \textit{ it is } \bar{P} \end{array}$$

whence it follows that *If* S *is* $\bar{R}$ *it is* Q, which expresses the disjunctive S *is either* Q *or* R. Such syllogisms are, however, of infrequent occurrence. As the order of the alternatives is indifferent it will be seen that the distinctions of figure have here no proper application.

CHAPTER XXI.

REDUCTION OF SYLLOGISMS.

1. **Function of Reduction.—Reduction is the process by which a given syllogistic argument is expressed in some other Figure or Mood.**

Reduction is generally confined to expressing in the First Figure arguments given in the other Figures, and though the process can be applied with equal ease to changing reasonings from any one figure to any other which contains the required conclusion, and even from one mood to another in the same figure, these processes are of no utility and need not be considered.

Nor is reduction to Figure I necessary to establish the validity of a syllogism expressed in another figure. As has been seen the figure of a syllogism is due to the nature of the propositions which form its premises, so that some arguments fall most naturally into figures other than the First, and to reduce them to that form is to substitute an awkward and unnatural expression for a simple and natural one. The validity of such arguments is as immediately obvious as is that of the moods of the First Figure, and, consequently, Reduction is unnecessary in order to prove validity.

It does not follow that Reduction has no legitimate place in syllogistic theory. It is true that the reasoning does not become more cogent by being reduced to the First Figure, but its distinctive character is more immediately obvious in that figure than in any other. Reduction thus makes evident the essential unity of all forms of syllogistic inference, and systematises the theory of syllogism by showing that all the various moods are, at bottom, expressions of but one principle.

2. **Explanation of the Mnemonic Lines.**—The primary intention of the mnemonic lines given in the last chapter is to indicate the processes by which syllogisms in figures other than the first can be reduced to that figure. This is ingeniously done by means of the consonants employed. For convenience of reference we will here repeat the lines—

*Barbara, Celarent, Darii, Ferio*que prioris:
Cesare, Camestres, Festino Baroco [or *Faksoko*], secundæ:
Tertia, *Darapti, Disamis, Datisi, Felapton,*
Bocardo [or *Doksamosk*], *Ferison,* habet: Quarta insuper addit.
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

The two additional names, given in square brackets, refer to the direct process of reduction, while *Baroco* and *Bocardo* indicate the indirect process adopted by the scholastic logicians.

Some writers replace the *c* in *Baroco* and *Bocardo* by *k*, but this letter is required for the two additional mnemonics for those moods, and cannot, therefore, be used in the older ones without confusion, as it would then denote two entirely different processes.

The initial letters of the moods in the First Figure are the first four consonants. The same initial letters in the moods of the other figures indicate that the moods so named can be reduced to the mood in the First Figure which has the same initial. In the other figures—

s denotes simple conversion of the preceding proposition.
p indicates that the preceding proposition is to be converted *per accidens.*
m signifies metathesis, or transposition, of the premises.
k denotes obversion of the preceding proposition.
ks indicates obversion followed by conversion—that is, contraposition—of the preceding proposition.
sk signifies that the simple converse of the preceding proposition is to be obverted.
c shows that the syllogism is to be reduced indirectly (*conversio syllogismi*, or change of the syllogism).

When one of these letters occurs in the middle of a word, one of the premises of the *original* syllogism is to undergo the process of eduction indicated. Now, when one of the changes indicated is the transposition of premises, the position of the extreme terms is reversed, and the major term of the original syllogism becomes the minor term of the new. The conclusion must, therefore, be converted to bring it to the original form.

Thus every word in which *m* occurs ends in *s*, *p*, or *sk*, and these letters indicate that the conclusion of the *new* syllogism is to be converted. It will be noticed that no other significant letter ends a word. The only meaningless letters are thus seen to be *r*, *t*, *l*, *n*, and *b* and *d* when they are not initial. Several attempts so to change the forms of the words as to omit meaningless letters, and to employ a distinctive letter for each mood have been made but none of them is likely to replace the traditional forms.

3. **Kinds of Reduction.**—It was indicated in the last section that there are two kinds of Reduction—*Direct* and *Indirect*, the latter being usually restricted to the moods *Baroco* and *Bocardo*.

(i) **Direct or Ostensive Reduction.**—*Reduction is direct when the original conclusion is deduced from premises derived from those given.* The original premises are changed by conversion, transposition, or obversion.

(*a*) *Conversion.*

(1) The moods *Cesare*, *Festino*, *Datisi*, *Ferison*, and *Fresison*, are reduced to the First Figure by simply converting one, or both, of the premises. For example, *Cesare* (Fig. II) becomes *Celarent* in Figure I—

$$\begin{array}{ccc} P\,e\,M & \text{———————} & M\,e\,P \\ S\,a\,M & & S\,a\,M \\ \hline \therefore S\,e\,P & & \therefore S\,e\,P \end{array}$$

and *Fresison* (Fig. IV) becomes *Ferio* (Fig. I)—

$$\begin{array}{rcl} P\ e\ M & \text{————————} & M\ e\ P \\ \underline{M\ i\ S} & \text{————————} & \underline{S\ i\ M} \\ \therefore S\ o\ P & & \therefore S\ o\ P \end{array}$$

(2) The moods *Darapti* and *Felapton* are reduced by converting the minor premise *per accidens.* Thus *Darapti* (Fig. III) becomes *Darii* (Fig. I)—

$$\begin{array}{rcl} M\ a\ P & & M\ a\ P \\ \underline{M\ a\ S} & \text{————————} & \underline{S\ i\ M} \\ \therefore S\ i\ P & & \therefore S\ i\ P \end{array}$$

(3) *Fesapo* (Fig. IV) is reduced to *Ferio* (Fig. I) by the simple conversion of its major, and the conversion *per accidens* of its minor premise—

$$\begin{array}{rcl} P\ e\ M & \text{————————} & M\ e\ P \\ \underline{M\ a\ S} & \text{————————} & \underline{S\ i\ M} \\ \therefore S\ o\ P & & \therefore S\ o\ P \end{array}$$

(*b*) *Transposition of premises.* This, as has been seen, involves conversion of the new conclusion.

(1) The moods *Bramantip*, *Camenes*, and *Dimaris*, all in Figure IV, reduce to the First Figure by merely transposing the premises. Thus *Bramantip* becomes *Barbara*—

$$\begin{array}{rcl} P\ a\ M & & M\ a\ S \\ \underline{M\ a\ S} & \times & \underline{P\ a\ M} \\ \therefore S\ i\ P & & \therefore P\ a\ S \\ & & \therefore \text{(by Conv.) } S\ i\ P \end{array}$$

(2) *Camestres* and *Disamis* are reduced to the First Figure by transposing one premise with the simple converse of the other. Thus, *Disamis* (Fig. III) becomes *Darii* (Fig. I)—

$$\begin{array}{rcl} M\ i\ P & & M\ a\ S \\ \underline{M\ a\ S} & \times & \underline{P\ i\ M} \\ \therefore S\ i\ P & & \therefore P\ i\ S \\ & & \therefore \text{(by conv.) } S\ i\ P \end{array}$$

(c) *Obversion.*

(1) The mnemonic *Faksoko* indicates that *Baroco* (Fig. II) may be reduced to *Ferio* (Fig. I) by contrapositing the major premise and obverting the minor. Thus—

$$
\begin{array}{lcl}
P\,a\,M & \text{———} & \bar{M}\,e\,P \\
S\,o\,M & \text{———} & S\,i\,\bar{M} \\
\hline
\therefore S\,o\,P & & \therefore S\,o\,P
\end{array}
$$

(2) Similarly *Doksamosk* signifies that *Bocardo* (Fig. III) may be reduced to *Darii* (Fig. I) by contrapositing the major premise and making it the minor, and then obverting the simple converse of the new conclusion. Thus—

$$
\begin{array}{lcr}
M\,o\,P & & M\,a\,S \\
M\,a\,S & & \bar{P}\,i\,M \\
\hline
\therefore S\,o\,P & & \therefore \bar{P}\,i\,S \\
 & & \therefore \text{(by conv.) } S\,i\,\bar{P} \\
 & & \therefore \text{(by obv.) } S\,o\,P
\end{array}
$$

(ii) **Indirect Reduction.**—*Reduction is indirect when a new syllogism is formed which establishes the validity of the original conclusion by showing the illegitimacy of its contradictory.* This method is also called *Reductio per impossibile* or *Reductio ad absurdum.* It can be applied to any mood, though in practice it is usually confined to *Baroco* and *Bocardo*; and this application is the only one contemplated in the original mnemonics. The method is founded on the Principle of Contradiction. When a conclusion is legitimately deduced from two given premises, it is formally true; when it is not so deduced from them, it is formally false. In judging of the validity of an inference, this formal truth, or self-consistency, is all we are concerned with. Now, if the conclusion is formally false, its contradictory must be formally true. If this contradictory is combined with one of the original premises, a new syllogism is formed whose conclusion will either be identical with, or will contradict, the remaining original

premise. If it contradicts it, it proves that the contradictory of the original conclusion was formally false, that is, that conclusion was formally true. Thus the validity of the original syllogism is established.

For example, *Baroco* is proved valid by a syllogism in *Barbara*. For if the conclusion, $S\ o\ P$, is formally false, then its contradictory, $S\ a\ P$, is formally true, that is, is an inference from the two premises $P\ a\ M$, $S\ o\ M$. Replacing the premise followed by *c* by this contradictory of the original conclusion, we get the following syllogism in *Barbara*, with P for its middle term—

$$\begin{array}{c} P\,a\,M \\ S\,o\,M \\ \hline \therefore S\,o\,P \end{array} \qquad\qquad \begin{array}{c} P\,a\,M \\ S\,a\,P \\ \hline \therefore S\,a\,M \end{array}$$

Thus, if $S\ a\ P$ is formally true so is $S\ a\ M$. But $S\ a\ M$ contradicts $S\ o\ M$ which is one of the original premises, and is, therefore, formally false. Hence, $S\ a\ P$ is also formally false; that is, the original conclusion, $S\ o\ P$, is formally true, and *Baroco* is a valid mood.

Similarly with *Bocardo*. If the conclusion, $S\ o\ P$, is formally false, its contradictory, $S\ a\ P$, is formally true. Replacing the premise followed by *c* by this proposition, we get a syllogism in *Barbara*, with S for its middle term—

$$\begin{array}{c} M\,o\,P \\ M\,a\,S \\ \hline \therefore S\,o\,P \end{array} \qquad\qquad \begin{array}{c} S\,a\,P \\ M\,a\,S \\ \hline \therefore M\,a\,P \end{array}$$

But $M\ a\ P$ contradicts the original major premise $M\ o\ P$. Therefore, $M\ a\ P$ is formally false, and this entails the formal falsity of $S\ a\ P$. Therefore, the original conclusion, $S\ o\ P$, is formally true, and *Bocardo* is a valid mood.

This process, though formally cumbrous, and, therefore, inferior for purposes of logical theory to the direct reduction by obversion, is a very effective weapon in controversy and is perhaps one of those most commonly employed. Euclid, as we all know, used it frequently as a method of proof.

4. Reduction of Pure Hypothetical Syllogisms.—The validity of the reduction of any syllogism depends upon the legitimacy of the processes of immediate inference involved. With hypothetical propositions all these processes are valid, consequently pure hypothetical syllogisms can be reduced in exactly the same way as categorical syllogisms. For example, the pure hypothetical syllogism corresponding to *Cesare* (Fig. II)[1] is reduced to the form in Figure I agreeing with *Celarent*, by simply converting the major premise, so that we get—

(conv. of orig. major) *If any S is X, then never is it P,*
If any S is M, then always it is X,

$\therefore$ *If any S is M, then never is it P.*

The form corresponding to *Bocardo* (Fig. III)[2] is directly reduced to that agreeing with *Darii* by contrapositing the major premise and transposing the premises. The new conclusion has then to be converted, and the converse obverted. We thus get—

(orig. minor) *If any S is X, then always it is M,*
(contrap. of orig. major) *If an S is $\bar{P}$, then sometimes it is X,*

If an S is $\bar{P}$, then sometimes it is M;
$\therefore$ (by conv.) *If an S is M, then sometimes it is $\bar{P}$,*
$\therefore$ (by obv.) *If an S is M, then sometimes it is not P.*

[1] See p. 248. [2] See p.248.

CHAPTER XXII.

MIXED SYLLOGISMS.

1. Mixed Hypothetical Syllogisms.—When one of the premises of a syllogism is a hypothetical and the other a categorical proposition, the former is called the major, as it furnishes the ground of the inference; while the latter is the minor, as it states a case in which the major is applicable. The inference conforms to the same principles whether the major premise is stated in the fundamental abstract connotative or in the derived concrete enumerative form, which we have called conditional.[1] But in the latter case the fundamental character of syllogistic inference—the application of a general principle to a special case—is perhaps more plainly seen than in the former. For, when the major premise is a conditional proposition, it lays down, in so many words, a general dependence of one phenomenon upon another, though it makes no assertion as to whether or not either of these phenomena occurs in any special instance. The categorical minor affirms, or denies, the occurrence of one of these phenomena in some special case, and thus enables us, by applying the general rule given in the major, to conclude as to the occurrence, or non-occurrence, of the other phenomenon in that same case. When, however, the major premise is stated in the abstract hypothetical form making explicit the ground for the connexion of content—*If S is M it is P*—then the application to reality is not made through some particular instance of *S*, but must be mediated by the ascertained nature of *S* itself; in other words the minor premise must be the generic judgment *S is M*, and the conclusion is the generic judgment *S is P*.

[1] See p. 107.

(i) **Basis of Mixed Syllogistic Reasoning from a hypothetical major premise.**—As the inference in these syllogisms is as purely formal as when both the premises are categorical, it must ultimately rest on the fundamental principles of thought.[1] There is a very distinct reference to the Principle of Sufficient Reason, which may indeed be regarded as the specific axiom of such syllogisms. This principle of thought and necessary postulate of knowledge compels us to grant the conclusion which follows from any data we have accepted. Applied to syllogisms with a hypothetical major premise this means that, if in the minor we assert the antecedent of the major to be true in fact, we must accept, as a conclusion, the truth of the consequent. But a stricter examination shows that this is an application of the Principle of Identity. On the other hand, if, in the minor, we deny the consequent of the major, we must, in the conclusion, reject the antecedent. For, by the Principle of Excluded Middle, the antecedent must be either true or false, and, if it were true, the consequent would be true; and by the Principle of Contradiction, neither the antecedent nor the consequent can be both true and false; therefore, the denial of the consequent necessitates that of the antecedent.

(ii) **Determination of Valid Moods.**—It is thus seen that the assertion of the truth of the antecedent of a hypothetical proposition justifies the assertion of the truth of the consequent, and the denial of the consequent necessitates the denial of the antecedent. But the same consequent may result from more than one antecedent; and, therefore, the denial of the given antecedent will not justify the denial of the consequent, nor will the assertion of the consequent warrant that of the given antecedent. For example, though if a man is shot through the heart he dies, yet men also die from other causes. The denial that he is shot through the heart will not, therefore, warrant the denial of his death; nor will the assertion of his death necessitate the statement that it was due to this particular cause. We may express symbolically the various ante-

[1] See Ch. II.

cedents which lead to the same consequent, using the most general formula of the hypothetical proposition, as in this respect it does not matter whether the consequent has the same subject as the antecedent or not—

If **A**, *then* **C**.
If **X**, *then* **C**.
If **Y**, *then* **C**.
If **Z**, *then* **C**.

Here, it is evident that if we deny **A**, we still leave open several possibilities of the occurrence of **C**, for either **X**, **Y**, or **Z**, may be true; and if we assert **C**, though we, thereby, assert *one* of its possible antecedents, we cannot tell *which one*; nor have we, indeed, in either case, any security that all the possible antecedents of **C** are known to us. If, indeed, **A** is the only possible antecedent of **C**, its denial is a material justification for the rejection of **C**, and the affirmation of **C** is a material warranty for that of **A**. But these material conditions do not hold in all cases, and we are not, therefore, justified in assuming them in any; in formal inference we can deal only with that which holds universally.

Now, as **C** may follow from several other antecedents besides **A**, it corresponds to an undistributed term. If, however, the denial of **C** were deduced from the denial of **A**, **C** would be used universally in the conclusion. Again, when **C** is affirmed, it is affirmed in one case only out of several possible ones; to posit **A** as a result of such affirmation of **C** would be to disregard this. Thus, the fallacy of denying the antecedent is analogous to an illicit process of the major term, and that of affirming the consequent bears a similar resemblance to an undistributed middle. In each, the unwarranted assumption is made, that the major premise embraces every case in which the consequent can be true.

There are thus two, and only two, valid processes of formal syllogistic inference from a hypothetical major premise. They are covered by the canon:

To affirm the antecedent is to affirm the consequent; to deny the consequent is to deny the antecedent.

In the former case the syllogism is said to be *Constructive*, or in the *Modus Ponens*; in the latter case, *Destructive*, or in the *Modus Tollens*.

When, in such a syllogism, the major premise is a negative hypothetical, it is more convenient, and equally natural, to regard the negation as belonging to the consequent. The major may, then, take any one of four forms, as both the antecedent and the consequent may be either affirmative or negative. There can, therefore, be four forms both of the *Modus Ponens* and of the *Modus Tollens*. But it must be remembered that these names have no reference to the quality either of the minor premise or of the conclusion, but simply to whether the minor enables us, in the conclusion, to affirm the consequent, or to deny the antecedent, of the major, whatever that antecedent or consequent may be. To each of these varieties of the two moods separate names are given by German logicians. These names, however, are based on the quality of the minor premise and the conclusion—*ponens* marking affirmative, and *tollens* negative, quality—and thus the same name may denote either a *Modus Ponens* or a *Modus Tollens*. Still using the one general formula to denote all forms of hypothetical propositions, these varieties of the two moods are thus expressed symbolically—

(*A*) **Modus Ponens.**

(1) *Modus ponendo ponens.*

If **A** *then* **C**,
A.

∴ **C**.

(2) *Modus ponendo tollens.*

If **A**, *then* **not C**,
A,

∴ **Not C**.

(3) *Modus tollendo ponens.*

If **not A**, *then* **C**,
Not A,

∴ **C**.

(4) *Modus tollendo tollens.*

If **not A**, *then* **not C**,
Not A.

∴ **Not C.**

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.*

If **A**, *then* **C**,
Not C,

∴ **Not A.**

(2) *Modus ponendo tollens.*

If **A**, *then* **not C**,
C,

∴ **Not A.**

(3) *Modus tollendo ponens.*

If **not A**, *then* **C**,
Not C,

∴ **A.**

(4) *Modus ponendo ponens.*

If **not A**, *then* **not C**,
C,

∴ **A.**

The identity of the names of the subordinate moods points out that the *Modus Ponens* and the *Modus Tollens* are, at bottom, identical. On comparing the majors of the moods with the same name it is seen that they are the obverted contrapositives of each other, with the antecedent and consequent transposed. It follows that, if we obvert the contrapositive of the major of any form of the *Modus Ponens*, we shall get the corresponding form of the *Modus Tollens*; and that the latter can be similarly reduced to the former. For example, if we take the *modus ponendo ponens* of the *Modus Ponens*

If **A**, *then* **C**,
A,

∴ **C,**

and obvert the contrapositive of its major, we get

If **not C**, *then* **not A**,
A,
———————
∴ **C**,

which is the *modus ponendo ponens* of the *Modus Tollens*. Similarly, if we take the *modus ponendo tollens* of the *Modus Tollens*,

If **A**, *then* **not C**,
C,
———————
∴ **Not A**,

by obverting the contrapositive of its major we get

If **C**, *then* **not A**,
C,
———————
∴ **Not A**,

which is the corresponding form of the *Modus Ponens*.

It must be borne in mind that as, in a hypothetical proposition when it is stated in the conditional or enumerative form, the subject of both the antecedent and the consequent are quantified, the minor may negate the consequent of the major by affirming either its contradictory or its contrary; in each case, however, we are only justified in merely denying the antecedent of the major; that is, in asserting its *contradictory* as our conclusion. Thus, from the premises 'If all prophets spoke the truth, some would be believed; but none are believed' we are only justified in inferring that 'some prophets do not speak the truth,' not that 'no prophets do so.'

(iii) **Examples.**—We will now give some material examples of the various forms of mixed hypothetical syllogisms:—

(*A*) **Modus Ponens.**

(1) *Modus ponendo ponens.* If any country increases in wealth, it increases in power; England is increasing in wealth; therefore, England is increasing in power.

(2) *Modus ponendo tollens.* If any import duty is imposed simply for revenue purposes, that duty is not protective; English import duties are imposed simply for purposes of revenue; therefore, English import duties are not protective.

(3) *Modus tollendo ponens.* If any swan is not white, it is black; Australian swans are not white; therefore, Australian swans are black.

(4) *Modus tollendo tollens.* If any war is not defensive, it is not just; the wars waged by Napoleon the Great were not defensive; therefore, those wars were not just.

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.* If any country is civilised it has a population among whom education is general; the people of Russia are not generally educated; therefore, Russia is not a civilised country.

(2) *Modus ponendo tollens.* If any social institution is justifiable, it oppresses no class of the community; slavery does oppress a class; therefore, slavery is not a justifiable social institution.

(3) *Modus tollendo ponens.* If any railway is not required in the district through which it runs, it is a financial failure; the great English lines are not financial failures; therefore, they are required in the districts through which they run.

(4) *Modus ponendo ponens.* If any country has no capital invested abroad, its imports will not exceed its exports; England's imports do exceed her exports; therefore England has capital invested abroad.

A few examples may be added of similar inferences when the hypothetical major has not been reduced to the fundamental form with the same subject to both antecedent and consequent.

(*A*) **Modus Ponens.**

(1) *Modus ponendo ponens.* If all men are fallible, all philosophers are fallible; but all men are fallible; therefore, all philosophers are fallible.

(2) *Modus ponendo tollens.* If all our acts are within our own control, no vice is involuntary; all our acts are within our own control; therefore, no vice is involuntary.

(3) *Modus tollendo ponens.* If vindictiveness is not a justifiable emotion, all punishment should be simply preventive; vindictiveness cannot be justified; therefore, all punishment should be simply preventive.

(4) *Modus tollendo tollens.* If seeking his own pleasure is not man's chief end, the egoist is not truly moral; the seeking his own pleasure is not man's chief end; therefore, the egoist is not truly moral.

(*B*) **Modus Tollens.**

(1) *Modus tollendo tollens.* If all prophets spoke the truth, some would be believed; but none are believed; therefore, some do not speak the truth.

(2) *Modus ponendo tollens.* If some of a man's deliberate acts are wholly determined by circumstances, he is not morally responsible for them; but a man is morally responsible for all his deliberate acts; therefore, no such acts are wholly determined by circumstances.

(3) *Modus tollendo ponens.* If no men were mad, lunatic asylums would be useless; but they are not useless; therefore, some men are mad.

(4) *Modus ponendo ponens.* If the earth did not rotate on its axis, there would be no alternation of day and night; there is such alternation; therefore the earth does rotate on its axis.

2. **Mixed Disjunctive Syllogisms.—A Mixed Disjunctive Syllogism, in the strict sense of the term, is one in which the inference is drawn from the disjunctive form of the major premise.**

(i) **Basis of syllogistic inference from a disjunctive major premise.**—If two alternatives are given in the major premise, the denial of one of them in the minor justifies the assertion of the other in the conclusion. Such an inference is purely formal, and is, therefore, based on the fundamental principles of thought. Though the common formula for a disjunctive proposition is *S is either P or Q*, yet even here the alternation is, at bottom, between the two propositions *S is P* and *S is Q*. And an alternation may be equally well asserted between two propositions with different subjects, as *Either S is P or M is Q*. If, then, we denote the alternative propositions by **X** and **Y**, we shall have the simple formula for disjunctive propositions—*Either* **X** *or* **Y**, which is more comprehensive than the customary *S is either P or Q*. Now, if we accept as the major premise the disjunctive proposition *Either* **X** *or* **Y**, we know that one, at least, of these alternatives must be true, *i.e.* **not X** ensures **Y**. If the minor premise denies **X**, it must, by the principle of Excluded Middle, affirm **not X**, and this justifies the affirmation of **Y**. But the alternatives may be both negative—*Either* **not X** *or* **not Y**, and this may be written *Not both* **X** *and* **Y**. Here again, if one of the alternatives is false, the other must be true; that is, **X** ensures **not Y**. If, then, the minor posits **X**, it must, by the Principle of Contradiction, deny **Y**, for, in this case, **X** and **Y** cannot be true together.

(ii) **Forms of Mixed Disjunctive Syllogisms.**—The denial of one alternative, then, justifies the affirmation of the other. And, if the number of alternatives is greater than two, the same rule holds—the denial of any number justifies the affirmation of the rest, categorically if only one is left, disjunctively if more than one remain. Thus—

Either **X** *or* **Y** *or* **Z**,
Neither **X** *nor* **Y**,

∴ **Z**.

and—

Either **X** *or* **Y** *or* **Z**,
Not **X**,

∴ *Either* **Y** *or* **Z**.

As a disjunctive proposition does not formally imply that the alternatives are mutually exclusive, we cannot infer the denial of one of them from the assertion of the other. Those logicians who hold the opposite view, of course, assert that this can be done. But, even if the exclusive view were right, and *S is either P or Q* implied that *S* could not be *both P and Q*, yet when it is inferred that *S* is not *Q* because it is *P*, the inference is plainly made from the categorical proposition, *No P is Q*, which the disjunctive major premise is held to imply, instead of from that major premise itself. Such an argument, therefore, even if valid, would not be a disjunctive syllogism. We may, then, give as the canon of syllogistic inferences from a disjunctive proposition:

To deny one member (or more) of any alternation is to affirm the other member or members.

This gives one mood only of mixed Disjunctive Syllogisms, commonly called the *Modus tollendo ponens* because it affirms one alternative by denying the other.

As, however, both the alternative members may be either affirmative or negative, this mood may take four forms, corresponding to the subordinate forms of the two more fundamental moods of mixed hypothetical syllogisms. Both minor premise and conclusion, therefore, may be either affirmative or negative categorical propositions. The forms are thus expressed symbolically, the first being the standard—

(1) *Either* **X** *or* **Y**,
Not **X**,

∴ **Y**.

(2) *Either* **X** *or* **not Y**,
Not **X**,

∴ **Not Y**.

(3) *Either* **not X** *or* **Y,**
X,

∴ Y.

(4) *Either* **not X** *or* **not Y,**
X.

∴ Not Y.

(iii) **Reduction of Mixed Disjunctive Syllogisms.**—As every disjunctive proposition may be expressed in hypothetical form[1] every disjunctive syllogism may be expressed as a mixed syllogism with a hypothetical major premise. When this is done, the above four forms are seen to be equivalent to (1) the *modus tollendo ponens*, (2) the *modus tollendo tollens*, (3) the *modus ponendo ponens*, and (4) the *modus ponendo tollens* of the *Modus Ponens* when the denial of the first alternative is taken as the antecedent of the hypothetical major premise, and to the same forms of the *Modus Tollens* when the denial of the second alternative is so taken. As every syllogism in the *Modus Ponens* is reducible to a categorical syllogism in the First Figure, and every syllogism in the *Modus Tollens* to a similar syllogism in the Second Figure, it follows that every disjunctive syllogism can be expressed at will as a categorical syllogism in either of these figures. This again illustrates the essential unity of the syllogistic process, though the reduction has no other value.

(iv) **Examples.**—As examples of the four possible forms of mixed disjunctive syllogisms we may give—

(1) Every tax which provokes general dissatisfaction is either onerous in amount, or unjust in its incidence; the unpopular Poll Tax of Richard II was not onerous in amount; therefore, it was unjust in its incidence.

(2) Any country which maintains a protective tariff either intends to subordinate present to future advantage, or fails to see its own interest clearly; America, in maintaining her protective policy,

[1] See pp. 110-111.

has no intention of subordinating the interests of the present to those of the future; therefore, she fails to see her own interests clearly.

(3) Every revolution is either unjustifiable, or is provoked by oppression; the French Revolution of 1789 was justifiable; therefore, it was provoked by oppression.

(4) Any penalty which fails to diminish the crime of which it is the appointed punishment, is either of insufficient severity, or is sometimes not incurred by the criminal; the penalty for murder thus fails, and being death, is of sufficient severity; therefore, its infliction on the culprit is not certain.

We will add a few examples in which the alternatives in the major premise have not the same subject—

(1) Either the ancient Athenians were highly civilised, or the highest artistic culture is possible amongst a people of inferior civilisation; but this latter alternative is impossible; therefore, the ancient Athenians were highly civilised.

(2) Either vice is voluntary, or man is not responsible for his actions; but man is so responsible; therefore, vice is voluntary.

(3) Either no man should be a slave, or some men are incapable of virtue; but no men are incapable of virtue; therefore, no man should be a slave.

(4) Either poverty is never due to misfortune, or desert sometimes goes unrewarded; but poverty is sometimes due to misfortune; therefore, desert does sometimes go unrewarded.

3. **Dilemmas.—A Dilemma is a syllogism with a compound hypothetical major premise and a disjunctive minor.**

In other words, the major contains a plurality either of antecedents or of consequents, which are either disjunctively affirmed, or disjunctively denied, in the minor. The

peculiar feature of a dilemmatic argument is the choice of alternatives which it thus offers; and, when it is used rhetorically, the aim is to make these alternatives of such a kind that, whilst one must be accepted, all lead to results equally disagreeable to an opponent. Hence arose the saying 'to be on the horns of a dilemma.'

Strictly speaking, a *Dilemma* contains only two alternatives; if three are offered we have a *Trilemma*; if four, a *Tetralemma*; and if more than four, a *Polylemma*. As these more complex forms are governed by the same principles as the dilemma, it will be sufficient to consider the latter.

(1) Forms of the Dilemma.

(*A*) *Determination of Forms.* Like all mixed hypothetical syllogisms, a dilemma may be either *Constructive*—when the antecedents are affirmed; or *Destructive*—when the consequents are denied. In the former case, there must, of necessity, be two antecedents in the major premise, as otherwise the minor premise could not be disjunctive; but there may be either a single consequent—which the conclusion will affirm in the same form, which is usually the simple categorical; or two consequents—when the conclusion will always be disjunctive. In the former case the dilemma is *Simple*; in the latter case *Complex*. Similarly, the major premise of a destructive dilemma must contain two consequents, which may have either one or two antecedents, the dilemma being again *Simple* or *Complex* accordingly. We thus get four main forms of the dilemma, which may be expressed by the following formulae, in which each letter represents a proposition—

(1) *Simple Constructive.*

(*a*) *If either* **A** *or* **B**, *then* **C**,
Either **A** *or* **B**,

∴ **C**.

(*b*) *If either* **A** *or* **B**, *then either* **C** *or* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**.

(2) *Simple Destructive.*

(*a*) *If* **A**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ **Not A.**

(*b*) *If both* **A** *and* **B**, *then both* **C** *and* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

(3) *Complex Constructive.*

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**.

(4) *Complex Destructive.*

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

The second form of the Simple Constructive dilemma is simple because the alternative hypotheticals which form the major premise have only one consequent. The conclusion is disjunctive because this single consequent is disjunctive in form. Similarly, the second form of the simple destructive dilemma is not complex, although it has a disjunctive conclusion, for that conclusion is merely the simple denial of the one single antecedent of the major premise. It thus appears that these forms are not fundamental, but are only special cases of somewhat greater complexity of the simple forms.[1]

It will be noticed that the major premise of both forms of the simple destructive dilemma has its consequent copulative, and not disjunctive, in form. The reason is that when two consequents are alternatives their disjunctive denial will not justify the denial of the antecedent; for, if one of two alternatives is false, the other must be true, and the truth of one consequent is all that the antecedent of such a proposition demands. It is necessary that *both*

[1] *Cf.* Keynes, *Formal Logic*, 3rd Ed., pp. 317-318, *notes.*

the consequents should be connected with the whole antecedent, in order that the denial of their conjunction may justify the rejection of the antecedent as a whole.

We will now illustrate each of the above forms.

(1) (*a*) *Simple Constructive.* The inhabitants of a besieged town might express their position in some such dilemma as this: 'If we hold out, we shall suffer loss by the bombardment destroying our property; if we surrender, we shall suffer loss through having to pay the enemy a heavy ransom; but we must adopt one or other of these two courses; therefore, whichever way we act, we are bound to suffer loss.'

(*b*) This form, which is more indefinite than the former, neither antecedent being limited to one consequent, is much less frequently employed. As an example of it we may give: 'If either England is over-populated or its industry is disorganised, many people must either emigrate or live in deep poverty; England at present suffers either from over-population or from disorganisation of industry; therefore, many Englishmen must either emigrate or live in deep poverty.'

(2) (*a*) *Simple Destructive.* Euclid's proof of Proposition VII of the First Book may be exhibited as a dilemma of this kind: 'If two triangles on the same base, and on the same side of it, have their conterminous sides equal, then two angles are both equal and unequal to each other; but they are either not equal or not unequal; therefore, the existence of two such triangles is impossible.'

Whately gives the following example of such an argument: "If we admit the popular objections against Political Economy, we must admit that it tends to an excessive increase of wealth: and also, that it tends to impoverishment; but it cannot do *both* of these; (*i.e.* either not the one, or, not the other) therefore we cannot admit the popular objections, etc."[1]

(*b*) This form is very seldom used. As an example we may give: 'If compulsory education is unnecessary and no

[1] Whately, *Elements of Logic*, 5th Ed., pp. 117-118.

legal regulation of the conditions of the labour of children is justifiable, then all guardians of children both understand and try to perform their duty to those under their charge; but some guardians either do not understand their duty to their young wards or do not try to perform it; therefore, either compulsory education is necessary or some legal regulation of the conditions of children's labour is justifiable.'

(3) *Complex Constructive.* A good example of this form of dilemma is found in the oration of Demosthenes *On the Crown,* where he argues: 'If Aeschines joined in the public rejoicings, he is inconsistent; if he did not, he is unpatriotic; but either he did or he did not; therefore, he is either inconsistent or unpatriotic.'

The following argument is in the same form: 'If the Czar of Russia is aware of the persecutions of the Jews in his country, he is a tyrant; if he is not aware of them, he neglects his duty; but either he is, or he is not, aware of them; therefore, either he is a tyrant or he neglects his duty.'

(4) *Complex Destructive.* This, again, is not a very common form. An example is: 'If the industry of England is well organised, there is work for every efficient labourer who seeks it, and if all labourers are industrious, all will seek work; but either some labourers cannot get work or they will not seek it; therefore, either the industry of England is not well organised or some labourers are idle.'

(*B*) *Mutual Convertibility of Forms.* Like the simpler mixed hypothetical syllogisms, the constructive and destructive dilemmas are, at bottom, identical; for any form of the one may be converted to the corresponding form of the other by obverting the contrapositive of the major premise. Thus, the complex destructive and complex constructive dilemmas are, fundamentally, the same, and each of the two forms of the simple destructive is mutually convertible with the corresponding form of the simple constructive dilemma. In illustration of this it will be sufficient to reduce each of the destructive to a constructive form.

Simple Destructive. (*a*) By obverting the contrapositive of the major premise and retaining the original minor, we get—

If either **not C** *or* **not D**, *then* **not A**,
Either **not C** *or* **not D**,

∴ **Not A**;

which is the simple constructive form with negative, instead of affirmative, elements.

(*b*) Similarly, by obverting the contrapositive of the second form of the simple destructive we get the second form of the simple constructive, with negative elements—

If either **not C** *or* **not D**, *then either* **not A** *or* **not B**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**.

Complex Destructive. The obverted contrapositive of the major premise being taken, we get—

If **not C**, *then* **not A**, *and if* **not D**, *then* **not B**,
Either **not C** *or* **not D**,

∴ *Either* **not A** *or* **not B**;

which is the complex constructive form with negative elements.

This convertibility may be illustrated by an example. A man in bad health, and who has no income but his salary, may argue that his recovery is hopeless, either in the simple destructive dilemma: 'If I am to regain health, I must both give up work and live generously; but I cannot do both of these (that is, either I cannot do one, or I cannot do the other); therefore, I cannot regain health'; or in the simple constructive: 'If I either continue to work, or live meagrely, I cannot regain health; but I must either continue to work or live meagrely; therefore, I cannot regain health.'

(ii) **Rebutting a Dilemma.**—The conclusiveness of a dilemma depends upon material, as well as formal, considerations. Not only must the connexion of antecedent and consequent be a real one, but the disjunction in the minor premise must exhaust every possible alternative.

The difficulty of securing this is the reason dilemmatic arguments are so often fallacious.

Very often a faulty dilemma can be *rebutted* or *retorted* by an equally cogent dilemma proving the opposite conclusion. In such a case, the consequents of the major change places. And their quality is changed. Thus

If **A**, *then* **C**, *and if* **B**, *then* **D**,
Either **A** *or* **B**,

∴ *Either* **C** *or* **D**,

may be rebutted by the dilemma

If **A**, *then* **not D**, *and if* **B**, *then* **not C**,
Either **A** *or* **B**,

∴ *Either* **not C** *or* **not D**.

But the conclusion proved is not really incompatible with that of the original dilemma, for both can be satisfied by **C** and **not D** or by **D** and **not C** being true together. Only the complex constructive forms of the dilemma lend themselves to this treatment (though destructive dilemmas can be reduced to the constructive form and then rebutted), and, of course, only those in which some flaw exists in the original argument; a valid dilemma cannot be rebutted. There are several classical examples of dilemmas thus rebutted, the consideration of which will tend to make the subject clear.

An Athenian mother is said to have advised her son not to enter public life; 'for,' said she, 'if you act justly men will hate you, and if you act unjustly the gods will hate you; but you must act either justly or unjustly; therefore, public life must lead to your being hated.' This argument he rebutted by the equally cogent dilemma: 'If I act justly the gods will love me, and if I act unjustly men will love me; therefore, entering public life will make me beloved.' But, according to the given premises, a public man must always be both hated and loved; the given conclusions are not, therefore, incompatible.

More famous is the *Litigiosus*. Protagoras agreed to train Euathlus as a lawyer, one-half the fee to be paid at

once, and the other half when Euathlus won his first case. As Euathlus engaged in no suit, Protagoras sued him, and confronted him with this dilemma: 'Most foolish young man, if you lose this suit you must pay me by order of the court, and if you gain it you must pay me by our contract.' To which Euathlus retorted: 'Most sapient master, I shall not pay you; for if I lose this suit I am free from payment by our contract, and if I gain it, I am exonerated by the judgment of the court.' Of this difficulty several solutions have been offered. The most reasonable seems to be this: As Euathlus had until then won no case, the condition of the bargain was not fulfilled, and the judges should have decided in his favour. It was then open to Protagoras to bring a fresh suit, when the judgment must have gone against Euathlus.

Somewhat similar is the *Crocodilus*. A crocodile had seized a child, but promised the mother that if she told him truly whether or not he was going to give it back, he would restore it. Fearing that if she said he was going to give it back, he would prove her wrong by devouring it, she answered, 'You will not give it back'; and argued: 'Now you must give it back—on the score of our agreement if my answer is true, and to prevent its becoming true if it is false.' But the crocodile answered: 'I cannot give it back, for if I did your answer would become false, and thus I should break our agreement; and even could your answer be correct I could not give it back, as that would make it false.' On this Lotze says: "There is no way out of this dilemma; as a matter of fact however both parties rest their cases on unthinkable grounds; for the answer really given can as little be true or untrue independently of the actual result as could the answer she might have given, an answer which only differs from this in being more fortunate."[1] For, had she said 'You will give it back,' then its restoration would both have made her answer true and have fulfilled the agreement.

[1] *Logic*, Eng. trans., vol. ii., p. 20.

CHAPTER XXIII.

ABRIDGED AND CONJOINED SYLLOGISMS.

1. Enthymemes.—An Enthymeme is a syllogism abridged in expression by the omission of one of the constituent propositions.

The most common form in which syllogistic arguments are met with is the enthymematic. The tendency of speech is always to state explicitly no more than is required for clearness; and as, in most cases, when two of the constituent propositions of a syllogism are given the third is sufficiently obvious, it would be mere pedantry to express it in ordinary discourse. It is, therefore, but seldom that fully expressed syllogisms are met with outside treatises on logic. This condensation, however, makes it both more easy to commit fallacy and more difficult to detect it. A false conclusion is often supported by a perfectly true premise. The fact that the implied premise is false or wanting in pertinence is not realised.

In reasoned discourse the omission of a premise is more usual than the suppression of the conclusion. The latter is essentially a rhetorical device and always carries a suggestion of feeling. As, however, any one of the three constituent propositions of a syllogism may be omitted, we can speak of enthymemes as of three *orders*—

First Order—when the major premise is omitted.
Second Order—when the minor premise is omitted.
Third Order—when the conclusion is omitted.

For example, the argument of the fully stated syllogism: 'All democratic governments are liable to frequent changes in foreign policy; the English government is democratic; therefore, the English government is liable

to frequent changes in foreign policy'—may be expressed by an enthymeme of each order:—

First Order. 'The English government is liable to frequent changes in foreign policy, because it is democratic.'

Second Order. 'The English government is liable to frequent changes in foreign policy, because all democratic governments are liable to this.'

Third Order. 'All democratic governments are liable to frequent changes in foreign policy, and the English government is democratic.'

When an enthymeme is of the first or second order, more frequently than not the conclusion is stated first, and the premise given in its support is introduced by some such illative particle as 'because' or 'since.'

When an enthymeme is of the third order, it is, of course, immediately obvious to which of the syllogistic figures it belongs. When it is of either the first or second order, this must be determined by the position in the given premise of either the minor or the major term. If the given premise contains the subject of the conclusion, it is, necessarily, the minor premise, and the enthymeme is of the first order; if it contains the predicate of the conclusion, the enthymeme is of the second order. Now, if both the given premise and the conclusion have the same subject, the enthymeme must be in either the First or the Second Figure; for in those figures only is S the subject of the minor premise. Similarly, if both the propositions have the same predicate, the figure is either the First or the Third, for in these P is predicate of the major premise. If the predicate of the conclusion is the subject of the given premise, the argument belongs either to Figure II or to Figure IV, in each of which P is the subject of the major premise. Finally, if the subject of the conclusion is the predicate of the given premise, the figure is either the Third or the Fourth, for in each of these S is the predicate of the minor premise.

2. Progressive and Regressive Chains of Reasoning.—A train of thought may be conducted by means of a succession of syllogisms in which the conclusion of each

syllogism supplies a premise to that which follows. Two cases may be distinguished. In the first the conclusion forms the major premise, and in the second the minor premise, of the syllogism which it precedes. Symbolically, therefore, this may be represented as follows—

$$
\begin{array}{lll}
(1) & Y\,a\,P & \text{(major)} \\
 & X\,a\,Y & \text{(minor)} \\ \hline
 & \therefore X\,a\,P & \text{(concl.)} \\
(2) & X\,a\,P & \text{(major)} \\
 & M\,a\,X & \text{(minor)} \\ \hline
 & \therefore M\,a\,P & \text{(concl.)} \\
(3) & M\,a\,P & \text{(major)} \\
 & S\,a\,M & \text{(minor)} \\ \hline
 & \therefore S\,a\,P & \text{(concl.)}
\end{array}
\qquad
\begin{array}{lll}
(1) & S\,a\,Y & \text{(minor)} \\
 & Y\,a\,X & \text{(major)} \\ \hline
 & \therefore S\,a\,X & \text{(concl.)} \\
(2) & S\,a\,X & \text{(minor)} \\
 & X\,a\,M & \text{(major)} \\ \hline
 & \therefore S\,a\,M & \text{(concl.)} \\
(3) & S\,a\,M & \text{(minor)} \\
 & M\,a\,P & \text{(major)} \\ \hline
 & \therefore S\,a\,P & \text{(concl.)}
\end{array}
$$

A common proposition connects each two members of such a train of syllogisms; and the syllogisms thus related are called respectively *Prosyllogism* and *Episyllogism* with respect to each other.

A Prosyllogism is a syllogism whose conclusion is a premise in the syllogism with which it is connected.

An Episyllogism is a syllogism one of whose premises is the conclusion of the syllogism with which it is connected.

In these trains of reasoning the progress of thought has been from prosyllogism to episyllogism. Such a demonstration is called **Progressive, Episyllogistic,** or **Synthetic**: it is very common in mathematics and is constantly used by Euclid in his direct proofs. It is the method by which the consequences of general principles are exhibited. It may consist either of categorical or of hypothetical propositions. A good example of the latter is given by Ueberweg: "If there is a medium obstructing the motion of the planets, then the path of the earth cannot be constant nor periodical, but must always become less: If this

be the case, then the existence of organisms on the earth cannot have been (nor can remain) eternal. Hence, if there is this medium, organisms must have at one time come into existence, and will wholly pass away. If organisms once existed for the first time on the earth, they must have arisen out of inorganic matter. If this is the case, there has been an original production (generatio aequivoca). Hence, if this obstructive medium exists, there has been an original production."[1]

But in physical science it more frequently happens that the highest and most general principles are the last to be discovered. "Certain general propositions are first discovered (as, *e.g.*, the laws of Kepler) under which the individual facts are syllogistically subsumed. The highest principles are discovered later (*e.g.* the Newtonian law of Gravitation) from which those general principles are necessary deductions."[2] In such a course of reasoning, thought advances from the episyllogism to the prosyllogism, going backwards further and further towards first principles. A demonstration of this kind is, therefore, called **Regressive, Prosyllogistic,** or **Analytic.** It may be thus represented symbolically, the episyllogism being stated first, as in such a train it comes first in the order of thought:—

(1)	$S\ a\ P$	(concl.)	(1)	$S\ a\ P$	(concl.)
	$\because M\ a\ P$	(major)		$\because M\ a\ P$	(major)
	$S\ a\ M$	(minor)		$S\ a\ M$	(minor)
(2)	$M\ a\ P$	(concl.)	(2)	$M\ a\ P$	(concl.)
	$\because X\ a\ P$	(major)		$\because X\ a\ P$	(major)
	$M\ a\ X$	(minor)		$M\ a\ X$	(minor)
(3)	$X\ a\ P$	(concl.)	(3)	$S\ a\ M$	(concl.)
	$\because Y\ a\ P$	(major)		$\because Y\ a\ M$	(major)
	$X\ a\ Y$	(minor)		$S\ a\ Y$	(minor)

Such a train of reasoning, whether progressive or regressive, is often called a *Polysyllogism.*

[1] Ueberweg: *Logic,* Eng. trans., p. 464. [2] *Ibid.*, p. 465.

3. **Sorites.—A Sorites is a progressive chain of reasoning whose expression is simplified by the omission of the conclusion of each of the prosyllogisms.**

The Sorites is, thus, a series of enthymemes, of which the first is of the third order, as both its premises are stated; and the last is of either the first or the second order, as one premise and the conclusion are given. But each of the intermediate enthymemes is represented by one premise alone, as the other premise is the omitted conclusion of the preceding prosyllogism. In somewhat different words, therefore, it may be said that a sorites is a series of enthymemes, in each of which, except the first, one premise is implied by a prosyllogism, and the other is explicitly stated. From this it follows that a full analysis of a sorites resolves it into a number of separate syllogisms, less by one than the total number of premises.

(i) **Kinds of Sorites.**—It was seen in the last section that the conclusion of a prosyllogism may form either the minor or the major premise of the episyllogism. There are, consequently, two forms of sorites—the *Aristotelian*, in which the suppressed conclusions form the minor premises of the following episyllogisms; and the *Goclenian*, in which they form the major premises. The symbolic expression of each may be thus given—

Aristotelian Sorites—*Every S is X*
Every X is Y
Every Y is Z
Every Z is P
∴ *Every S is P*

Goclenian Sorites—*Every Z is P*
Every Y is Z
Every X is Y
Every S is X
∴ *Every S is P*

If both forms are analysed into their constituent syllogisms, it will be seen that in the Aristotelian form the omitted conclusions—which we enclose in square brackets

—form the minor, and in the Goclenian form the major, premises of the succeeding episyllogisms.

Analysis of Aristotelian Sorites.

$$\begin{array}{lll}
(1) & \textit{Every } X \textit{ is } Y & \text{(major)} \\
 & \textit{Every } S \textit{ is } X & \text{(minor)} \\
\hline
\therefore & [\textit{Every } S \textit{ is } Y] & \text{(concl.)} \\
(2) & \textit{Every } Y \textit{ is } Z & \text{(major)} \\
 & [\textit{Every } S \textit{ is } Y] & \text{(minor)} \\
\hline
\therefore & [\textit{Every } S \textit{ is } Z] & \text{(concl.)} \\
(3) & \textit{Every } Z \textit{ is } P & \text{(major)} \\
 & [\textit{Every } S \textit{ is } Z] & \text{(minor)} \\
\hline
\therefore & \textit{Every } S \textit{ is } P & \text{(concl.)}
\end{array}$$

Analysis of Goclenian Sorites.

$$\begin{array}{lll}
(1) & \textit{Every } Z \textit{ is } P & \text{(major)} \\
 & \textit{Every } Y \textit{ is } Z & \text{(minor)} \\
\hline
\therefore & [\textit{Every } Y \textit{ is } P] & \text{(concl.)} \\
(2) & [\textit{Every } Y \textit{ is } P] & \text{(major)} \\
 & \textit{Every } X \textit{ is } Y & \text{(minor)} \\
\hline
\therefore & [\textit{Every } X \textit{ is } P] & \text{(concl.)} \\
(3) & [\textit{Every } X \textit{ is } P] & \text{(major)} \\
 & \textit{Every } S \textit{ is } X & \text{(minor)} \\
\hline
\therefore & \textit{Every } S \textit{ is } P & \text{(concl.)}
\end{array}$$

It is evident that the two forms agree in the fact that each omitted conclusion is a premise of the following syllogism. Now, this advance from previous to consequent inferences is the characteristic of progressive reasoning; it is, therefore, an error to speak of the Goclenian Sorites, as some logicians have done, as a regressive form of reasoning.

Either form of sorites may be entirely composed of hypothetical propositions. In the Goclenian Sorites the last premise may be categorical, and then the concluding enthymeme is the abridged form of a mixed syllogism, in

which the categorical minor premise either affirms the antecedent, or denies the consequent, of the implied conclusion of the preceding prosyllogism; *e.g.*—

If **C**, *then* **D**,	*If* **C**, *then* **D**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **A**, *then* **B**,	*If* **A**, *then* **B**,
A,	**Not D**,
∴ **D**.	∴ **Not A**.

In the Aristotelian Sorites, however, the same result can only be obtained by adding to the sorites a categorical minor premise, and then regarding the implied conclusion of the preceding prosyllogism as the major, instead of the minor premise of the last episyllogism. In other words, a mixed syllogism at the end of a sorites must, in all cases, correspond to the Goclenian form; *e.g.*—

If **A**, *then* **B**,	*If* **A**, *then* **B**,
If **B**, *then* **C**,	*If* **B**, *then* **C**,
If **C**, *then* **D**,	*If* **C**, *then* **D**,
A,	**Not D**,
∴ **D**.	∴ **Not A**.

The following example of a categorical Aristotelian Sorites may be quoted: 'Action is that in which happiness lies; what contains happiness is the end and aim; the end and aim is what is highest; therefore, action is what is highest.'[1] As an instance of a similar sorites composed of hypothetical propositions we may give: 'If any man is avaricious, he is intent on increasing his wealth; if he is so intent, he is discontented; if he is discontented, he is unhappy; therefore, if any man is avaricious, that man is unhappy.' In the following the last syllogism is a mixed hypothetical: 'If the soul thinks, it is active; if it is active, it has strength; if it has strength, it is a substance; now the soul thinks; therefore, the soul is a substance.' In all these cases, if we reverse the order of premises, we get a sorites of the Goclenian form.

[1] Aristotle: *Poetics*, vi.

(ii) Special Rules of the Sorites.

(*a*) *The Aristotelian Sorites.* In this form of sorites, the predicate of the last premise is, in the conclusion, affirmed or denied of the first subject, through one or more intermediate propositions. Each intermediate term must, therefore, be affirmatively predicable of the whole of the preceding one, or the chain of connexion is broken. This gives us the two following as

Special Rules of the Aristotelian Sorites:

1. *Only one premise, and that the last, can be negative.*
2. *Only one premise, and that the first, can be particular.*

The necessity of these rules is evident when the sorites is analysed into its constituent syllogisms.

Rule 1. More than one premise cannot be negative; for, as a negative premise in any syllogism necessitates a negative conclusion, if more than one premise in the sorites were negative, one of the constituent syllogisms would contain two negative premises.

If any premise in the sorites is negative, the conclusion must be negative; therefore, the predicate of the conclusion must be distributed in the last premise, of which it is the predicate; that is, the last predicate must be negative.

Rule 2. As every premise except the last must be affirmative, it is evident that if any, except the first, were particular, it would involve the fallacy of undistributed middle.

(*b*) *The Goclenian Sorites.* In this form of sorites the predicate of the first premise is, in the conclusion, either affirmed, or denied, of the subject of the last, through one or more intermediate propositions. Each intermediate term must, therefore, be affirmed universally of the succeeding one, or the necessary connexion will not be secured. We, thus, get the two following as

Special Rules of the Goclenian Sorites:

1. *Only one premise, and that the first, can be negative.*
2. *Only one premise, and that the last, can be particular.*

A consideration of the constituent syllogisms again shows the necessity of these rules.

Rule 1. As in the Aristotelian sorites, a plurality of negative premises would result in one of the syllogisms containing two negative premises.

If any premise is negative, the conclusion must be negative; therefore its predicate must be distributed in the first premise, of which it is the predicate; that is, the first premise must be negative.

Rule 2. If any premise but the last were particular, the conclusion of the syllogism in which it occurred would also be particular, and, as that proposition would be the major premise of the succeeding syllogism, we should have the fallacy of undistributed middle.

The above rules assume, in each case, that the sorites is entirely in the First Figure; that is, that each of the constituent syllogisms is in that figure.

4. **Epicheiremas.—An Epicheirema is a regressive chain of reasoning abridged by the omission of one of the premises of each prosyllogism.**

Each prosyllogism, therefore, appears in the epicheirema as an enthymeme, though the episyllogism is stated in full. Each prosyllogism furnishes a reason in support of one of the premises of the episyllogism, and the whole epicheirema may be described as a syllogism with a reason given in support of one or both of its premises.

When one premise only is thus supported, the epicheirema is *Single*; when both are furnished with reasons, it is *Double*; and when those reasons themselves have other reasons attached to them, it is *Complex*. The progress of thought in an epicheirema is from the episyllogism to the prosyllogisms on which it depends; from the conclusion to the principles which support it.

Symbolic examples of the Double Epicheirema are—

(1) *Every M is P, because it is X,*
Every S is M, because it is Y,

$\therefore$ *Every S is P.*

(2) *Every M is P, because every A is,*
Every S is M, because every B is,
$\therefore$ *Every S is P.*

In the first case the enthymemes expressing the reasons are both of the first order, the suppressed major premises being *Every X is P*, and *Every Y is M*. In the second case both the enthymemes are of the second order, the implied minor premises being *Every M is A*, and *Every S is B*. Of course, both need not be of the same order. If we leave out one of the reasons in either of the above examples we have a single epicheirema.

We will now illustrate what has been said by material examples of the two forms of double epicheirema given above.

(1) 'All unnecessary duties on imports are impolitic, as they impede the trade of the country; the American protective duties are unnecessary, as they support industries which are quite able to stand alone; therefore, the American protective tariff is impolitic.'

(2) 'All Malays are cruel, because all savages are; all the aboriginal inhabitants of Singapore are Malays, because all the natives of that part of Asia are; therefore, all the natives of Singapore are cruel.'

CHAPTER XXIV.

FUNCTIONS OF THE SYLLOGISM.

1. Universal Element in Deductive Reasoning.—The essential feature of syllogistic reasoning is the subsumption of a particular case under a general rule; in other words, every deductive inference must rest on a universal element. This necessity has been denied by certain writers, of whom Mill may be taken as representative. He says: "All inference is from particulars to particulars: General propositions are merely registers of such inferences already made, and short formulae for making more: The major premise of a syllogism, consequently, is a formula of this description: and the conclusion is not an inference drawn *from* the formula, but an inference drawn *according to* the formula: the real logical antecedent, or premise, being the particular facts from which the general proposition was collected by induction."[1] To which he adds in a later passage that the "universal type of the reasoning process" is "resolvable in all cases into the following elements: Certain individuals have a given attribute; an individual or individuals resemble the former in certain other attributes; therefore they resemble them also in the given attribute."[2]

We frequently do reason by analogy from our experience of particulars to another particular instance, and such reasoning is fairly described in the last sentence quoted above from Mill, though we must demur to the claim that it is the "universal type of the reasoning process." Such arguments are often fallacious, but even when they are valid, on what do they really rest? Surely on an implicit universal, that is, on the presence in all the cases of an identical

[1] *Logic*, Book II., Ch. iii., § 4. [2] *Ibid.*, § 7.

element. It is not from the concrete case in all its aspects —that is, as a particular—that the conclusion is drawn, but only from this point of identity which relates that case to the new one. And such a common bond is what is meant by a universal. Similarity implies such identity amid some diversity. When we conclude from one case to other cases similar to it, it is the identity, not the difference, which carries our thought from the one to the other.

This is evident upon a careful examination of an example Mill himself gives: "It is not only the village matron, who, when called to a consultation upon the case of a neighbour's child, pronounces on the evil and its remedy simply on the recollection and authority of what she accounts the similar case of her Lucy."[1] But why does she account it "a similar case"? Is it not because she regards the symptoms observed in both cases as marks of the presence of one and the same disease? But if so, she is reasoning, not from her Lucy as an individual but, from the universal connexion between a certain disease and the symptoms Lucy exhibited in her sickness; and thence she infers that the remedies which proved efficacious in that case will prove equally beneficial. And she will be ready if need arises to make this inference, not only in this new case of the neighbour's child but, in all similar cases which may be brought under her notice. Though, then, she may never have formulated her belief in a generalised statement, yet she thinks it implicitly as a universal, and shows her thought by her readiness to act.

Thus, even in cases where the inference at first sight seems to be founded on one or more particular experiences, it is really based on the recognition of the universal element in which they agree; and this may be expressed in a general proposition which forms the major premise of a syllogism.

2. **Validity of Syllogistic Reasoning.**—Not only has the syllogistic process been asserted to be valueless, but its very validity has been frequently denied, on the ground

[1] *Ibid.*, § 3.

that it involves the fallacy of *petitio principii.* Strictly speaking, this should mean that the conclusion of every syllogism is itself assumed as one of the premises; but, more loosely, it is held to imply that the premises presuppose the truth of the conclusion, and cannot, therefore, be used to establish it. This argument was advanced, in the third century, by Sextus Empiricus, who said that the major premise must result from a complete testing of every instance which can come under it, and that, therefore, to deduce an individual fact from a general principle is to argue in a circle. The same argument has been adopted by the empiricist school generally. Thus, Mill says: "It must be granted that in every syllogism, considered as an argument to prove the conclusion, there is a *petitio principii; . . .* that no reasoning from generals to particulars can, as such, prove anything: since from a general principle we cannot infer any particulars, but those which the principle itself assumes as known."[1] Mill then proceeds to argue, as we have seen, that the real inference is from particulars to particulars, and that the syllogism is merely a guarantee of the validity of those inferences.

If a universal proposition were a mere 'universal of fact,' or summary of examined particulars, the cogency of this objection to the syllogism could not be denied. But in our examination of the universal judgment we saw that this was not so, and that the ground for such a judgment is not a complete counting but an analysis of content which establishes a necessary connexion of attributes. So the objection falls to the ground. Indeed it is itself a *Petitio,* as it assumes that the universal judgment originates in the only way which could lay it open to the charge. The truth of such a proposition is recognised, and even held to be necessary, before the totality of instances which come under it have been examined, or are, indeed, known. For instance, the laws of Kepler are syllogistically applied to all newly discovered planets and satellites without a doubt of the accuracy of the conclusion. Similarly, the universal validity of the law of gravitation was held to be

[1] *Logic,* Book II., Ch. iii., § 2.

so certain, that when the observed orbit of the planet Uranus appeared to violate it, the existence of a disturbing cause was inferred—an inference which led to the discovery of the planet Neptune.

Again, a syllogistic inference requires the combination of both premises, but the objection we are considering involves the tacit assumption that the minor is unnecessary. If, when you have admitted the major premise, you have asserted the conclusion, the minor premise is superfluous. But the necessity of the minor premise is granted, and this necessity is a proof that such an inference is not a *petitio principii.*

If the syllogism were really open to the charge of *petitio principii*, it would, of course, follow that no advance could be made in knowledge by its means. But the objection springs from a too objective view of logic; from neglecting to remember the difference between what *is* in the facts of the external world, and what *we know to be* in them. Inference cannot, of course, give us more than already exists in the world, but it may help us to see and understand more. It is, indeed, our imperfect knowledge which makes inference of new truths possible. Were our knowledge complete, all truths would lie open before us, and such inference would be both unnecessary and impossible. For the truth of the conclusion is, in fact, concomitant with that of the premises from which we deduce it; it does not succeed them, though our perception of it may follow our perception of them.

For, though the objectors to the syllogism deny the fact, it is certainly possible to accept the premises without deducing the conclusion. The shortness of the syllogistic process, and the triteness of the examples of it commonly given, disguise this possibility, and give plausibility to the assertion that no advance in knowledge is really made by syllogism. But, because, as statements of fact, the premises contain the conclusion, it by no means follows that "in studying how to draw the conclusion, we [are] studying to know what we knew before. All the propositions of pure geometry, which multiply so fast that it is only a small and isolated class even among mathematicians who

know all that has been done in that science, are certainly contained in, that is necessarily deducible from, a very few simple notions. But *to be known from* these premises is very different from being *known with* them. Another form of the assertion is that consequences are *virtually* contained in the premises, or (I suppose) *as good as* contained in the premises. Persons not spoiled by sophistry will smile when they are told that knowing two straight lines cannot enclose a space, the whole is greater than its part, etc.—they as good as know that the three intersections of opposite sides of a hexagon inscribed in a circle must be in the same straight line. Many of my readers will learn this now for the first time; it will comfort them much to be assured, on many high authorities, that they virtually knew it ever since their childhood. They can now ponder upon the distinction, as to the state of their own minds, between virtual knowledge and absolute ignorance."[1]

Nor, indeed, even were this objection true would it be to the point. It is a psychological, not a logical, objection. A proof does not cease to be a proof because it is thoroughly familiar to any individual mind. The conclusions of geometry, for example, do not cease to be inferences from mathematical axioms and definitions because the process of reasoning by which they are reached is understood and remembered. We may, indeed, look upon a formally stated syllogism as an analysis of the mode of deductive inference, and as such an analysis it makes explicit elements which, in the actual drawing of the inference, may be implicit, and so escape superficial observation. But, as was shown in the last section, in all deductive inference there must be an application of a universal judgment to a particular case; in other words, the elements of syllogism must be present though each may not separately engage attention.

We may, then, sum up our answer to the charge of *petitio principii* brought against the syllogism under four heads—the major is essentially not a mere summation of

[1] De Morgan, *Formal Logic*, pp. 44-45.

observed instances; the minor is a necessary part of every syllogism; it is possible to accept the premises without drawing the conclusion, and hence to make progress in knowledge by means of syllogism; and the fact of inference depends on the rigidity of the proof, not on its novelty.

3. **Limitations of Syllogistic Reasoning.**—Having shown the validity and value of the syllogism, we have now to enquire whether it is the *only* type of valid mediate inference. This has been strongly asserted by many logicians.

In opposition to these claims it has been pointed out that the syllogism deals only with propositions which express the relation of subject and attribute, and that inferences from other relations, though they may be perfectly valid, not only are not made syllogistically but, cannot be satisfactorily expressed in that form. Such, for example, is the *argumentum à fortiori*—*A is greater than B, B is greater than C; therefore, A is greater than C*. Various attempts have been made to express such arguments syllogistically, the most successful of which is Mansel's:[1]

"*Whatever is greater than a greater than C is greater than C,*

"*A is greater than a greater than C,*

"*Therefore, A is greater than C.*"

But the whole argument is really assumed in the major premise, and the inference is, therefore, invalidated by a *petitio principii;* moreover, *B* does not appear in the premises, which cannot, therefore, express the whole argument.

If, then, account is to be taken of all valid inferences, we need a *Logic of Relatives* which "shall take account of relations generally, instead of those merely which are indicated by the ordinary logical copula 'is.'"[2] Such a logic has never been worked out, and, perhaps, never will

[1] *Artis Logicae Rudimenta*, 3rd Ed., p. 198.
[2] Venn, *Symbolic Logic*, p. 400.

be; for, as Dr. Venn says: "the attempt to construct a Logic of Relatives seems . . . altogether hopeless owing to the extreme vagueness and generality of this conception of a Relation."[1] Mr. Bradley, however, gives a list of relations, which though it "does not pretend to be complete" is yet, probably, the best classification which has yet been put forward. He calls them "principles of inference," and enumerates five—

"(1) *Synthesis of Subject and Attribute.* Under this all syllogistic inferences can be brought.

"(2) *Synthesis of Identity.* Where one term has one and the same point in common with two or more terms, there these others have the same point in common . . . as 'If *A* is the brother of *B*, and *B* of *C*, and *C* is the sister of *D*, then *A* is the brother of *D*.'

"(3) *Synthesis of Degree.* When one term does, by virtue of one and the same point in it, stand in a relation of degree with two or more other terms, then these others are also related in degree . . . as '*A* is hotter than *B* and *B* than *C*, therefore *A* than *C*.'" The *argumentum à fortiori* comes under this head.

"(4 and 5) *Syntheses of Time and Space.* When one and the same term stands to two or more other terms in any relation of time or space, there we must have a relation of time or space between these others. *Examples;* '*A* is north of *B* and *B* west of *C*, therefore *C* south-east of *A*'; '*A* is a day before *B*, *B* contemporary with *C*, therefore *C* a day after *A*.'"[2]

The validity of the arguments in classes (2) to (5) may be granted at once, as may the fact that they are not syllogistic. But it must be pointed out that, strictly speaking, neither are they deductive; for in them is no subordination of a special case under a general principle. They depend upon definite and abstract relations, such

[1] *Ibid.*, p. 403. [2] *Logic*, pp. 243-244.

that the mere combination of the given elements reveals the whole system. If *A* is north of *B* and *C* is an equal distance east of *B* we know that *A* is north-east of *C* by the mere examination of these given relations. To put it into syllogistic form would be both misleading and absurd. The abstract relations of north and east are not, therefore, the implied major premises of the arguments.

Of course, the recognition of the validity of such inferences does not mean the rejection of the syllogism, but only its restriction to its proper sphere. As Leibniz says: "The discovery of the syllogism is one of the most beautiful and greatest ever made by the human mind; it is a kind of universal mathematic whose importance is not sufficiently known, and when we know and are able to use it well, we may say that it has a kind of infallibility:—nothing can be more important than the art of formal argumentation according to true logic."[1]

4. **Syllogistic Fallacies.**

(i) **Abstract.**—Violation of any of the fundamental rules of syllogisms may be either open, or hidden by the ambiguities of language. In the former case we have abstract syllogistic fallacies, and in the latter case concrete syllogistic fallacies. The former we have already dealt with in connexion with our consideration of the rules of syllogism.[2] They take one of the three forms—

(*a*) Undistributed Middle.

(*b*) Illicit Process of the Major Term.

(*c*) Illicit Process of the Minor Term.

It is sufficient here to add to what was there said that these are all at bottom instances of the most general syllogistic fallacy of *four terms*. To have an undistributed middle is formally equivalent to having two distinct middle terms, for there is no assurance that the reference is in both cases to the same part of the denotation of that term, and we cannot, therefore, assume in any case that it is so. Similarly, if illicit process, either of the major or of the minor term, is committed, we have again four terms

[1] *Nouveaux Essais*, iv., 17, § 4. [2] See pp. 225-227.

in the apparent syllogism, for the extreme term which is illicitly distributed in the conclusion is different in its reference from the corresponding extreme term contained in the premises. In every case of syllogistic fallacy, therefore, the apparent syllogism is really a 'logical quadruped.'

(ii) **Concrete.**—Not different in the ultimate form of the error are the concrete syllogistic fallacies, though in them the fallacy is veiled by the use of ambiguous language. But when these ambiguities have been pierced through, the fallacy in every case stands revealed as an instance of four terms. All the cases of fallacy due to ambiguities of language which we have already considered[1] lead to such syllogistic fallacy when the propositions containing them are used in syllogistic argument. The ambiguity is most frequently found in the middle term, though this is not always the case. The fallacy of composition or division, for instance, is generally based on a confusion between the collective and distributive use of the minor term. It is unnecessary to discuss at length this class of fallacies, but the reader will be well advised to examine each form of ambiguity as committed in relation to each of the terms of syllogism and to decide to which of the three formal syllogistic fallacies it leads.

[1] See Chapters VI., X.

CHAPTER XXV.

GENERAL NATURE OF INDUCTION.

1. Basis and Aim of Induction.—As an instrument of thought the syllogism is restricted in its scope. It endeavours to secure the consistency of its conclusion with the premises from which it is derived. Whether the premises are true or false is immaterial to its formal validity. As soon as any serious investigation of experience is undertaken its inadequacy to express the course which thought actually takes in the attainment of truth becomes obvious. If we suppose the principles themselves to be questioned, or if thought is concerned about matters on which no recognised principles have been formulated and seeks to find them, the syllogism affords no test by which to establish or reject such propositions. Now the first condition of scientific thought is that it is throughout subject to the control of fact. The fact may be an event in the outer world attested by observation or by testimony, or it may be an occurrence in the mind, or a relation found to obtain in the actual world. But everywhere the appeal is to fact. And this appeal the formal process of the syllogism is powerless to make. This does not imply that the syllogism is discarded; it forms, as we shall see, an integral part of inductive enquiry besides having a value of its own in the setting forth of proofs.

'Induction' in this sense includes the whole method by which modern science attains to knowledge. Its aim is to arrive at universal propositions which as expressions of the essential character of phenomena are known as laws. Thus in political economy we have the law of diminishing returns which states that after a certain point is reached, other things remaining the same, the returns to successive

applications of labour and capital to any portion of land will continuously diminish; in physics we have Boyle's law that the volume of a gas varies inversely as the pressure to which it is subject at a fixed temperature.

Now it is obvious that neither of these laws can be matter of direct experience. For we do not know land except in particular portions, nor gas except in its varieties: how then can we make statements about land or gas in general? Consider the case of a farmer who has found in the cultivation of a certain field that an increase in the labour and capital expended on it is bringing in a less proportionate return year by year. This series of events is unique and will never recur. It relates to a particular object, in a particular place at a particular time. As a record it is merely a matter of history: a narrative of what occurred in the past. Yet he will not treat it so. He will use his experience as a guide to his treatment of the rest of the farm. In other words he will roughly generalise what in itself was unique. And the political economist, meeting with such instances, will formulate the generalisation with greater exactness into a law. Moreover, he will use the law confidently to interpret economic riddles of the past and to shape a policy for the future.

Now what are the assumptions which underlie this procedure? First of all, while the particular fact is individual and therefore different from every other fact, it nevertheless results from certain conditions without which it would not have been at all. On some of these conditions depend the special features of the fact which are being investigated—in the case of the land, for instance, the diminished ratio of the returns to the expenditure. Other aspects are for the time being neglected, and if the conditions of those under consideration can be exactly determined, we assume that wherever the like conditions recur another instance of the phenomena will be found. The statement of these conditions is purely abstract, for it simply asserts that whenever they occur a determinate result will follow—symbolically, *If* A *then* B. The particular is thus regarded as exemplifying the universal which can be read out of it by a process of

thought. The extension to cases that have not been investigated is justified by an appeal to the principles of causation and of the unity of nature. These two principles are the pre-suppositions of all induction.[1]

The universal is not easily discerned in the particular; it does not leap to the eye of the observer however careful he may be. His observations must be guided and interpreted by a methodical train of thought, by reasoned conjectures, and by an ever fresh scrutiny of the relevant facts.

An easier and more natural way of making a generalisation would seem to be to count the instances in which the relation is present and to argue from its uncontradicted frequency to its universal prevalence. The argument would run that since the relation has held good in every instance that has been met with, it will continue to hold good in all further examples of the same kind.

Aristotle drew attention to this process of seeking the universal on the basis of a careful enumeration and expressed the conditions which it must fulfil if it is to be of any value. He also set forth its relation to the doctrine of the syllogism. Taking as an example the absence of bile in certain long-lived animals, he exhibited the induction as a syllogism in Figure III.

Man, horse, mule, etc., are long-lived.
Man, horse, mule, etc., are bileless.
∴ Bileless animals are long-lived.

Now this is formally invalid unless the class of bileless animals is co-extensive with the animals enumerated in the subject of the minor premise; so that the term 'bileless' may be read as 'all bileless animals,' thus making possible the simple conversion of the minor. For Aristotle such a complete enumeration was quite feasible since in accordance with the current view it was always possible to name the species of a genus; and man, horse, mule, etc., are not individual things but species of the genus animal. His analysis, therefore, shows that he did not regard the element of counting as equally important with that of connexion of content.

[1] See next chapter.

Later logicians tended to subordinate this rational element in the doctrine of Aristotle to that of enumeration. Thus arose the distinction between '*Perfect Induction*,' in which the subject is known to be a complete enumeration of the denotation of the predicate of the minor premise, and '*Imperfect Induction*,' in which this cannot be assured.

To the so-called perfect induction the title of inference has been denied by many modern logicians. For in it the conclusion merely states in a more convenient form what has been already expressed in the premises. Thus in speaking of the books ranged on a certain shelf I may say

a b c d......are bound in red,
a b c d......are all the books on the fourth shelf,
∴ all the books on the fourth shelf are bound in red.

Instead of tediously reciting the titles of the works and predicating red binding of them severally, I may use the conclusion as a serviceable abridgment. The step is not one of inference from the given premises, but a summary of the information they convey.

Moreover, it is rarely possible, even if it were sufficient, to enumerate completely the particular things under investigation, and if the enumeration is not complete, then any inference based on it is more or less uncertain, and is always liable to be overthrown. As Bacon pointed out: "That induction which proceeds by simple enumeration is a puerile thing, and concludes uncertainly, and is exposed to danger from any contradictory instance, and for the most part pronounces from fewer instances than it ought, and of these only from such as are at hand."[1] The utmost we are ever entitled to say as the result of such a method at its best is that certain things always have been and are. We can never conclude that they necessarily must be. At most there is a probability which in certain cases may amount to practical certainty that the expectation aroused will be realised in any further case that may come under observation. Some other way, then, than that of enumeration must be found for the detection of the universal, and

[1] *Novum Organon*, Bk. I., p. 105.

in modern science enumeration is always subordinate to analysis.

Now the data of experience are concrete, and as concrete, exceedingly complex. Thought assumes that each element in this complexity is the necessary expression of universal law, so that every particular is, as it were, the centre of *many universals. To discover any one of them it is* necessary to isolate it from the rest in thought; to create, as it has been expressed, a kind of void around it so that it may stand out naked and clear of the details in which it is embedded and which are irrelevant *to it.* These details may be different in every instance of the phenomena, and it is only when they are set aside that the essential conditions on which the universal depends become apparent. Consequently, the process by which we reach the knowledge of the exact form of any law of relation is one of analysis, and one which offers abundant opportunities for error. The relations as we conceive them in thought may not be the relations which actually hold. Then, sooner or later, we come to contradiction in our explanations, and see the necessity for a revision of our mental construction.

It is in relation to analysis that the advantage of a plurality of instances is found. The form of any universal law is often more clearly and easily seen when comparison of instances which are known, or believed, to exemplify it enables the observer to disregard the unessential elements in each concrete particular. It is here that number of instances has a place in induction. They assist us in analysing the conditions of the given, but it is from those conditions, and not from the instances as such, that the inference is made. If the conditions can be exactly ascertained in a single instance—as in the case of many chemical experiments—then no plurality of instances is needed, unless, indeed, there is some doubt as to the accuracy of the experiment, when, of course, its repetition may be necessary to remove that doubt. The only cases in which an inference is made from number of instances as such is when it is impossible—at any rate for the time—to ascertain the conditions of the phenomenon in question; and

then the inference only yields a certain amount of probability.[1]

We see then that the aim of induction is knowledge of universal laws: that such laws are abstract, for concrete reality only presents us with them encumbered with extraneous detail; that, consequently, we can reach them only by an analysis of facts, and in this analysis comparison of instances may be helpful but is not essential; that abstraction offers opportunity for error, and that, therefore, the form in which we think universal laws may be subject to modification, till they are conceived as consistently interrelated within a system.

2. **Method of Induction.**—How then is the analysis of phenomena to be conducted? Since the universal is not directly open to observation, it can only be grasped by thought. A first observation gives acquaintance with the facts. These have no meaning apart from the universals which they exemplify. Hence the mind is led to conjecture a universal law of relation which will make the facts significant, or in other words explain them. Such a supposition is called a hypothesis, and before it can be accepted it must be subjected to a careful examination in the light not only of the phenomena it is intended to explain, but also of all established knowledge which seems related to them. Indeed the method of induction throughout consists in the framing of hypotheses to explain phenomena given in experience, and the verification of those hypotheses by constant appeal to facts. In the process of examination and verification it will rarely happen that the original hypothesis stands as first conceived. In many cases it will be rejected outright; in others it will be pruned and re-fashioned until it expresses with greater accuracy the true nature of the data that experience affords. When its agreement with known reality is exact then its truth is held to be established.

The testing, re-shaping, and verifying of hypotheses may proceed mainly by either of two methods, the one

[1] *Cf.* Ch. 33.

direct, the other indirect. The causal relation may be such that the sequence lies open to observation, or the cause may so far admit of manipulation as to be controlled by experiment. Then the behaviour of the cause in relation to the effect can be watched to some extent directly. It is here that the methods associated with the name of Mill have their chief importance.[1] They all aim at exhibiting to observation a simple causal sequence by means of a definite variation of accompanying circumstances.

On the other hand the indirect method makes use of deduction. When a hypothesis has been formed its consequences are inferred. These are then compared with the phenomena under investigation to see if there is agreement between the two. Such agreement is the minimum test to be satisfied if the hypothesis is to be accepted as the statement of a real causal relation. This method may be used as well as the direct method in those cases where, though the cause is of such a nature as to make experiment impossible, nevertheless the causal sequence is open to observation amid diverse surroundings. But its special sphere is found where the direct methods of observation and experiment are inapplicable. The causal relation is often hidden from view, and then there is no other resource but to devise a hypothesis which, by the consequences it necessitates, explains the facts as they are. This method, then, is necessarily used where we are compelled to start from certain given effects, and from them infer the causes, because direct observation of the causal sequence is then impossible. This is well illustrated in the explanation of present-day geological phenomena by the geologist; or in the causes assigned by the historian for the growth of an institution such as Parliament. Indeed geology, history and the social sciences are almost entirely dependent on the indirect method as a method of investigation.

The essence of the method of induction is clearly put by De Morgan: "Modern discoveries have not been made by large collections of facts with subsequent discussion, separation, and resulting deduction of a truth thus ren-

[1] See Ch. 31.

dered perceptible. A few facts have suggested an *hypothesis*, which means a *supposition* proper to explain them. The necessary results of this supposition are worked out, and then, and not till then, other facts are examined to see if these ulterior results are found in nature. The trial of the hypothesis is the *special object*; prior to which hypotheses must have been started, not by rule, but by that sagacity of which no description can be given, precisely because the very owners of it do not act under laws perceptible to themselves. The inventor of hypotheses, if pressed to explain his method, must answer as did Zerah Colburn, when asked for his mode of instantaneous calculation. When the poor boy had been bothered for some time in this manner, he cried out in a huff, 'God put it into my head, and I can't put it into yours.' Wrong hypotheses, rightly worked from, have produced more useful results than unguided observation."[1]

The essential steps in the inductive method then are—

(1) A first observation of facts.

(2) The formation of a hypothesis suggested by this observation.

(3) The deduction of the consequences of this hypothesis.

(4) The testing of these consequences by a careful analysis of phenomena, and the consequent exact formulation of the hypothesis, which then, as expressing the true universal nature of reality, is verified and received as an established theory or law.

[1] *Budget of Paradoxes*, pp. 55-56.

CHAPTER XXVI.

THE POSTULATES OF INDUCTION.

1. **The Bases of Generalisation.**—So far as the system of knowledge is constituted by means of inductive inference a new problem arises. Induction begins with particulars and establishes universals. The relation which is exemplified here and now is extended to cover all cases of the same kind, past, present, and future. The statement "this or that fire burns" is generalised into "fire burns," and wherever fire may be met with we are confident that its nature will be the same. But what right have we to affirm on the ground of past experience that such will be the case? How can we know that the fire that is kindled to-morrow will not cool instead of scorch the hand that is thrust too near it, or will not burst into fiercer flame as water is poured upon it? The answer is that if such things were possible the organisation of experience could not be attempted, so life would be impossible. Science refuses to limit itself to a mere registration of phenomena that have come under its observation, and denies that the validity of the principles it discovers is restricted to the cases from which they were drawn. It grants that all the instances cannot be examined, but it asserts that this need not deter us from concluding from one to all, if the nature of the one is thoroughly known.

Now this position rests on two assumptions, (i) that nature is a unity and therefore uniform, (ii) that every change must have a cause. Without these assumptions no system of knowledge of reality could be established. They are the postulates of induction, and because of their fundamental importance they must be considered at length.

2. **Unity of Nature.**—We can think the universe only as a system which remains identical with itself, and which, as a whole, does not change. At the same time we know that there are unceasing changes in the relations between its parts. These, then, must be thought as necessitated and determined by the nature of the whole. The universe, for example, is of such a nature that all material bodies attract each other directly as their mass and inversely as the square of their distance. If the universe as a whole were different, the changes within it would be different: the facts, for instance, which now exemplify the law of gravitation would be at least partly changed, or they might cease to exist. Similarly if the nature of the universe were different in one place from what it is in another, then the relations which hold in one place would not necessarily hold in another. The apple might fall to the ground in China and soar to the sky in Peru. We are compelled then to conceive nature as a unity which determines every relation, and consequently every change of relation, in such a way that those relations hold true everywhere and always of all identical facts. In other words, the idea of unity implies that of uniformity.

We may add that if nature were not uniform man could neither utilise it in fact nor conquer it in thought. There would be no guarantee that the iron fashioned to-day into a serviceable support might not to-morrow possess the qualities of pulp, nor that the structure planned in reliance on the stability of the present qualities of its proposed materials would ever be built. Each thing and phenomenon would tell of nothing but itself. We could never pass with confidence from one to another, never plan a moment ahead, never gather from experience a system of universal relations, since no necessary relation would exist. Experience would then be a never-ending surprise of novelty or chance resemblance. Uniformity is thus seen to be a form without which experience would not be intelligible.

(i) **Origin of Principle.**—It has been held that the principle of uniformity is derived from the repetition of phenomena in sensuous experience. From the recurrence of similar facts in similar circumstances, it is said, the

conclusion follows that nature acts in a uniform way. But this would only follow if observation gave us nothing but instances of uniformity. Now this is exactly what observation does not do. "Every person's consciousness assures him that he does not always expect uniformity in the course of events. . . . The course of nature, in truth, is not only uniform, it is also infinitely various. Some phenomena are always seen to recur in the very same combinations in which we met with them at first, others seem altogether capricious; while some, which we had been accustomed to regard as bound down exclusively to a particular set of combinations, we unexpectedly find detached from some of the elements with which we had hitherto found them conjoined, and united to others of quite a contrary description."[1] But as nature is thus at once uniform and multiform in the sequences of phenomena which she presents to man's observation, it seems obvious that the idea that multiformity is after all only apparent, and that if we examine deeper we shall find uniformity, cannot have been derived simply from observation understood as mere sensuous experience.

Moreover, observation in this sense could only give us an expectation that under similar conditions phenomena already observed will be repeated. And our expectation might be disappointed. It would be only a habit that had arisen from the recurrence of similar instances in the past. Now "habit is within us, not outside us,"[2] and we have no right to assume that what we have formed the habit of expecting will actually come to pass. There would be no warrant that our mental disposition represented the course of nature, and hence a system of objective knowledge could not be built up on so slender a foundation.

Such a derivation of the principle is open to the further objection that it is arrived at by a process of simple enumeration. The recurrent sequences are counted, and on no other ground than their frequency, they are held to indicate uniformity. But this method of reasoning is most

[1] Mill, *Logic*, Bk. III., Ch. iii., § 2.
[2] Liard, *La Logique*, p. 157.

unsatisfactory, and a principle that owed all its validity to it could not offer a firm basis for rigid scientific induction.[1] The cogency of any such enumeration is altogether impaired by the existence of known instances to the contrary, and nature is far from exhibiting an uncontradicted uniformity. It is true that the apparent chaos vanishes upon deeper investigation. "But it is just this 'interrogation' that has to be accounted for: it is only upon the supposition of uniformity that we make the interrogation. How can this be, if the supposition is only derived from the observation of uniformity, an observation which presupposes the interrogation?"[2]

It is evident, then, that the principle of uniformity is not derived from experience in the sense of a mere observation of facts. Nor is it a principle born ready-made with every man, on which he consciously or unconsciously acts. If it were innate in this sense the practices of superstition could not have arisen, and the fabric of savage beliefs would have been differently woven. It would have been impossible to expect the horse-shoe to bring good luck, or the damaged shadow to work harm to the man. Breaches in popular maxims would long ago have destroyed their validity as popular guides to action. It is certain that even to-day a thorough-going belief in uniformity is the exception rather than the rule. As Sigwart says: "If we had needed merely to open our eyes in order to see 'uniformity in the course of Nature' everywhere before us, belief in the thorough-going constancy of the way in which causes act would not have been so slow to arise nor have been still only a scientific and not a popular belief; nor would the tendency to make capricious powers, demons and gods, responsible for what happens in the universe have been so deeply rooted."[3]

The truth is that uniformity is implicit in experience. It is there long before men become conscious of it. Indeed life itself, even in its most rudimentary form, could not go on without it. When we make it explicit we pass from

[1] Cf. pp. 297-299. [2] Green, *Phil. Works*, vol. ii., p. 283.
[3] *Logic*, Eng. Trans., vol. ii., p. 111.

experience as felt to experience as thought. We then recognise that in any valid generalisation from the observation of particular facts there is an implicit assumption of uniformity, and that knowledge, which is constituted on the basis of such generalisations, stands or falls with its truth. Since the belief that any particular sequence is uniform involves the general principle, the claim that the principle itself arises from the repetition of particular uniformities cannot be accepted.

It has become abundantly clear from our discussion that the principle of uniformity originates in the attempt to systematise experience. There is, of course, a unity in the experience of each individual which follows from the fact that it belongs to him. But practical life cannot be carried far without the necessity arising for a different kind of unity. It is important for the purposes of action to know what relations hold in the real world, and so the individual is compelled to organise parts, at any rate, of his experience: for example, the hunter acts on certain practical generalisations which so far unify the facts relative to the chase as to enable him when acting on them to bring down his prey with greater certainty than he could without them. However simple and imperfect these generalisations may be, they are formed and acted on with the underlying assumption that nature is so far constant that in similar circumstances the same measures will bring about the same results.

It is the same when the experiences which are organised cover a wider and yet wider range of reality; and when the conception is reached of a system of knowledge in which the world shall be known as a totality, with a thorough-going interrelation of parts, it is seen that it cannot be constituted except on the supposition that uniformity can everywhere be taken for granted. The point of view is no longer that of the individual seeking with more or less completeness to reduce his personal experience to unity: it is that of science which aims at unifying all experience. It is now nature as a whole which is dealt with, and the uniformity demanded no longer applies only within a narrow department of fact: it must be operative

throughout the universe which we attempt to explain. Thus the principle is a development of the reduction of individual experience to a unity, and is meant to express the truth that if we set before ourselves as the aim of induction the explanation of all experience through generalisation, that aim cannot be realised without taking the uniformity of nature as a necessary postulate.

(ii) **Meaning of Principle.**—Briefly the principle means that wherever there is identity of conditions exactly similar phenomena will be found. Wherever, for example, hydrogen and oxygen in certain proportions are ignited by an electric spark water will be formed. This meaning must be carefully distinguished from that which bases the uniformity on resemblance. Mill, for instance, expresses it "that the unknown will be similar to the known, that the future will resemble the past."[1] Such a general assumption that the future will resemble the past we have no right to make, and there is no necessity to make it. "The future," says Green, "might be exceedingly unlike the past (in the ordinary sense of the words) without any violation of the principle of inductive reasoning, rightly understood. If the 'likeness' means that the experiences of sensitive beings in the future will be like what they have been in the past, there is reason to think otherwise. Present experience of this sort is very different from what it was in the time of the ichthyosaurus."[2] Indeed the uniformity of nature does not mean monotony of experience. Probably particular events are never repeated exactly: there is always some difference, at any rate in surrounding circumstances, which gives the event its individuality. Yet this does not prevent us from examining the various aspects of events to discover the identity of conditions which underlie their common features. No two thunderstorms are alike, but the conditions on which the thunder depends may be ascertained.

The other half of Mill's assumption—that "the unknown will be similar to the known"—also gives us no basis for demonstrative inference. From mere likeness

[1] Bk. III., Ch. iii., § 2. [2] *Op. cit.*, pp. 282-283.

of isolated phenomena we can draw no safe conclusions at any time. The argument from resemblance falls short of proof: there must be, as we have said, identity of conditions.

Nor would it be correct to accept uniformities, but reject the unity of nature. To do so would be to abandon the possibility of knowledge. No doubt we do seek to find such uniformities and to express them as laws of nature. Yet we do not rest satisfied with a series of independent sequences; we hold that the variety which appears to observation is just as much a part of the total system of the universe as are the instances of uniformity, and that it is equally as necessary. To bring the uniformities together we endeavour to show that they are parts of the one system of nature within which they are inter-related. These considerations lead us to prefer to speak of the 'unity' rather than the 'uniformity' of nature as the postulate of induction.

(iii) **Scope of the Principle.**—The importance of this fundamental principle to the organisation of knowledge is well expressed by Mach. He says: "In the infinite variety of nature many ordinary events occur; while others appear uncommon, perplexing, astonishing, or even contradictory to the ordinary run of things. As long as this is the case we do not possess a well-settled and unitary conception of nature. Thence is imposed the task of everywhere seeking out in the natural phenomena those elements that are the same, and that amid all multiplicity are ever present. By this means, on the one hand, the most economical and briefest description and communication are rendered possible; and on the other, when once a person has acquired the skill of recognising these permanent elements throughout the greatest range and variety of phenomena, of seeing them in the same, this ability leads to a *comprehensive, compact, consistent,* and *facile conception of the facts.* When once we have reached the point where we are everywhere able to detect the *same* few simple elements, combining in the ordinary manner, then they appear to us as things that are familiar; we are no longer surprised, there is nothing new or strange to us in

the phenomena, we feel at home with them, they no longer perplex us, they are explained."[1]

As the expression of the ground of all uniformity this general postulate lies at the root of all classification as well as of all formulation of law. The very idea of species under a genus involves that different classes of things partake in the same general nature, and that they are distinguished from each other by essential and regular differences. Unless there were constancy of this kind knowledge would be impossible. If, for example, the attributes of iron did not persistently co-exist in a fixed and definite way we could not speak of iron at all, for we should never know what was meant by the term. The organisation of experience could not even begin, for certainty would be unattainable.

3. **Function of the Concept of Causation.**—One aspect of the uniformity of nature is so important that it requires separate treatment. It is that of the uniformity which characterises the mutually determined changes of relation amongst the elements of the universe. This leads us to the discussion of the second assumption underlying inductive inference—that every change must have a cause. The principle of causation is not independent of that of uniformity but rather the most important exemplification of it.

Whenever we attempt to think experience we have to take account of the two factors of persistence and change. Things persist, and the persisting things change. If we regarded change as arising out of nothing and passing into nothing then the mind would receive a series of isolated impressions which could in no way be related in a system of knowledge. Every event would be a miracle: and experience would be unintelligible. To understand experience we must seek the origin of change in the nature of the changing thing and in its relations to other things. It is an assumption that we shall find it there, but since we cannot think experience in any other way, the assumption is necessary.

[1] *Science of Mechanics*, Eng. Trans., pp. 5-6.

Causation, then, is one of the forms in which alone we can think experience. It is, therefore, involved in all experience That is not the same thing as saying that it is consciously recognised in every case where it is acted on. Long before men thought about causation as such, the relation of cause and effect was the basis of practical activities. In very early times the hunter sharpened his arrow, and the farmer sowed his seed in the expectation of definite desirable results. So often does a modern workman go through industrial processes to-day.

It is a far cry from such implicit recognition of causation in practical life to its acceptance in the fullest sense as a universal determinant of experience. Indeed, corresponding to the degree in which individual knowledge is organised there are great variations in the fullness with which the idea of causation is grasped by different people. It is only the man of scientific thought who recognises that every event is the outcome of uniform laws, even though the complexity of their mode of action is hidden from him. "Even thoughtful men usually receive with surprise the suggestion, that the form of the curl of every wave that breaks, wind-driven, on the sea-shore, and the direction of every particle of foam that flies before the gale, are the exact effects of definite causes; and, as such, must be capable of being determined, deductively, from the laws of motion and the properties of air and water."[1] The conscious acceptance and application of the general axioms of causation are, therefore, a higher phase in the process of interpreting and unifying experience than that in which the existence and regularity of causation are recognised in particular instances.

In order to explain the changes in the world about us, which at first seem so chaotic, it is necessary first of all to ascertain what follows what, and then to account for this sequence by defining clearly the conditions on which it depends. What this general statement of the problem involves can only be made clear by a fuller examination of the nature of causation.

[1] Huxley: *Hume*, p. 122.

4. **Nature of Causation.**—The essence of the idea of cause is that which has power to produce change, and of effect that it is a change which has been produced and not one which has merely happened accidentally. Under causation man first thinks his own efforts to do things: *he* is the efficient cause of the changes in the world due to his efforts. To know what changes follow each kind of effort is evidently the only way in which he can begin to organise his activities. To conceive *all* change as ordered by conditions which necessitate it is only a refinement and extension of this primitive idea.

In this extension and refinement there are many stages, so that the popular idea of cause differs a good deal from the scientific conception. In the former, as is natural, one of the chief distinctions drawn between cause and effect is that they differ in time. Cause precedes effect, and effect follows cause. Observation of uniform sequences gives this, and there the plain man stops. But to regard such observations as proving a universal bond would be to trust for our generalisation to mere enumeration of instances. The scientific thinker must go deeper and lay bare the hidden bond of identity which underlies the observed uniform sequences.

Here, as always, science starts from common thought. Now such thought regards some event as a cause and a subsequent event as an effect. The one may follow the other immediately—as death on the blowing out of the brains, or at a longer or less interval—as death on the taking of poison. Similarly, unjust government may be said to cause rebellion—yet the people may suffer for many years before they rise against their tyrants. If we consider such cases we see that we are grouping together vast numbers of facts and events as the cause, and another great mass of happenings as an effect. Amid such complexity we cannot expect to find uniformity between one instance and another except in the most abstract sense—and an 'abstract' sense means a very incomplete sense and one very inadequate for explaining this particular set of occurrences. Tyranny has been shown in an enormously large number of ways differing

widely from each other: rebellion has taken one form in this country, time, district; another in that. Tyranny and rebellion are very abstract terms, and tell us nothing definite about actual events. So with death. It is an abstract term denoting one aspect of occurrences all of which have not only their own peculiar features, but other common features which make it possible to classify them. So a doctor in giving a certificate of death specifies the cause as such and such a disease, mere senile decay, poison, a gun shot, and so on. That is to say, he substitutes for the very general and abstract effect 'death' the more specific effect 'death of such a character because due to such a cause.'

Evidently he reaches this more definite conclusion by analysing the case before him. But this analysis of necessity tends to abolish, so far as it is pushed, the time interval between 'cause' and 'effect.' When poison, or a disease, or mere old age, eventuates in death, it is because a certain condition of the body has been reached—and that condition does not precede death—it *is* death. Doubtless that condition was sequent on other conditions—for all existence is continuous change. But the *mere* condition at one moment cannot by itself give rise to anything beyond itself: it is not as *condition* but as *process* that it passes from form to form. The reasons why the process goes on in this way rather than in that must be sought in the permanent and persistent natures of the factors that enter into it. When a man takes poison, for instance, the properties of the poison brought into effective relation with the bodily organism, determine a change in the bodily process which would otherwise have gone on healthily. The culmination of this series may be death, or it may be merely a more or less severe illness. If an antidote be taken in time this series of changes may again be diverted, and the process be bent in yet another direction which may end in re-established health.

In such cases the taking of poison or of antidote on the one hand, and the death, illness, or recovery on the other, are well marked events in time, so we make them the beginning and end of the event, and label them 'cause' and

'effect' respectively. Such a method is the only way in which we can successfully grapple with the complexities of existence. But it must be noted that the whole constitution of the event is an arbitrary act on our part. The taking of the poison did not begin the life-processes which it modified, nor did death either begin or end the processes of bodily disintegration. Nor are there always two such well-marked points. If death ensues from senile decay, who can say when the decadence of old age began?

What we have, then, is the continuous changing universe and the power of selecting this or that element of the change for examination. But if the change is continuous it is clear that the common distinction between cause and effect is only a matter of convenience and can be placed where it is best adapted for the purpose in hand. A doctor in detailing the progress of a disease—a physicist or chemist in describing a natural process—will put between the 'cause' and the 'effect' of the common man an indefinitely large number of intermediate steps, any one of which ensues on changes which have gone before, and precedes those that come after, and may, therefore, be called indifferently effect or cause according to whether it is thought in relation to the former or to the latter. In some simple cases of common life this is obviously the case. A drop of ink on paper 'causes' a blot; the blot is the 'effect' of that dropping, and so we name the same event according to whether we are looking forwards to what is to come, or backwards to what has been. But the contact of ink with paper *is* the blot. Cause and effect are different names for the same 'event,' appropriate because we can think of that event either as a fact to be accomplished or as one already accomplished.

The more analysis is pushed home the nearer this point is reached, at which cause and effect are seen to be the same content regarded from different standpoints. Whenever the state of things—or totality of conditions—which we call 'cause' is reached there we have the phenomenon which we call 'effect.' Whichever we name it, it is just one momentary phase of the continuous change which is what really exists. The distinction between cause and

effect is in every case purely arbitrary so far as this continuous world-process is concerned. It refers always just to some specific problem—to the attempt to understand an arbitrarily selected piece of the constant change we see around us, and of which we ourselves form part.

Our analysis will have enabled us to see that when we speak of an effect persisting after the cause has ended we are using the terms only in a loose and popular sense. *Cessante causa, cessat effectus,* said the old logical axiom; and in the deeper analysis to which we have been led, we see that in any sense in which the cause may be said to cease, the effect of necessity ceases. Even in the looser use of terms some instances seem to exemplify this. For instance, the withdrawal of the weight from one scale of an evenly poised balance destroys the equilibrium of the two scales, which was the 'effect' of the equality of the two weights. But in most cases the opposite seems to hold, for the changed conditions persist in a way open to observation. The most striking and instructive examples are found in mechanics. A blow sets a body in motion, and the motion not only continues after the blow is ended, but, according to the law of inertia, tends to endure for ever, and would actually do so were it not counteracted by opposing 'causes,' such as gravity and friction. Again, if a moving body comes into contact with one which is immovable, the former is brought to a state of rest, and remains in that state so long as no other influence is exerted upon it. In this case we may speak of the fixed body as the 'cause' of the cessation of movement in the previously moving body, just as accurately as we may speak of the blow in the preceding example as the cause of the motion, or of poison as the cause of death. But in none of the cases are we using the word with scientific accuracy; it is the changed spatial relation combined with the permanent nature of each of the objects involved that is the true cause of the result, and that result is in each case nothing but the actual occurrence of that changed spatial relationship and the interaction of the permanent natures of the objects concerned which it involves.

Our discussion, then, has led us to the conclusion that

the distinction between cause and effect is one of point of view. In existence they are not two but one. Nevertheless, in the attempt to understand the one continuous process which constitutes an event both the plain man and the scientific thinker find it well to distinguish these two aspects. The difference is that while the plain man is content with a loose temporal sequence the scientist seeks to define exactly the conditions which determine the event. No doubt, as in the case of the plain man, science has often to be satisfied with 'causes' which are to some extent removed from the 'effect.' One cannot in practice make an infinite number of cross-sections in a process to show at each one the identity of the two. But the ideal of science is to reveal that identity in one and the same content and to show that it may be looked at from the two points of view of that which determines and of that which is determined. Thus the combination of hydrogen and oxygen in the quantitative ratio of two to one determines that the effect shall be water, and the character of that effect is determined by the character of the elements which are combined. But the combined elements and the water are one and the same identical substance, and this substance is the content both of the cause and of the effect.

5. **Axioms of Causation.**—Similarly, as has been said, our constitution of an event with a definite beginning and end and a definite content is arbitrary, and determined by the purpose of the investigation we have in hand. The present arises out of the past, the future out of the present without a break. Of course, in many cases, the boundaries of what we choose to call an instance of causation are plainly marked for observation, even as a strongly marked point has been seen to form often a convenient dividing line between cause and effect in the popular view of causation as temporal sequence. Thus when we observe an acid and an alkali combine to form a salt the event stands out prominently from its surroundings. But the presence of such well-marked boundaries does not indicate any breach in nature. Every stage in a process may be regarded

either as a cause when we are looking forwards or as an effect if we look backwards. Thus, if we assign a cause to any event it is always possible to seek the cause of which the cause assigned is the effect: and the process may be repeated in a regress which is practically endless. For example, we may attribute famine to a failure of crops, the failure of crops to excessive cold, excessive cold to atmospheric changes, and so on. Only when we reached a conception of nature as a whole should we find the terminus of our thought and the explanation of the series of causes and effects through which we had passed. Since this is impossible we are compelled to consider the process as it were in sections, and to make limits where no limits exist.

This means not only an arbitrary determination of starting-point and conclusion of the causal sequence, but a limitation of the elements which we take into consideration. The universe is such that every part determines every other part, but it is obvious that the totality of circumstances attending any change can never be taken into account in our analysis of the conditions of that change. Some of the circumstances are more immediately relevant than others. In a sense it may be said that the early history of the Roman Republic helped to determine the course of the French Revolution: but no one would think of going back so far in his analysis of causes, even if it were possible. The historian would probably content himself with analysing the condition of France in the years immediately preceding the Revolution, without, perhaps, asking very closely how that condition came to be what it was. So, too, the inventions of the eighteenth century were undoubtedly links in the chain of events which now end in a cotton strike, but there are conditions much nearer at hand which for all practical purposes sufficiently explain it. Hence the discovery of causes virtually resolves itself into an attempt to find out what elements in the total conditions under which a change takes place are essential and material to that change and what can be disregarded.

Now the selection of relevant conditions is usually made in such a way that the effect is seen to follow on the cause.

But in our discussion of temporal sequence we have seen that cause and effect are identical in content, and, therefore, cannot be successive. The apparent contradiction vanishes when we remember that causation, so far as it is susceptible of close analysis, is found in a continuous process in which we can at any number of points find identities of cause and effect. These have been arbitrarily grouped together in thought, and the point at which we choose to fix the end of the cause is taken to be the beginning of the effect which also includes a series of identities of cause and effect. At the junction of the two which we separate for purposes of thought there is no real break in nature. By thus choosing our limits we introduce the idea of temporal sequence which is not an integral part of causation. "The cause acts in time: the effect goes on in time. The times occupied by the cause and by the effect succeed each other, the one ending at the point of time at which the other begins."[1] In proportion as analysis is more thorough 'cause' and 'effect' are brought nearer and nearer together till, with the completion of the analysis, their identity of content is revealed and time-sequence disappears.

The process of selecting the essential conditions is made harder by the fact that causal connexions are never given to us pure in nature. With the conditions relevant to the problem in hand are mingled many which are irrelevant, and it is not easy to distinguish between them. Hence there are the dangers, on the one hand, of omitting those which are essential and, on the other hand, of including those which are accidental: indeed, both faults may be committed at the same time. Common reasoning often fails to avoid one or other of these faults, and it is one of the results of the deeper thought of scientific minds to refine and make accurate the judgments to which such thinking gives rise. "Trade follows the flag" is partly true and partly false. Economic analysis shows that trade depends on many other conditions besides such as security and good government, which the flag may be taken to connote.

[1] Whewell, *History of Scientific Ideas*, Vol. I., pp. 197-198.

This practical uncertainty in determining conditions, conjoined with the loose use of the terms cause and effect, has led to the denial that statements of cases of causation can be regarded as formally reciprocal. That "the same cause always produces the same effect" is universally recognised as the fundamental axiom of causation: that "the same effect is always due to the same cause" is questioned. It is asserted that the same effect may be produced in many ways. For example, a charge of gun-cotton, of dynamite, or of cordite will blast a rock. You cannot, therefore, equate one cause to one effect: indeed for any given effect you may find a plurality of causes. This was the doctrine of Mill; he said: "It is not true . . . that one effect must be connected with only one cause, or assemblage of conditions: that each phenomenon can be produced only in one way. There are often several independent modes in which the same phenomenon could have originated. One fact may be the consequent in several invariable sequences; it may follow with equal uniformity, any one of several antecedents or collections of antecedents. Many causes . . . may produce death."[1] The example with which Mill ends shows that he is using 'effect' in a general and abstract way. We have already seen that though "many causes may produce death," yet in every case just this form of death is the expression of just this set of conditions—otherwise coroners' inquests would be futile. Indeed, if it be once grasped that 'cause' and 'effect' are simply different names for the same reality according to whether we are looking forwards or backwards; that cause is the totality of conditions looked at as conditioning, while effect is the same totality regarded as conditioned, then it is plain that cause and effect are exactly equated. A change in the totality of conditions *is* a change, whether we call the totality cause or whether we name it effect.

True, in any particular instance we may feel that we are not justified in regarding our statement of the causal relation as reciprocal, but that is because we are not certain that it is adequate—that the whole of the essential con-

[1] *Logic*, Bk. III., x., § 1.

ditions have been correctly sorted out. When we do feel that certainty we do regard the statement of the causal relation as reciprocal. Thus the statement that the combination of hydrogen with oxygen in certain definite proportions always produces water is held to state a reciprocal relation, that is, to be simply convertible, and we feel equal confidence in asserting that pure water can be completely resolved into those two gases in those exact proportions. It is the same in all cases of scientific experiment where it is possible to make sure that all the essential conditions are known and are secured, and that no others are present which can modify the result. Thus in every case where the establishment of a causal relation is certain, that relation is established as a reciprocal one.

The popular idea of the non-reciprocal character of the fundamental axiom of causation is due to the fact that the 'cause' is much more frequently analysed than the 'effect'—using those words in the popular sense of temporal antecedent and consequent phenomena.

The causal relation is determined most exactly when the amount of change can be measured. This is frequently possible in the physical sciences, and two principles are assumed to aid in this determination—the indestructibility of matter and the conservation of energy. The first states that whatever re-distribution of matter results from the change the total amount must remain the same. This principle is most serviceable in chemistry: thus when sodium and chlorine combine to form common salt, which differs considerably in its properties from both its constituents, the weight of the compound must equal the sum of the weights of its separate elements. The other principle expresses the fact that in any physical system the amount of energy remains the same whatever the changes that take place within it. The system has a certain capacity for work before the change: it possesses the same capacity for work after the change. The energy need not continue in the same form. The motion of a body, for instance, impinging on another body is partly transformed into sound and heat. If each form of energy is measured in the same unit the number of units before impact can be

equated with the number after impact. The two principles may be summed up in the statement that the cause equals the effect.

The axioms of causation may be briefly stated as follows—

1. Every event must have a cause.
2. The same cause always produces the same effect.
3. The same effect is always due to the same cause.
4. Cause and effect are equal in amount of energy.

CHAPTER XXVII.

OBSERVATION.

1. **The Basis of Science.**—All the sciences aim at explaining the facts of experience. It is, therefore, evident that they must rest on an exact knowledge of those facts. When the facts are wrongly thought, the theories invented to explain them will be erroneous. But knowledge of facts is gained in the ultimate resort only by observation. Not only at the beginning of an investigation but throughout its whole course, the enquirer must get in touch with facts.

Hence, logic must recognise that observation is an integral part of an inductive enquiry, and though no formal rules can be laid down for its conduct, yet its nature must be examined. So will appear the general conditions of sound and fruitful observation, and the general ways in which observation is most liable to error.

Now when we simply observe, without attempting to change in any way what we are observing, we have observation pure and simple—observation in its most direct form. The analysis of the phenomena we are studying is purely mental. When we attempt to modify and determine what we are observing we have observation brought into bondage to the purposes of our thought, and made to answer definite questions. This is experiment, the great hand-maiden of science.

Science is, however, a co-operative movement in which each worker avails himself of the results of others. Such results are communicated in speech or writing. Sometimes they can be tested by fresh observations, at others they cannot. Then the whole basis of the structure built upon them rests on the accuracy of the reports of the original

observations. With this also logic is concerned. So our enquiry branches out in a three-fold direction, and we must consider in turn the nature of observation, of experiment, and of testimony.

2. **Simple Observation.**—To observe seems simplicity itself, and in any particular case we find it difficult to believe that "the testimony of our senses" cannot be implicitly trusted. Doubtless we grant that we *have* made mistakes —we have taken a stranger for a friend; we have misunderstood what was said to us; we have read words in the printed page which were not there instead of those that were. But at the moment of experience nothing seems more certain than this testimony. What we see, hear, touch, taste, or smell, has an impressive directness which admits of doubt much less readily than do constructions of our thought which are not determined by immediate sense-experience. So also we find it hard to believe that our memories of such experiences are at fault; yet careful examination has proved that this is so to the extent of at least twenty-five per cent. Indeed, there is nothing more common than for "the senses to play us false" as we say. It is a commonplace that only the artist's eye observes the colours of a scene, light and shade, as they are. Every illusion tells the same tale. "When I look at a brick viaduct, a mile or two off, I appear to myself to recognise its redness. In fact, however, the impression of colour which I receive from the object is not that of brick red at all, but a much less decided tint, which I may easily prove by bending my head downwards and letting the scene image itself on the retina in an unusual way."[1] Moreover, the observer tends to experience what he expects to experience. "An officer who superintended the exhuming of a coffin rendered necessary through a suspicion of crime, declared that he already experienced the odour of decomposition, though it was afterwards discovered that the coffin was empty."[2] It is the same with observations carried out for scientific purposes. The

[1] Sully, *Illusions*, p. 88. [2] *Ibid.*, p. 108.

astronomer who is watching for the appearance of a particular star will tend to see the first ray of light before it comes within his line of vision. Important points are equally likely to be omitted. Before the labours of Venetz many keen geologists had worked in valleys once occupied by glaciers, and failed to see the evident marks of glacial action. There is, therefore, more in observation than the bare reception of impressions through the senses. It involves also interpretation and selection.

It is here that we find the possibility of error. It is certain that physiologically each external stimulus leads to the appropriate nervous response. If we mistake a stranger for a friend it is not because there is a wrong image on the retina: it is that the image actually there has evoked the wrong set of ideas in the mind. This evoking of ideas is the one and only manner in which what is present to the body is also present to the mind. The majority of things which surround us every moment fail to evoke any clear ideas: we do not notice—or observe—them at all.

If we examine the process we see that it is inferential—that is, that from what is directly given a conclusion is drawn which is not directly given. Take the simplest case of all—that of recognition. If a man looks at an orange and says "That's a Jaffa," he is stating what is certainly not given directly by the sense impressions which reach his eye. Yet the judgment is immediate, and does not point to anything beyond itself which would justify the statement. But if the question were asked, How do you know? he might proceed to call attention to the shape, and to assert that all Jaffa oranges have the same shape, namely that of the example before him, and hence that his original assertion was correct. To strengthen his conclusion he might add that it could not be any other variety, for instance a Tangerine, because of the absence of certain characteristics of size and so forth by which that variety is distinguished. Now although the judgment "That's a Jaffa," is seen on analysis to involve inference, we do not commonly call it an inference because it contains no direct reference to the grounds on which it

is made. When, however, in response to some challenge the reasons for the judgment are given explicitly, we have the fully developed inference with which logic is concerned.

When this inference is made explicit it is seen to take the form of a syllogism in the second figure—

> Jaffas have qualities *a b c*,
> This orange has the qualities *a b c*,
> Therefore, this orange is a Jaffa.

The fact that the middle term is undistributed shows that formally the conclusion is only one of a greater or less amount of probability: hence, the possibility of error.

We have dealt with the very simple case of recognition because it is evident that if that involves implicit inference, and is liable to error, much more will this be the case with the more elaborate observations on which science is built. Throughout, observation is seen to depend not simply or even mainly on the integrity and firmness of the senses, but on the accuracy of the interpretation given by the mind.

We have pointed out that always the greater part of the sense impressions we receive pass unnoticed: in other words that all observation is *selective*. What we select to observe is determined by the purpose in hand, that is by the explicit or implicit question our mind asks of reality. The fruitfulness of our observation will depend on the perfection of our choice. And this is also inference, guided by pertinent knowledge. For we may disregard what is really essential. In the days when chemistry was almost exclusively qualitative, the weight of the products of chemical action, though occasionally recorded, was regarded as unimportant. It was not until the time of Cavendish and Lavoisier that this quality was selected as of the first importance in chemical enquiry. Nor would it have occurred to them to give such prominence to the balance had they not been expert chemists as well as men of insight.

On the other hand the selection may be vitiated by laying too much stress on what is trivial and incidental.

But how can the selection be rightly made? There are no rules. Both the intelligence with which the phenomena to be observed are selected, and the power of finding meaning in the observations when they are made, depend upon the character of the mind of the observer—not only upon previous knowledge, though that is essential, but also upon that special insight which marks the true discoverer. A strict impartiality open to every impression would defeat itself, but to appraise correctly what is presented requires abundant knowledge in the department of science to which the investigation belongs. Moreover, the growing recognition of a fundamental unity in the sciences emphasises strongly the need for a considerable acquaintance with sciences which are collateral with the particular one which the worker has made his own. Only so can the facts be appreciated at their true worth. What is found in any object examined is largely determined by what the mind brings to the examination. A piece of machinery means more to an engineer than to an observer with equally good sense-organs who is ignorant of machinery; the former, indeed, actually *sees* more in it than does the latter.

It is the same in the domain of scientific research. Count Rumford in 1798 "while engaged in the boring of brass cannon at the military arsenal in Munich . . . was struck by the high temperature of the metallic chips thrown off, and by the excessive development of heat during the process."[1] The observation was not barren, for it led him to make certain experiments by which he showed that there was a relation between work done and the amount of heat generated, though it was left to Joule to determine the relation exactly. Yet the facts must have been perfectly familiar to the workmen engaged: to the mind of the Count, prepared by wider knowledge, they meant more.

After some time spent in the pursuit of any branch of knowledge, an investigator comes to recognise almost intuitively the kind of conditions to which it will be

[1] Preston, *Theory of Heat*, p. 39.

well for him to give heed. His increasing knowledge makes it possible for him to detect with greater readiness the bearing of the facts as they are given, and so to isolate in thought the essential from the unessential. But there will always remain individual differences in knowledge and in insight. "The rôle of the mere observer must always be a humble one, even in the case of those sciences which offer him the most abundant scope. The true 'seer,' indeed, is the rarest of all discoverers: but the true seer is one who brings to his observation more than he finds in it. The drudgery of the patient interrogator of nature is made divine only when it is inspired by ideas which are not objects of observation." [1]

3. **Danger of Bias.**—Since the mind has so large a share in the practice of observation it is difficult, even with the utmost candour and a sincere desire for truth, to distinguish what is perceived from what is inferred. The difficulty is rendered greater by the liability of most men to be unconsciously the victims of bias and prejudice. Beliefs that have been handed down by tradition or that are the outcome of previous investigation become hardened by use and custom until they are acted on as inviolable truths. Prepossessed by such shadows of knowledge men are apt to attend only to those facts which support their prejudices or preconceived theories, and to neglect those that tell against them, or even it may be to explain them away in the interests of their prior notions. Before the discovery of oxygen bodies when heated were thought to give off a special substance named 'phlogiston.' When it was found that metals so heated became heavier, the apparent contradiction was resolved by supposing phlogiston to have a negative weight which diminished the true weight of compounds in which it was present. So the theory was saved, and the progress of chemistry retarded.[2]

The difficulty is especially felt when a possible explana-

[1] Mackenzie, *Introd. to Social Philosophy*, p. 13.
[2] See Von Meyer, *History of Chemistry*, Ch. iv.

tion of certain phenomena has been conceived, and an appeal is being made to the facts to establish the truth or falsity of the conception. The mind is apt to hold fast with a singular tenacity to theories of its own invention, and "it is difficult to find persons who can with perfect fairness register facts for and against their own peculiar views."[1] It is, therefore, essential to cultivate an attitude of detachment towards the hypotheses which guide for the time being the course of enquiry. This implies an equal readiness to give due weight to instances which appear to negate the suggested explanation as well as to those which strengthen it, and, if need be, to banish ruthlessly the most cherished theory which runs counter to the facts.

The qualities of mind which disregard the influence of bias cannot be brought under logical rules. They may be summed up in the requirement that the observer should incessantly submit himself to the discipline of fact. "Man must submit his idea to nature as it were tremblingly, and if nature rejects it, he must not stiffen himself against her, and take upon himself, which were a useless thing to do, to lesson her, laying down for her the law: he must yield to nature and renounce his idea."[2]

4. **Scientific Instruments.**—If men were confined to their unaided senses as means of observation the area of knowledge would be considerably contracted. Each sense has but a limited range, and there are some natural phenomena, such as those of electricity, for the direct detection of which no sense exists. But what cannot be directly perceived may be prepared for observation by the use of scientific instruments. An object too small or too distant to be seen, a sound too low to be heard, are rendered evident by microscope, telescope, or microphone. So also an event too swift, a scene too extensive to leave a clear impression on the retina may be faithfully recorded on the photographic plate: the composite structure of light which the eye has no power to decompose is revealed through the spectroscope.

[1] Jevons, *Principles of Science*, p. 402.
[2] Rabier, *Logique*, p. 103.

To extend the range of observation is not the only service which instruments can render: they also increase its exactness. We can judge weight by the hand, but not with the accuracy of the balance: we can distinguish temperature by the skin as greater or less, but not with the certainty of the thermometer. And in many ways phenomena may be registered and measured which would either escape detection altogether or which would be imperfectly realised in their quantitative relations.

Now all scientific instruments have grown out of previous knowledge, and embody much knowledge in themselves. They contribute greatly, therefore, to the advance of science. But their use is dependent for its accuracy and fruitfulness on the qualities of the observer. Only a man of great skill and well-versed in science can employ many of the most accurate instruments, because he alone can turn to practical account the knowledge they embody, and can detect and allow for errors incidental to their use. "Skill in modern laboratory work is as far out of the reach of the untaught as performance on a musical instrument."[1]

In many cases the use of instruments makes no change in the phenomena, but only in the conditions under which they are observed. It is still observation, but it implies not only the inference implicit in all observation, but also that involved in applying the knowledge embodied in the instrument. Thus we may have to infer that the object focussed in the microscope is one hundred times smaller than it looks, or that a certain deflexion of the galvanometer indicates a current of a certain strength. The accuracy of the observation depends upon the correctness with which the instrument expresses ascertained truth, as well as on the competence of the observer.

5. **Observation by Experiment.**—The use of scientific instruments is one aspect of the transition from simple observation to experiment. It is better to regard it in this light than to speak of the modified arrival of sense-impressions of phenomena through instruments as being

[1] Sir T. Clifford Allbutt: Article on *Medicine* in *Enc. Brit.*, 11th edn.

really experiment. The latter name is not appropriate unless the instrument modifies the object which is being observed. Thus we invariably speak of *observing* with a telescope or a microscope. The distinction is very clearly put by Dr. Bosanquet: "The fact is then that experiment is not merely observation under artificial and determinable conditions but *observation under determinate conditions which constitute an integral part of the image or product to be observed.* Thus common dissection is not experiment, though it introduces conditions in the way of separation and demarcation as definite as anything can be: but vivisection is experiment, because the determinate conditions it produces enter as factors into the action of the organism observed."[1] This raises the further question as to the relation between observation in general and experiment.

The end aimed at in observation is a full and exact knowledge of all the conditions without which the phenomena observed would not occur. If these conditions were presented in isolation, the task of the observer would be comparatively simple. But this is not the case. They are as it were overlaid with many other elements which obscure them from view, and often an analysis which is purely mental is not sufficient to get rid of these factors which do not directly contribute to the result under investigation. By varying as much as possible the circumstances under which the observation is made something may be done to eliminate what is unessential. If at different times and in different places with varied surroundings we find the same features recurring again and again, there is some probability that their persistence amid considerable change is an indication that among them the true conditions are at least included. To that extent the area of enquiry is narrowed.

Such observation under varied circumstances is closely akin to experiment, and differs from it only in degree. This has been well expressed by Jevons, who gives to it the name of "natural experiment," and thus describes his

[1] *Logic*, Vol. ii., p. 145.

meaning: "It may readily be seen that we pass upwards by insensible gradations from pure observation to determinate experiment. When the earliest astronomers simply noticed the ordinary motions of the sun, moon, and planets upon the face of the starry heavens, they were pure observers. But astronomers now select precise times and places for important observations of stellar parallax, or the transits of planets. They make the earth's orbit the basis of a well arranged *natural experiment*, as it were, and take well considered advantage of motions which they cannot control. Meteorology might seem to be a science of pure observation, because we cannot possibly govern the changes of weather which we record. Nevertheless we may ascend mountains or rise in balloons, like Guy-Lussac and Glaisher, and may thus so vary the points of observation as to render our procedure experimental. We are wholly unable either to produce or prevent earth-currents of electricity, but when we construct long lines of telegraph, we gather such strong currents during periods of disturbance as to render them capable of easy observation."[1]

But the observer, whether he has recourse to simple observation or to "natural experiment," is hampered in his endeavours to discover the essential conditions of phenomena. He cannot go far by those means alone, for he is compelled to wait on nature. Now nature may present him with instances in abundance, but of such complexity that he cannot disentangle them. On the other hand, they may be of such rarity that adequate investigation becomes difficult or impossible. Again, there are processes of nature as swift and transient as the lightning, and others so slow and gentle that they escape observation. As Lavoisier remarked, the decomposition of water has been continually going on, though none had observed it before his time.

But however alert and active the mind of the observer may be, his activity in every case is limited to the examination of conditions that he can neither determine nor modify. He must make the best of what is given him

[1] *Principles of Science*, pp. 400-401.

It is here that experiment comes to the aid of simple observation by supplying a means of control over natural conditions. By isolation and combination of physical agents it can so manipulate them as to determine in many cases the conditions under which the phenomena to be examined occur. Such definitely determined observation is what is meant by experiment.

Wherever experiment is possible it will clearly be advantageous to resort to it in preference to simple observation. Instead of the medley of conditions, some known, others unknown which nature presents, there will be an assemblage of conditions in greater part determined by the observer, and therefore known. Whatever is irrelevant or likely in any way to interfere with the phenomena under investigation will be as far as possible excluded. Factors believed to affect the result will, so far as they are under control, be varied in intensity and introduced or removed at will; and the differences, if any, will be carefully noted. In this way those conditions which are unessential will be eliminated with greater accuracy and certainty than would be possible to a purely mental analysis.

Moreover there is no long waiting on nature. Experiments can be repeated as often as is necessary. They may be devised so as to produce effects that throw an important light on phenomena, although these effects are never produced by nature herself. "Experiment *invents* original phenomena which nature left to herself never realises; for example, the fall of bodies in a vacuum, the liquefaction of hydrogen and oxygen. The chemist creates in his laboratory many compound bodies which do not exist outside it."[1] In this way knowledge advances more surely and rapidly than would be possible were men confined to simple observation. It is only by the use of experiment that instances can be sufficiently varied in the points of identity and difference to show clearly and simply what the essential conditions are. And these can often be determined with a quantitative precision.

It would therefore be a mistake to set up any opposition

[1] Rabier, *La Logique*, p. 115.

between observation and experiment. Both aim at establishing the exact conditions upon which phenomena depend, but the element of control in the latter case makes observation more efficient than it would otherwise be. Indeed it is not too much to say that without experiment physics and chemistry would have been unable to make any serious advance. There are, however, other departments of knowledge in which observation must always be the prime resource. Some natural processes are so slow that experiment is impossible. The geologist, for example in tracing the history of the earth is confined to observation as to the structure and formation of rocks and the occurrence of fossils; the biologist is unable to experiment on the evolution of species; the historian cannot experiment with the past. On other grounds the doctor and the politician are limited in the experiments it is permissible to make.

6. **Aim of Experiment.**—Appeal to experiment is, then, necessary whenever simple observation alone will not make plain all the essential conditions of a phenomenon; and its object is to eliminate all conditions which are not specially operative in the particular case under consideration. When this has been done successfully we have one or more hypothetical judgments of the form *If S is a then it is x*. Here x will stand for a known element in the phenomenon and a for the condition upon which it invariably and necessarily depends. Now we can only be sure that this relation is invariable and necessary if we establish the reciprocal proposition: *If S is x then it is a* as well as the pure hypothetical, *If S is a it is x*. There is no other way of doing this than by examining instances of the absence of a ($\bar{a}$), and so trying to show that when a is absent, x is also absent ($\bar{x}$); in other words to establish the judgment *If S is* $\bar{a}$ *then it is* $\bar{x}$. Hence in every conclusive experiment there is comparison of the phenomenon both in the presence and in the absence of that particular condition we are investigating.

This latter—which is what chemists sometimes call a *Blind Experiment*—is absolutely essential to the establish-

ment of the reciprocal judgment. But it is more difficult than the positive experiment; for sometimes the effect may really be present but be so small that it either escapes notice altogether or is included in a larger effect and confounded with it. A striking instance of this is the fact, already referred to, that though many experiments had been worked on the constitution of the atmosphere, yet argon so long escaped notice. Jevons quotes a very instructive example of the difficulties of negative experiments. He says: "A curious instance of false negative inference is furnished by experiments on light. Euler rejected the corpuscular theory on the ground that particles of matter moving with the immense velocity of light would possess momentum, of which there was no evidence. Bennet had attempted to detect the momentum of light by concentrating the rays of the sun upon a delicately balanced body. Observing no result, it was considered to be proved that light had no momentum. Mr. Crookes, however, having suspended thin vanes, blacked on one side, in a nearly vacuous globe, found that they move under the influence of light. It is now allowed that this effect can be explained in accordance with the undulatory theory of light, and the molecular theory of gases. It comes to this—that Bennet failed to detect an effect which he might have detected with a better method of experimenting; but if he had found it, the phenomenon would have confirmed, not the corpuscular theory of light, as was expected, but the rival undulatory theory. The conclusion drawn from Bennet's experiment was falsely drawn, but it was nevertheless true in matter."[1] Again, some conditions—*e.g.* gravity—cannot be removed, and in experimenting on phenomena in which such permanent conditions are essential, the experimenter is reduced to varying their intensity as far as possible.

To render a negative experiment conclusive is therefore difficult, though it is at the same time essential. Such a conclusion can be justified only on the assumption that our analysis excludes all inoperative elements and them

[1] *Principles of Science*, p. 435.

only. But in every investigation, owing to the incompleteness of our knowledge, there is a residue of unanalysed phenomena which is regarded as indifferent to the relation we are trying to establish; and this in addition to those elements which are disregarded because previous knowledge of their nature makes it improbable that they need be considered in the given case. The exclusion of possible conditions in either of these directions may be at fault. Such an arbitrary limitation can only be made with any probability of justification by one who has large and varied, as well as systematic knowledge, both of the subject under investigation and of kindred subjects. For effective experiment, even more than effective observation, is conditioned by the knowledge and insight of the man of science who devises the experiment, whether he carries it out himself or not.

In all cases where the phenomena are complex this limitation through exclusion is, therefore, liable to error. Such error can only be detected by extremely careful and varied experiments to determine whether any condition is operative which had not been suspected, and had, therefore, been relegated to the realm of concomitants that it is thought need not be taken into consideration. An experiment of Pouchet affords a striking example. "He filled a bottle with boiling water, hermetically sealed it with the greatest care, and plunged it upside down into a basin of mercury. When the water was quite cold he uncorked the bottle under the metal, and introduced into it half-a-litre of pure oxygen gas . . . [He] then introduced a minute bunch of hay which had been enclosed in a corked bottle, and exposed in a stove for a long time to a temperature of more than 100 degrees."[1] Living organisms developed in the hay. He concluded that since he had excluded all germs from the water, the hay, and the oxygen, life had been spontaneously generated. "Pasteur thus criticised the experiment of Pouchet: 'This experiment is irreproachable, but irreproachable only on those points which have attracted the attention of its author. I will demon-

[1] *Louis Pasteur*, p. 95.

strate before you that there is a cause of error which M. Pouchet has not perceived, which he has not in the least suspected. . . . I will prove to you, in short, that it is the mercury which carries the germs into the vessels.' . . ."[1] A number of experiments were carried out which established the truth of this assertion.

7. **Fallacies incident to Observation.**—As all inductive inference starts from concrete phenomena as the basis of every hypothesis, and continually returns to concrete phenomena as the test of accuracy, it is evident that its validity depends directly upon the correctness and perfection of the observations of reality upon which it is based. But observation of given phenomena is a process of extreme difficulty and delicacy, and, consequently, one in which it is very easy to go wrong. The conditions which must be fulfilled by a perfect observation have already been set forth. We will now consider the main kinds of error to which observation is liable. Error which is due to mere personal idiosyncrasy or to carelessness, we are, of course, not concerned with. Logic only considers those sources of error to which observers in general are subject.

In the first place, then, as Mill points out, "a fallacy of misobservation may be either negative or positive; either Non-observation or Mal-observation. It is non-observation when all the error consists in overlooking, or neglecting, facts or particulars which ought to have been observed. It is mal-observation when something is not simply unseen, but seen wrong; when the fact or phenomenon, instead of being recognised for what it is in reality, is mistaken for something else."[2] It will be most convenient to consider these two classes of imperfect observation in succession.

(i) **Non-observation.**—It has been pointed out that all observation implies selection and isolation of phenomena. In making this selection it is evidently easy to overlook or to neglect either instances or conditions which are perti-

[1] *Louis Pasteur*, p. 96. [2] *Logic*, V., iv., § 1.

nent to the matter in hand. The former would be most likely to happen in the earliest stage of inductive enquiry, when by simple enumeration of instances an attempt is made to determine exactly what is the character of the phenomena to be explained. The latter would be most easily committed at a later stage of the process, when an analysis of phenomena has been entered on for the purpose of testing and moulding a hypothesis.

(*a*) *Neglect of Instances.* Probably the most fertile source of the error of passing over instances pertinent to the subject in hand is bias. We have a natural tendency to dwell upon instances which make for the theory we would like to see established, or which we have hitherto held, and to treat as of no importance those which make against it, or even to neglect to consider them at all. A striking example of this is mentioned by Mill: "The opponents of Copernicus argued that the earth did not move, because if it did, a stone let fall from the top of a high tower would not reach the ground at the foot of the tower, but at a little distance from it, in a contrary direction to the earth's course; in the same manner (said they) as, if a ball is let drop from the mast-head while the ship is in full sail, it does not fall exactly at the foot of the mast, but nearer to the stern of the vessel. The Copernicans would have silenced these objectors at once if they had *tried* dropping a ball from the mast-head, since they would have found that it does fall exactly at the foot, as the theory requires: but no; they admitted the spurious fact and struggled vainly to make out a difference between the two cases."[1] This influence of bias is not surprising when it is remembered how largely observation involves implicit inference and is guided by previous knowledge.

A very common form in which non-observation of instances appears is when attention is directed to positive instances, whilst negative instances are neglected. This originates in a kind of natural tendency of the mind to be impressed by any occurrence and to omit to notice non-occurrence. To this are due many superstitions which take

[1] *Logic,* V., iv., § 3.

the general form of attributing causal connexion to what is merely casual coincidence. For instance, the belief in the prophetic character of dreams is based upon taking note of the few cases in which a dream bears some resemblance to succeeding or simultaneous events, and passing over entirely those infinitely more numerous cases in which the dream is not 'fulfilled.' The theory of probability would prepare us to expect some such coincidences—especially as a small degree of resemblance is always liberally interpreted—and that without any necessary connexion. "In former generations," remarks Dr. Fowler, "'coincidences' of this kind were regarded not simply as 'curious' and 'remarkable,' but as proofs of some causal connexion between the events. To talk of a person was supposed to render his presence more likely; a verified prediction was regarded as evidence of second-sight; and a comet which was observed to be followed by a war was supposed to be, if not the cause of the war, at least a messenger sent from Heaven to proclaim its approach."[1]

Here the unconscious influence of bias and prejudice plays an important part—"He answered well, who, when the pictures of those who had fulfilled their vows after escaping the peril of shipwreck were shown him hung up in a temple, and he was pressed with the question, did he not after this acknowledge the Providence of the Gods, asked in his turn, 'But where are they painted who, after vowing, perished?' The same is the method of almost every superstition, as in astrology, in dreams, omens, judgments, and the like; in which men who take pleasure in such vanities as these attend to the event when it is a fulfilment; but where they fail, (though it be much the more frequent case,) there they neglect the instance, and pass it by."[2]

One other danger of non-observation must be noticed—the tendency to infer that because a phenomenon has never been noticed it is non-existent. To what extent such an inference is justifiable depends upon the improbability that the phenomenon would have escaped observation had it

[1] *Inductive Logic*, p. 256.

[2] Bacon, *Novum Organon*, tr. by Kitchin, § 46.

existed. As Jevons says, "The earth's surface has been sufficiently searched to render it highly improbable that any terrestrial animals of the size of a camel remain to be discovered."[1] But the smaller the phenomenon the more likely it is to escape observation. A good instance of this is the length of time during which the existence of argon, as a constituent of the atmosphere, was unsuspected.

When a proposition is accepted on merely negative evidence, care should be taken to make that evidence as complete as possible. An excellent example of this care is seen in the investigations which led Darwin, on purely negative evidence, to conclude that certain orchids secrete no nectar. Many observations both of German naturalists and of Darwin himself seemed to point to this conclusion, but, says Darwin: "Notwithstanding these several facts I still suspected that nectar must be secreted by our common Orchids, and I determined to examine *O. morio* rigorously. As soon as many flowers were open, I began to examine them for twenty-three consecutive days: I looked at them after hot sunshine, after rain, and at all hours: I kept the spikes in water, and examined them at midnight, and early the next morning: I irritated the nectaries with a bristle, and exposed them to irritating vapours: I took flowers which had lately had their pollinia removed by insects, of which fact I had independent proof on one occasion by finding grains of some foreign pollen within the nectary; and I took other flowers which, judging from their position on the spike, would soon have had their pollinia removed; but the nectary was invariably quite dry. . . . I one day saw various kinds of bees visiting repeatedly the flowers of this same Orchid, so that this was evidently the proper time to examine their nectaries; but I failed to detect under the microscope even the minutest drop of nectar. So it was with the nectaries of *O. maculata* at a time when I repeatedly saw flies of the genus Empis keeping their proboscides inserted into them for a considerable length of time. *Orchis pyramidalis* was examined with equal care

[1] *Op. cit.*, p. 412.

with the same result, for the glittering points within the nectary were absolutely dry. We may therefore safely conclude that the nectaries of the above-named Orchids neither in this country nor in Germany ever contain nectar."[1]

(*b*) *Neglect of Operative Conditions.* In every examination of concrete reality a residue of unanalysed elements remains, and this opens out a possibility of leaving out of consideration some element which is an essential part of the phenomena we are investigating. The whole intention of all inductive methods is to eliminate those elements, and those elements only, which are not operative in respect to the aspect of reality under examination. In a science still so largely in the empirical stage as medicine, this fallacy is still not unknown and was formerly extremely common. Take as an instance Kenelm Digby's sympathetic powder which in the seventeenth century attracted considerable attention even from scientific men. "The sympathetic powder was that which cured by anointing the weapon with its salve instead of the wound. I have long been convinced that it was efficacious. The directions were to keep the wound clean and cool, and to take care of diet, rubbing the salve on the knife or sword. If we remember the dreadful notions upon drugs which prevailed, both as to quantity and quality, we shall readily see that any way of *not* dressing the wound would have been useful."[2] Those who attributed the cure to the powder committed the error of overlooking the essential circumstance that nature was left to her own devices under favourable conditions for effecting a cure.

Owing to the complexity of the phenomena, all inductions connected with social and economic subjects are particularly liable to this form of fallacy. An increase in the number of convictions for any particular crime must not be taken as a necessary proof of an increase in that particular form of criminality; it may be due to greater vigilance on the part of the police. An excellent example,

[1] *Fertilization of Orchids*, pp. 38-39.
[2] De Morgan, *Budget of Paradoxes*, p. 66.

given by Mill, is the following: "Take, for instance, the common notion, so plausible at the first glance, of the encouragement given to industry by lavish expenditure. A, who spends his whole income, and even his capital, in expensive living, is supposed to give great employment to labour. B, who lives on a small portion, and invests the remainder in the funds, is thought to give little or no employment. For everybody sees the gains which are made by A's tradesmen, servants, and others, while his money is spending. B's savings, on the contrary, pass into the hands of the person whose stock he purchased, who with it pays a debt he owed to some banker, who lends it again to some merchant or manufacturer; and the capital being laid out in hiring spinners and weavers, or carriers and the crews of merchant vessels, not only gives immediate employment to at least as much industry as A employs during the whole of his career, but, coming back with increase by the sale of the goods which have been manufactured or imported, forms a fund for the employment of the same and perhaps a greater quantity of labour in perpetuity. But the observer does not see, and therefore does not consider what becomes of B's money; he does see what is done with A's: he observes the amount of industry which A's profusion feeds; he observes not the far greater quantity which it prevents from being fed; and thence the prejudice, universal to the time of Adam Smith, that prodigality encourages industry, and parsimony is a discouragement to it."[1]

(ii) **Mal-observation.**—By mal-observation is meant the wrong interpretation of sense impressions. It has been pointed out that all observation involves implicit inference, or the interpretation of what is received through the senses by referring it to some part of the knowledge previously possessed. When this reference is wrongly performed, we have a case of mal-observation. The rustic who takes a tombstone brightened by the rays of the moon for a ghost, or who interprets a donkey's bray as the voice of a departed ancestor, falls into the fallacy we are now

[1] *Op. cit.*, V., iv., § 4.

considering. All 'conjurer's tricks' appeal to the innate facility with which mankind observes badly. It is, however, never the senses that are wrong; the sensuous impression as received is just what it should be under the circumstances; in the meaning given to that impression the error will always be found to lurk. One of the most striking examples of mal-observation was the universal acceptance of the Ptolemaic solar system for many centuries. As Mill remarks, "People fancied they *saw* the sun rise and set, the stars revolve in circles round the pole. We now know that they saw no such thing; what they really saw was a set of appearances equally reconcilable with the theory they held and with a totally different one."[1]

Sometimes we find ourselves practically unable to interpret our sense impressions rightly, and we are then said to labour under an illusion. "To give one or two . . . examples of the kind of illusion which the senses practise on us or rather which we practise on ourselves, by a misinterpretation of their evidence: the moon at its rising and setting appears much larger than when high up in the sky. This is, however, a mere erroneous judgment; for when we come to measure its diameter, so far from finding our conclusion borne out by fact, we actually find it to measure materially less. Here is eyesight opposed to eyesight, with the advantage of deliberate measurement. In ventriloquism we have the hearing at variance with all the other senses, and especially with the sight, which is sometimes contradicted by it in a very extraordinary and surprising manner, as when the voice is made to seem to issue from an inanimate and motionless object. If we plunge our hands, one into ice-cold water, and the other into water as hot as can be borne, and, after letting them stay awhile, suddenly transfer them both to a vessel full of water at blood-heat, the one will feel a sensation of heat, the other of cold. And if we cross the two first fingers of one hand, and place a pea in the fork between them, moving and rolling it about on a table, we shall

[1] *Op. cit.*, V., iv., § 5.

(especially if we close our eyes) be fully persuaded we have two peas. If the nose be held while we are eating cinnamon, we shall perceive no difference between its flavour and that of a deal shaving."[1]

The history of science presents us with many instances of mal-observation. "A vague and loose mode of looking at facts very easily observable, left men for a long time under the belief that a body, ten times as heavy as another, falls ten times as fast;—that objects immersed in water are always magnified, without regard to the form of the surface;—that the magnet exerts an irresistible force;—that crystal is always found associated with ice;—and the like. These and many others are examples how blind and careless men can be, even in observation of the plainest and commonest appearances; and they show us that the mere faculties of perception, although constantly exercised upon innumerable objects, may long fail in leading to any exact knowledge."[2]

Dangerous as this fallacy is, logic can neither give special rules for avoiding it, nor determine when it has been committed. The latter point is decided by further advance in that particular branch of knowledge. On the former the only rule that can be given is the very general one that every observer should equip himself with as much definite knowledge as he possibly can both of the subject under investigation and of cognate subjects.

[1] Herschel, *Natural Philosophy*, § 72.
[2] Whewell, *Novum Organon Renovatum*, p. 61.

CHAPTER XXVIII.

TESTIMONY.

1. **Importance of Testimony.**—The range of direct experience even for the most versatile and gifted of men is extremely small. Were the individual dependent upon it alone for his knowledge his mental outlook would be narrow, and he could neither impart the results of his experience to others nor become acquainted with their achievements; the co-operative movement of science as well as a large part of ordinary life would be non-existent. But men communicate the knowledge they acquire to one another, and so contribute both to individual good and to general progress. Where the communication relates to observations that any competent person may repeat, there is an independent check on accuracy. The first announcement of the discovery of radium and certain of its properties is not the sole authority on which the facts have been accepted: many workers have tested and confirmed the properties for themselves.

So vast is the field of knowledge that no individual worker can cover the whole of the ground even in that portion which he seeks to make his own. He must accept many of his data on authority, and leave the corroboration of them to others of his fellow-labourers. The geographer could not possibly check the widely-scattered observations upon which his generalisations are founded, though each of them might be capable of verification. So, too, in ordinary life, we are compelled to receive and act upon much of which we have neither the time, capacity, nor opportunity to make sure for ourselves. We accept London as a fact long before we ever see it, and when we do see it we do not confine our conception of it to the parts that we actually observe. Thus a good deal of the working knowledge both of life and of science is based on testimony.

Further, there are many occurrences which from their nature can never be repeated. Where the check of independent repetition fails us, we are forced to a closer scrutiny of the source from which the information comes. In no other way, for example, can we sift the facts which form the foundation of history. The past is past, and is only known to us so far as some record of it remains. In studying it we cannot get into direct touch with the experiences we seek to understand: we know them only through the medium of one or many witnesses, who must go bail for the accuracy of the facts which they avouch. If no reliance could be placed on testimony when it is backed up by no evidence but itself, it is unnecessary to say how large a part of historical and sociological knowledge would crumble away for want of support.

2. **Value of Testimony.**—Now it is obvious that testimony varies considerably in value. Some we accept without hesitation, some of it as unhesitatingly we reject, while towards a good deal of it our attitude is one of suspended judgment. Every piece of testimony which is examined resolves itself, if only we can carry the analysis far enough, into an observation and a communication of the observation. Each affords numerous opportunities for error. Even the careful scientific observer, who in the quiet of his laboratory, or amid the restful scenes of nature, is most favourably situated for a calm concentration of his mind on the matter before him, is liable as we have seen to error. The danger of mistake is far greater when, as so often happens, the witness on whose testimony we are asked to rely was an active participator in scenes which he afterwards portrays. Rarely has the narrator of an historical event observed it with no other intent than to make a record for posterity. Add to this that any account even of the recent past will almost invariably reveal gaps due to imperfect recollection, and we see that a writer or speaker with a sincere desire to make his statements accurate may easily deceive himself, and hence in all good faith propagate the false as true. Unfortunately not only may a man be unwittingly deceived, he may of set purpose endeavour to

deceive. In any particular case we must be satisfied beyond reasonable doubt that deception of either kind is absent before testimony can be accepted as a guarantee that what is testified is true. So great is the improbability of any individual being free from one or other of the two faults that it is commonly said that one witness to an event is no witness at all. Nor is convergent testimony by any means so conclusive as to be accepted at its face value.

When we turn from the observer to the written records based upon his observations a fresh series of problems presents itself. The book or manuscript may not be what it purports to be. The relation of the writer to the sources of his information may be of all degrees of nearness, from that of eye-witness to that of the mere transcriber of oral tradition and legend. He may have idiosyncrasies of style or character which cast doubt on the accuracy of all he writes.

Enough has been said to show that in whatever form testimony reaches us it must be subjected to careful sifting. The aim is that the fact it attests be re-constituted for thought as it really was in the first instance. This ideal is often impracticable. But in any case such criticism can only attain a measure of success where special knowledge and skill are brought to bear on the task.

3. **Criticism of Testimony.**—As we have seen, there are two grounds upon which we may have reason to distrust a piece of testimony. There is the possibility that the author may know the truth, and yet deliberately misrepresent it. On the other hand he may in all good faith present a tissue of truth and falsity to us which he himself believes to be solely true. We are bound, therefore, to doubt whatever comes to us through this source alone, until we are convinced of the sincerity and accuracy of the witness who attests it.

But how can we determine whether a man is sincere or not in his statement of what came under his own observation? Obviously the enquiry is one of great difficulty. If we know something of the character of the man, we can estimate his general level of credibility; but as a rule

the material for such an estimate is either scanty or, even if copious, conflicting.

There are, however, certain general motives which induce men to lie, and we may ask if anything in the character or circumstances of the witness makes it likely that they were operative. There are some men with a love of exaggeration which leads to habitual distortion of fact; others have an itching to speak, and cannot refrain from supplementing the defects of experience by the products of inference and imagination. Many are swollen with self-importance and allow vanity to credit them with fictions that enhance their reputation. Nor is it uncommon for a man to recount the false and marvellous simply because they are attractive to him. But perhaps the most powerful motive to lying is some form of self-interest. There may be the hope of personal gain, or the fear of impending harm; there may be a party to serve or a cause to condemn. Truth ceases to be of importance if only the end in view can be gained.

There is often no surface test by which to distinguish the false from the true: both wear an air of sincerity. Indeed in the case of the false this will add to the deception. Literary skill is in itself a snare to a writer. The gorgeous epithet, the rounded phrase and eloquent period, are easily coined from the clippings of veracity. Men differ in the control they exercise over tendencies to insincerity, and all do not feel them in the same form or in the same strength. But whenever sincerity is jeopardised the reason should be ascertained if possible, and allowed for so far as it invalidates the extent to which the testimony can be received. If what is vouched for can be established or rejected by other means, then the good faith of the author ceases to have any logical interest. We are only concerned with it when it claims to be part of the guarantee of truth.

Sincerity is logically useless without *accuracy*, and that is not easy to secure. All direct testimony refers to something observed, and may be marred by any or all of the defects to which observation is subject. It is especially important to determine how far the recorder of a fact was

in a position to observe correctly. It is seldom that testimony is confined to a single incident or series of incidents well within the range of an individual observer. Frequently the events described, either because of their complexity, or because of the wide area over which they are spread, cannot be perceived by one man alone. The description of a siege, for example, would be fragmentary and disconnected if confined to the direct experience of one of those engaged in the struggle. It would be a physical impossibility for him to take in all the essential details of say an assault or a sortie, and his duties might or might not have given him leisure for a general survey. Moreover, many things would happen of which few would be cognisant at the time: as, for example, the daily consultations and plans of the leaders. Hence the necessity of scrutinising the 'narrative of an eye-witness' in order to separate the details which, because of his location or circumstances, must have been supplied indirectly from those which in all probability came under his personal notice. Men differ too in their capacity to seize the essential features of a situation. One man carries away a confused impression where another swiftly marshals the details into an orderly whole.

Prejudice may vitiate testimony as well as observation. A fact cannot be presented naked. It must be clothed in words, and in this way we have a second source of inaccuracy. For even where the facts have been apprehended correctly, they may be woven together so as to create a false impression. We must discount, therefore, the sub-conscious bias from which few, if any, authors are entirely free. Enquiry should be made how far the writer has been led by his preconceived opinions to select particular aspects for description to the exclusion of others of equal importance, or how far he has grouped his facts to illustrate an idea which he did not find in them but brought to them. We must disrobe the works of the 'whig historian' of their whiggish dress before we can see the truth they contain; nor would it be fair to accept without question the story of the downfall of Carthage as told by Roman writers. Prejudice may peep out in an

epithet, and we must cut away all such disguised comment from what purports to be a faithful narration of fact. No matter what may be the protestations of authors to the contrary, love and hatred often exert an influence on their judgment and imagination of which they are quite unconscious.[1] And it is imperative to ascertain their likes and dislikes in order to eliminate all traces of partiality from what they record.

Memory in most men is notoriously treacherous. Its efficiency as a rule decreases with the lapse of time from the events which are recalled. For this reason it is a sound demand in all the natural sciences that observations should be noted down at the time when they are made. But it would be impossible to insist on a similar requirement in the case of all testimony. The men whose writings furnish much of the raw material of history did not mingle in the affairs of life, whether as actors or spectators, note-book in hand: such a procedure is left to the professional reporter. Whether rough notes are made or not the written document is the fruit of recollection. Then how many are the gaps that have to be filled in! Inference and imagination lend their aid. Jagged edges are smoothed away, and a continuous story is composed which has all the air of a transcript from actual experience. The longer the interval, the more likely is the element of construction to preponderate. An old man's memories suffuse the past with a mellow light, and facts often recalled soon cease to be distinguished from accretions and confusions which the years have brought with them. It is so even for much shorter periods. A few hours or a few days are often sufficient to blur a recollection. We must, therefore, ask how near to the actual occurrence is the writer's description of it, and how far was the event of such a kind that it would be recalled with ease and distinctness; and further, are there any indications that the writer's memory was in general trustworthy? Add to this a critical scrutiny of the statement itself, and we may often make a fair estimate of the probability that certain

[1] Cf. Casteleyn, *Logique*, p. 405.

details are due to the filling out of imperfect recollection by inference, or by imagination guided perhaps by a later opinion or prejudice.

So far we have spoken of the testimony of a single person. But there may be several witnesses to the same fact. Taken singly each one may be open to doubt on the score of want of good faith and accuracy, and yet the total evidence may be strong. The untrustworthiness of each witness is not necessarily cumulative. That they agree or disagree is in the nature of additional evidence. Supposing it to be established that there has been no collusion between them, and that in no case are their statements derived from a common source, then their agreement on essential points indicates a strong probability that their testimony is true, while any discrepancy proves that one statement at least is false. This is all the more likely to be the case if there is considerable variation in detail. Half a dozen people narrating the story of a fire are sure to differ in describing the several episodes: but their diverse stories would be strong evidence as to the fact of the fire. If certain witnesses agree while others disagree the conflicting accounts must be carefully compared. It will not do merely to count the number for and against. Every witness is not equally competent in all matters. The opinion of the layman is usually of inferior weight to that of the expert.

There are some facts of so improbable a nature that no amount of concordant testimony would lead us to accept them. Crowds as well as individuals are subject to illusion, and in all good faith report portents and prodigies. What at first sight is contrary to the order of nature as we know it is rightly subjected to severer tests than what appears as an exemplification or extension of laws with which we are familiar, though men may sometimes reject as incredible what is afterwards accounted true. This antecedent probability or improbability of a fact affects the value of testimony. "In certain cases the probability of the fact and the probability of testimony reinforce each other; but in other cases the improbability of the fact and the probability of testimony are counterbalanced: and indeed

it may happen that the former will be sufficiently strong to destroy entirely the latter."[1]

4. **Criticism of Indirect Testimony.**—The greater part of testimony reaches us indirectly. It is not often that the statement of an original witness lies before us. The historian as a rule presents material from various sources. Even when he states facts on his own authority we may reasonably doubt whether more than a small proportion of them are due to him as an eye-witness. Contemporary evidence is valuable because it is given by one who is under the influence of the age in which he lived, and one very often in a position to ascertain the truth. So far as the testimony is that of an eye-witness it must satisfy the requirements we have already set forth. But if it represents facts that have been transmitted through the medium of one or more persons we must endeavour to trace them back to the source from whence they came.

The first question to ask about any document will therefore be—Who was its author? The problem may be perfectly simple when we are dealing with a modern work, but in earlier times when an author did not scruple to issue his thoughts under a better known name and forgeries were common, and in all cases of anonymous writings, a critical examination of style and content is essential. To this must be added, more especially in the case of manuscripts, an investigation directed to the establishment of the text approximately as the writer wrote it. The errors that are made in copying works that have come down to us in manuscript form are numerous and need special knowledge to detect. Closely allied with these two enquiries will be the determination of the time when the document was first written. All these matters may scarcely present themselves for special investigation in the case of comparatively recent works. But we cannot make an estimate of the value of testimony to fact in any writing until we have satisfied ourselves about its authenticity, integrity, and date.

Then we may begin the task of tracking to their source

[1] Rabier, *Logique*, p. 323.

the statements of fact which the work contains. As a preliminary to this we shall endeavour to divest them of the comment or amplification in which they lie embedded. It is not often that the exact origin of the facts can be determined so that we are face to face with an original bit of testimony. For the most part the authority on which the author has relied will remain unknown, and hence it cannot be tested. We are not in a position to assess at their true worth the sincerity and accuracy of persons about whom we know nothing. At first sight it might seem a safe rule to reject anonymous statements as unworthy of credence. But this must be taken with much abatement. It would, for example, reject all newspaper records, which even the most sceptical among us would hardly be prepared to do. Events do not stand in isolation from one another, nor do they come and go without leaving any trace behind. So we can often accept the evidence of one or more anonymous witnesses, because what is attested accords with the knowledge we have of the times or helps to explain the past more satisfactorily. Such testimony may even give the key to the results of an age which without it would be unintelligible. Moreover we are sometimes in a position to test particular statements by an examination of monuments, coins, or other remains. And if the author stands this test successfully it gives a greater or less probability, according to circumstances, that the other parts of his evidence are trustworthy.

The monuments, medals, coins, vases and other products of man which survive from the past must themselves be subject to criticism. They too may flatter and lie or be misdated. Inscriptions are not always veracious. They may err by excess of panegyric or by actual falsehood. The English struck a medal commemorating the capture of Carthagena in 1740 by Admiral Vernon, though as a matter of fact the gallant admiral had raised the siege.[1]

Where testimony takes the form of traditions handed down orally from generation to generation, there is often

[1] Cf. Rabier *op. cit.*, p. 326.

no way in which we can be sure whether they had their origin in truth or in fancy. Scarcely anyone can receive and transmit faithfully an accurate account of even very simple transactions; and after a few repetitions the original is apt to be distorted almost beyond recognition. Even where tradition is embodied in poetic form errors in transmission accumulate. To find, as some profess to do, a kernel of truth in traditions whose source is lost in remote antiquity can never be justified as other than more or less reasonable conjecture. At most we can say that story and legend of this kind are valuable as affording some indication of customs, beliefs, and mode of life, in the time to which they owe their origin.

Not only do men examine what an author has said, but not infrequently inferences are made from what he has not said. This is a tempting procedure whenever information on points of importance is scanty. It generally appears in the claim that an author must have mentioned a certain fact if such fact had been in existence. But the omissions of authors are apt to confound all conjecture. Logically the argument from silence is only admissible where the purpose of the writer was such that had the alleged fact omitted been true it would have been vital to his statement and would on no conceivable grounds have been suppressed.

CHAPTER XXIX.

NATURE OF HYPOTHESES.

1. **Meaning of Hypothesis.**—Observation and testimony furnish facts which need interpretation if they are to find their true place in a system of knowledge. To interpret them is to determine the relations which they hold to one another and to the rest of the sphere of knowledge to which they belong. The mind must supply the interpretation, and so proceeds by inventing some provisional explanation which seems likely to account for the facts as they are. If on further examination this explanation is proved to be true, it becomes a part of knowledge: if false, it is rejected. In either case the explanation exists in the first instance as a supposition. Such a supposed relation suggested by the facts is termed a hypothesis.

We are continually forming hypotheses. Every supposition we make so as to account for any event whatever is a hypothesis. No doubt, it is customary to restrict the name to suppositions made in the scientific investigation of any subject; but these do not differ in original character from the hypotheses of common life. The difference comes in later; the hypotheses of every-day life answer their purpose in a more or less rough and imperfect form, but the hypotheses of scientific thought need to be determined as completely and accurately as possible, till at length they satisfy all the demands made upon them as explanations of actual phenomena, and are accepted as demonstrated and established theories.

To such hypotheses our discussion will be confined, for logic deals only with scientific—that is, with exact and accurate—thought. It must be remembered, however, that,

as Clifford reminds us, "Scientific thought does not mean thought about scientific subjects with long names. There are no scientific subjects. The subject of science is the human universe; that is to say, everything that is, or has been, or may be related to man."[1] In scientific thought—no matter what the subject-matter may be—every regularity or irregularity in the occurrence of events for which no reason is apparent, every imperfectly understood or unexplained fact, is a call for a hypothesis. And every hypothesis is a guide to further enquiry till the ultimate goal of explanation is reached.

2. **Origin of Hypotheses.**—The modes in which hypotheses are suggested cannot be tabulated. The process of suggestion is purely individual, and does not admit of general rules. Some hypotheses are so obvious that an acquaintance with the facts almost inevitably gives birth to them. Others, and often the most far-reaching, require the insight of genius to fashion them. Thus men differ much in their power and readiness to frame hypotheses. One laboriously determines facts, but is slow to see their meaning. Another has to prune the luxuriance of conjecture: to him everything is suggestive. The man of scientific genius is fertile in suppositions, and quick to see their probable value. A few of his discoveries may come as it were unsought, but the bulk of them are the result of definite lines of enquiry set on foot to test his own suppositions. We cannot, then, eliminate the personal factor.

But although the suggestion of hypotheses depends so much upon the insight of the individual who conceives them, it is generally to the man with a wide knowledge of the order of facts under investigation that they occur. Brilliant conjectures may flash across the unprepared mind, but they are apt to remain fruitless. Perraudin, an intelligent Swiss hunter of chamois, conceived the idea that the huge blocks of rock scattered about the valley with which he was most familiar had been transported there by glacial action. But without the special knowledge and

[1] *Essays*, p. 86.

interests of the geologist he was unable to perceive the true significance of his observation.[1] Pasteur prefaced his long investigation into the causes of disease among silkworms by making himself master of all that was then known on the subject. It was this, added to his own great ability, which made him ready to see the bearing of the facts revealed by his microscopic examinations, and which gave point to the suppositions he founded on them. Knowledge of facts and laws previously ascertained gives the power to detect what is new or exceptional, and so in need of further explanation. It enables a man to give steadiness to his conjectures by basing them on what has already been done.

It would be wrong to regard the hypothesis as a new creation. It always incorporates past knowledge, and arises out of it. It is for this reason that men of the same epoch often make the same discoveries or enunciate similar theories. The doctrine of evolution was taking shape in the minds of Darwin and Wallace at about the same time; Adams and Leverrier both predicted the existence of the planet Neptune. There is a general progress of science which while carried on by individuals is in a sense independent of particular individuals. Thus it has been said that the invention of the calculus was bound to come about the time it did even though neither Newton nor Leibniz had thought of it. On the other hand a bold conjecture may be before its time, and remain in oblivion because the state of knowledge does not admit of its development. Thus Copernicus found the germ of his solar theory in an ancient writer. The hypotheses then that are destined to be fruitful are in close relation to the knowledge of the time, and it is obvious that a well-equipped mind is most likely to conceive them.

This is evident, too, when the origin of the hypothesis has a largely accidental character. For example, the laws of the internal structure of crystals were suggested to Haüy by his observing that in a crystal accidentally broken the fracture showed regular geometrical faces. Similarly,

[1] Cf. Naville, *La Logique de l'Hypothèse*, p. 42.

But, in so far as our instances differ, these differences call for explanation, and when this explanation shows that they are not essential, but merely accidental, in relation to the purpose in view, the argument from analogy is confirmed; though it must be borne in mind, that this confirmation can never amount to demonstration while the inference remains analogical. Thus, in the example just quoted, the striking differences of the different varieties were shown on investigation not to be essential; that is, they were proved to be due to the course of evolution and not to the original common nature which was the subject of investigation.

(iii) **Fallacies incident to Analogy.**—It is evident that the use of analogy is peculiarly liable to lead to fallacious inference. If the analogy is a false one, the hypothesis suggested by it will be wrong and the enquiry will be started on a false scent. This is continually happening in scientific investigations; further examination of phenomena constantly leads to modification or rejection of the hypothesis. In the former case the analogy on which the hypothesis was based was partly, and in the latter case wholly, erroneous.

When an analogy leads to a hypothesis which is afterwards found to be only partially true, and, therefore, to need *modification*, it is because the force of the analogy has been wrongly estimated. The analogy is really present, and really suggests a hypothesis of this general character. But the points of identity have been allowed too much weight relatively to the points of difference with which they are bound up, and, as a consequence, the hypothesis is found to break down in a detailed application. Here, it will be seen, we have really a fallacy of non-observation of conditions as the starting-point of our false analogy.

One special case of false estimation of the force of an analogy is when the exact point of the analogy is missed. In this case, points of identity which are important relatively to one purpose are made the basis of an analogy directed to another purpose. The importance of points of identity we have already seen to be relative to purpose.

tions from known laws has led to many new discoveries. But this source of suggestion of hypotheses is obviously open only to those already versed in the science to which the facts belong.

It must not be supposed that the right hypothesis is necessarily suggested at once even to the man of knowledge and insight. As a rule there is rather a plurality of hypotheses which must be successively tested till all but one are, by disagreement with the facts, rejected. These alternative hypotheses may be simultaneously or successively suggested, but only one is taken up at a time. Thus Kepler records that he advanced nineteen hypotheses which he afterwards disproved, before he arrived at the true statement of the laws of planetary motion. He was handicapped by the backward state of algebra in his day. Had that study been more advanced than it actually was, it is probable that he would much sooner have hit upon his third law—that the squares of the periodic times of the several planets are proportional to the cubes of their mean distances from the sun. For, as Whewell says, "The process of connecting two classes of quantities by comparing their *powers* is obvious only to those who are already familiar with general algebraic views."[1]

The process of selecting out of the several suggestions that occur to the mind the one which finally does lead to a full explanation of the facts may go on rapidly or slowly. Sometimes a false hypothesis is dwelt on and its consequences worked out in detail before its inadmissibility becomes apparent, at others it is rejected almost as soon as formed. Wrong hypotheses generally precede right ones; and that frequently in the same mind. "To try wrong guesses is, with most persons, the only way to hit upon right ones."[2] It is not, however, pure unguided guesswork. In most cases the attempts of previous enquirers have shown more or less plainly in what direction explanation must be sought; either by partial establishment of some hypothesis, or by making manifest the inadmissibility of others.

[1] *Hist. of Ind. Sciences*, vol. i., p. 323.
[2] Whewell, *Nov. Org. Ren.*, p. 79.

3. **Testing of Hypotheses.**—Every hypothesis, then, must be held subject to revision. The truly scientific thinker has none of the spirit which says "If the facts do not agree with the theory, so much the worse for the facts." It was against this spirit—this assumption of hypotheses on little or no evidence, and obstinate adherence to them without much direct reference to the world of fact—that Newton protested when he said *hypotheses non fingo.*[1] On the contrary, to the true thinker, as Brown says, hypotheses "are of use . . . not as superseding investigation, but as directing investigation to certain objects,—not as telling us what we are to believe, but as pointing out to us what we are to endeavour to ascertain."[2]

As striking examples of the true scientific spirit, Whewell quotes Kepler and Newton. The former not only most earnestly and carefully worked out the results of his various hypotheses as to the orbit of Mars, but "never allowed the labour he had spent upon any conjecture to produce any reluctance in abandoning the hypothesis, as soon as he had evidence of its inaccuracy."[3]

A similar spirit was shown by Newton in respect to his hypothesis that the moon is retained in her orbit by the force of gravity. From this hypothesis, he calculated that the moon ought to be deflected from the tangent of its orbit something more than fifteen feet every minute. But the apparent deflection was only thirteen feet. This discrepancy, comparatively small though it was, Newton accepted as a disproof of his hypothesis, and "laid aside at that time any further thoughts of this matter," critical though the supposition was to the theory of cosmical gravitation. But some fifteen years later, the distance of the moon from the earth had been more exactly ascertained; and Newton repeated his calculations, working with these new values. The agreement between the calculated and the normal actual deflection was then seen to be remarkably precise, and the hypothesis became an established theory.

[1] *I do not invent hypotheses.*
[2] *Philosophy of the Human Mind,* vol. i., pp. 230-231.
[3] *Op. cit.*

4. **Descriptive and Working Hypotheses.**—All hypotheses, then, must be held subject to revision, modification, or even rejection. But it does not follow that a hypothesis which is finally disproved has been of no service. Every well imagined hypothesis, suggested by a real knowledge of the facts dealt with, opens out a line of enquiry which is rarely fruitless. As an example we may take the Ptolemaic hypothesis of cycles and epicycles to account for the motions of the sun, moon, and planets. This hypothesis enabled the ancient astronomers to calculate those motions with considerable accuracy, and was, therefore, "of immense value to the progress of astronomical science; for it enabled men to express and reason upon many important truths which they discovered respecting the motion of the stars up to the time of Kepler."[1] Yet the hypothesis was essentially false as an explanation; for it assumed the earth to be the centre round which all heavenly bodies revolve, and the motions of all such bodies to be circular.

A hypothesis may be of a purely descriptive character. When the relations embodied in phenomena are unknown it is often possible to describe the facts through their resemblance to the characteristics of something with whose properties we are familiar. The description is recognised as rather symbolic than explanatory. Such a hypothesis was that which spoke of the "electric fluid," when the term 'fluid' was used as merely a convenient way of describing some of the known phenomena of electricity, but with a full consciousness that electricity is not a fluid in the correct meaning of that word. So, too, the atom in chemistry was for long regarded by many more as a figure of what was real than real in itself. For "it does not appear that philosophers considered the existence and usefulness of chemical formulae as a proof of the physical existence of atoms, or of smallest indivisible particles of matter, in the older sense of the theory."[2]

When a hypothesis is provisionally assumed as a guide

[1] Whewell, *op. cit.*, p. 84.

[2] Merz, *History of European Thought in the Nineteenth Century*, i., pp. 419-420.

to enquiry, it is termed a *working hypothesis*. It furnishes an expression of the facts which is recognised as only partially adequate, and yet seems indispensable for further advance. As Jevons says: "When [Professor Huxley] advocates the use of 'working hypotheses' he means no doubt that any hypothesis is better than none, and that we cannot avoid being guided in our observations by some hypothesis or other."[1] Thus the corpuscular theory of light and the Ptolemaic theory of astronomy were both essentially working hypotheses although at one time each was mistaken for a real explanation.

Of such hypotheses more than one may for a time appear to be equally true: the facts in question may be expressed in more than one way. Many of the observed facts of the motions of the heavenly bodies could, for example, be as accurately formulated on the Cartesian hypothesis of vortices as on the Newtonian theory of gravitation. But, as other facts were accurately observed and measured, the hypothesis of vortices required a continual series of modifications each of which made it more complex than before, while the Newtonian theory was found to explain these new facts as well as those for whose interpretation it had been originally suggested. Thus, the necessity of relating the original facts to other series of phenomena furnishes a test of the truth of a working hypothesis, and as it is only in such a relation that explanation can be found, so only a working hypothesis which ultimately admits of explanation can be true.

However, a working hypothesis discarded as untrue may still be retained as a ready means of expressing many of the facts. "In optics, the so-called corpuscular theory of light is still used with advantage as a convenient means of summarising the laws of reflection and refraction."[2]

5. Conditions of Validity of Hypotheses.

(*a*) *Statement of Conditions.* Every hypothesis is an attempt to find meaning in observed phenomena, to constitute reality in a rational way. It follows that the fundamental

[1] *Principles of Science*, p. 509. [2] Merz, *op. cit.*, p. 422.

condition of a valid hypothesis is that it should explain and give meaning to the facts of observation. And it can only do this if it embraces those facts in that systematic whole which is the one form under which it is possible to think the universe. This general condition, then, may be considered as involving three subordinate conditions—

(1) That the hypothesis be self-consistent, and in harmony with laws included in the rest of the conceived system of reality.
(2) That it furnish a basis for rigorous deductive inference of consequences.
(3) That these inferred consequences be in agreement with fact.

Of these conditions, the first two are applicable to the formation of every hypothesis, no matter how provisional a character it may have; the third is a condition of the acceptance of a hypothesis as true.

(*b*) *Examination of Conditions.* **First Condition.**—The requirement that a hypothesis do not involve consequences which would be inconsistent with itself is obviously necessary. That it should not be in conflict with the rest of reality as known must be interpreted with caution. There is always a presumption that any hypothesis which contradicts a law which has been found to hold good over a wide range of fact is scarcely worth investigation. We should be rightly disinclined to entertain a suggestion which implied the complete reversal of the law of gravitation. Yet theories have been received as true, which later thought has shown to be false. In this way the Copernican theory of the heavens was substituted for the Ptolemaic. When the hypothesis is first formulated there is an initial inconsistency which constitutes a definite claim to modify or to reject in the interests of truth what has been previously accepted as an explanation of the facts. In no case is the new hypothesis held concurrently with theories inconsistent with itself; on the contrary, the previously accepted theories are modified and brought into harmony with the new view necessitated by the fuller study of reality. As

science advances, the probability that necessity will arise for any such thorough-going revision of our concepts as was involved in the acceptance of the Copernican theory or the doctrine of evolution becomes continually smaller.

Second Condition.—It is impossible to infer any consequences from the absolutely unknown. As Jevons says: "We can only infer what would happen under supposed conditions by applying the knowledge of nature we possess to those conditions."[1] Hence, our hypothesis must be always in accordance with some analogy, or based on some experience; otherwise we can draw no conclusions from it. To quote Jevons again: "When we attempt to explain the passage of light and heat radiations through space unoccupied by matter, we imagine the existence of the so-called *ether*. But if this ether were wholly different from anything else known to us, we should in vain try to reason about it. We must apply to it at least the laws of motion, that is, we must so far liken it to matter. And as, when applying those laws to the elastic medium air, we are able to infer the phenomena of sound, so by arguing in a similar manner concerning ether we are able to infer the existence of light phenomena corresponding to what do occur. All that we do is to take an elastic substance, increase its elasticity immensely, and denude it of gravity and some other properties of matter, but we must retain sufficient likeness to matter to allow of deductive calculations."[2]

Hypotheses, then, should not be mere arbitrary fictions. Indeed, if they turn out to be true, they will always be hypotheses with a *vera causa*.[3] For a true cause is one whose assumption is necessary to reconcile observed data. It need not be already known to exist: it may be conceived for the first time when suggested as a possible explanation. Nor need it be open to direct perception: that would rule out such fruitful theories as the atomic constitution of matter and the assumption of a luminiferous ether. But it must be in real and necessary relation to the facts we are considering. If a hypothesis deals

[1] *Op. cit.*, pp. 511-512. [2] *Ibid.* [3] I.e. *true cause*.

with a true cause fulfilling that requirement then it will necessarily offer a basis of deductive inference to the facts dependent on the true cause to which it refers.

Third Condition.—As the very aim of every hypothesis is to express the relations which exist in reality, it is obviously essential that the hypothesis should be verified by comparison of the results deduced from it with facts of observation. In order that this may be done it is essential that the consequences of the hypothesis should be inferred with the utmost precision, and that the comparison of these consequences with the facts should be made with great care and accuracy. The hypothesis must, of course, in the first place agree with the phenomena it was invented to explain. But we must not rest satisfied with this. It must be compared with facts of the greatest possible variety, and exemplifying every possible case which can be brought under it. A single absolute disagreement with facts is fatal to a hypothesis.

Nevertheless, a hypothesis is not to be hastily abandoned on the first *prima facie* conflict with reality. It must be made clear first that the opposing facts have been rightly grasped—that they really are in conflict with the hypothesis. If this turns out to be the case, enquiry should be made as to whether these facts have not been partly determined by the existence of interfering conditions, in the absence of which they would be found in agreement with the hypothesis. And, finally, if, after making all such allowances, the facts are still in conflict with the hypothesis, it must be considered whether a modification of the latter will meet the case, or whether it must be absolutely rejected.

6. **Extension of Hypotheses.**—If the hypothesis is true, it will generally be possible to infer deductively from it facts which have not been before explained or which have even been unobserved. Still, Comte's saying that "prevision is the test of true theory" cannot be accepted absolutely as a hypothesis may be true even though it does not enable the discovery of new facts to be predicted, and, on the other hand, predictions may sometimes be made

successfully from a false hypothesis. Yet, it is, doubtless, one of the tests of the validity of a hypothesis.

The history of science is full of such extension and prediction. Thus, for example, the discovery of Neptune was predicted by deductive reasoning from the principle of gravitation. Other instances of deductions from the same theory are thus summarised by Whewell: "The attraction of the sun accounted for the motions of the planets; the attraction of the planets was the cause of the motion of the satellites. But this being assumed, the perturbations, and the motions of the nodes and aphelia, only made it requisite to extend the attraction of the sun to the satellites, and that of the planets to each other;—the tides, the spheroidal form of the earth, the precession, still required nothing more than that the moon and sun should attract the parts of the earth, and these should attract each other;—so that all the suppositions resolved themselves into the single one, of the universal gravitation of all matter."[1]

Another example is Whewell's own prediction that as the tides of the German Ocean consist of interfering tidal waves, one coming round the North of Scotland and the other through the English Channel, there would be a point about midway between Lowestoft and Brill on the coast of Holland, where no tide would be found, the two interfering waves exactly neutralising each other. The accuracy of this prediction was established during a survey of that sea.

The next instance we will take is a striking one as it shows how it is permissible to maintain a clearly conceived hypothesis, even though results may be deduced from it which are at variance with present experience, if it can be shown that it explains these exceptions. It is thus recorded by Herschel: "When Dr. Hutton expounded his theory of the consolidation of rocks by the application of heat, at a great depth below the bed of the ocean, and especially that of marble by actual fusion; it was objected that, whatever might be the case with others, with calcareous or marble rocks, at least, it was impossible to grant

[1] *Op. cit.*, p. 92.

such a cause of consolidation, since heat decomposes their substance and converts it into quicklime, by driving off the carbonic acid, and leaving a substance perfectly infusible, and incapable even of agglutination by heat. To this he replied, that the pressure under which the heat was applied would prevent the escape of the carbonic acid; and that being retained, it might be expected to give that fusibility to the compound which the simple quicklime wanted. The next generation saw this anticipation converted into an observed fact, and verified by the direct experiments of Sir James Hall, who actually succeeded in melting marble, by retaining its carbonic acid under violent pressure."[1]

As a last example we will take one from the theory of Electricity, quoted by Jevons. "As soon as Wheatstone had proved experimentally that the conduction of electricity occupies time, Faraday remarked in 1838, with wonderful sagacity, that if the conducting wires were connected with the coatings of a large Leyden jar, the rapidity of conduction would be lessened. This prediction remained unverified for sixteen years, until the submarine cable was laid beneath the Channel. A considerable retardation of the electric spark was then detected, and Faraday at once pointed out that the wire surrounded by water resembles a Leyden jar on a large scale, so that each message sent through the cable verified his remark of 1838."[2]

7. **Crucial Instances.**—It happens sometimes that two or even more different hypotheses will explain a great number of the phenomena in question. In such a case, it is necessary to look for some instance which can be explained on only one of these rival hypotheses. As Ueberweg puts it: "One single circumstance, which admits of one explanation *only*, is more decisive than a hundred others which agree in all points with one's own hypothesis, but are equally well explained on an opposite hypothesis."[3] Such a test case is called a *Crucial Instance*, and if arrived at by experiment, the experiment is named *Experimentum Crucis*. The essence of such an instance is that it should

[1] *Natural Philosophy*, § 299. [2] *Principles of Science*, p. 543.
[3] *Logic*, Eng. Trans., p. 513.

absolutely negative one hypothesis, and at the same time confirm another.

The most remarkable instance in modern science in which many phenomena were equally well explainable on either of two incompatible hypotheses, was, perhaps, in the case of the long rivalry between the corpuscular and the undulatory hypotheses of light. "It is remarkable," says Jevons, "in how plausible a manner both these theories agreed with the ordinary laws of geometrical optics, relating to reflection and refraction."[1] But "if the undulatory theory be true, light must move more slowly in a dense refracting medium than in a rarer one; but the Newtonian theory assumed that the attraction of the dense medium caused the particles of light to move more rapidly than in the rare medium. On this point, then, there was complete discrepancy between the theories, and observation was required to show which theory was to be preferred. Now by simply cutting a uniform plate of glass into two pieces, and slightly inclining one piece so as to increase the length of the path of a ray passing through it, experimenters were able to show that light does move more slowly in glass than in air."[2]

Another *experimentum crucis* in connexion with the same two hypotheses is related by Herschel. He says: "When two very clean glasses are laid one on the other, if they be not perfectly flat, but one or both in an almost imperceptible degree convex or prominent, beautiful and vivid colours will be seen between them; and if these be viewed through a red glass, their appearance will be that of alternate dark and bright stripes. These stripes are formed *between* the two surfaces in apparent contact, as anyone may satisfy himself by using, instead of a flat *plate* of glass for the upper one, a triangular-shaped piece, called a prism, like a three-cornered stick, and looking through the inclined side of it next the eye, by which arrangement the reflexion of light from the upper surface is prevented from intermixing with that from the surfaces in contact. Now the coloured stripes thus produced are explicable on both

[1] *Op. cit.*, p. 520. [2] *Ibid.*, p. 521.

theories . . . but there is a difference in one circumstance according as one or the other theory is employed to explain them. In the case of the Huyghenian [the undulatory] doctrine, the intervals between the bright stripes ought to appear *absolutely black*; in the other, *half bright*, when so viewed through a prism. The curious case of difference was tried as soon as the opposing consequences of the two theories were noted by M. Fresnel, and the result is stated by him to be decisive in favour of that theory which makes light to consist in the vibrations of an elastic medium."[1]

But it is not only between great rival theories that *experimenta crucis* may be called to decide. Whenever two explanations of a phenomenon are possible, such an experiment is required. For instance, in investigating the nature of the molecular structure of a gas: If the gas is compressed are the molecules themselves compressed, or are they only brought nearer to each other? Now, if the former is the case, the rate of vibration of the molecules will be altered. But a gas intercepts the colours whose rates of vibration correspond with those of its molecules. If, therefore, the rates of vibration are changed by compression of the gas, the gas will no longer intercept the same colours as before; but if the molecules are simply brought nearer together, the same colour waves will be intercepted, but to a greater extent. The latter being shown to be the case by experiment, it is proved that compression of the gas does not affect the size of the individual molecules of which it is composed, but simply brings them closer together.

[1] *Natural Philosophy*, § 218.

CHAPTER XXX.

THE INCEPTION OF HYPOTHESES.

1. **The Beginnings of Induction.**—Hypotheses are suggested to the mind by observed experience, but they occur only to that mind which can use what it already knows to throw light on the facts. The aim of every hypothesis is to bind together into an intelligible whole a vast mass of experiences otherwise disconnected. Thus at the basis of each is a real or imagined likeness between things or events.

The likeness which first strikes the mind is that of external observable resemblance. The mind tends to generalise uncontradicted experience into universal law. Logically this is the first suggestion of a hypothesis that such resemblance is really universal—a hypothesis which deserves consideration but which cannot be taken as proved. The attempt to show that it is true must proceed through examination of the nature and relations of the facts involved. So is reached likeness of a more fundamental kind—one not always open to direct observation. Our thought has passed from obvious resemblance which can be enumerated to that deeper likeness which is strictly called analogy. But the fact that a suggested explanation has its analogue in some other relation known to exist does not prove its truth. We are still, then, in the realm of hypothesis; the establishment of the supposition as true has yet many stages to pass through.

These two initial stages, however, are of sufficient importance to deserve our serious consideration of their logical value and function.

2. **Enumerative Induction.**—Generally it may be said that every observed regularity of connexion between phenomena suggests a question as to whether it is universal. To take a simple example from mathematics—it is easily seen by simple inspection that $1 + 3 = 2^2$, $1 + 3 + 5 = 3^2$, and so on. This suggests the hypothesis that in every case the sum of the first n odd numbers will be equal to n^2. Such a hypothesis may be tested by additional examples, and as it is found to hold in a continually increasing number of cases, the probability of its holding universally is strengthened. But it can never be more than an empirical law—that is, a description of what relation actually does hold—until its necessity is established by a consideration of the essential properties of numbers, and this takes us beyond mere enumeration.

Such inductive inference does not start from wholly unorganised experience. Ordinary speech carries with it an enormous amount of preliminary organisation. In learning general terms we are learning to class things, actions, and qualities together, so that their points of resemblance are, as it were, forced on our notice. Were it not for the general name 'dog,' for instance, would the thinking together, as members of one class, animals which show so many differences of size, form, and colour, be as general as it is? Evidently it would not be so readily acquired.

Here we meet the first difficulty of induction. The things marked out by common speech can be thought in various relations of likeness to other things, and therefore classed in various ways. Wheat, for example, may be thought of as food, as a grass, as a plant, as an import into this country or an export from that, as a product of the temperate regions, and so on. So according to the purpose of knowledge at the time we must determine under which of these sets of relations the particulars we are examining shall be thought. This determination is itself a preliminary hypothesis—that the ground of the relation we seek to establish will be found within that set of relations. Here error is very possible. We should, for example, be unjustified in inferring the excellence of

all pictures from the recognised excellence of a few we have examined, even had none but admirable examples come within our purview. But if we take as a narrower class within the genus 'picture,' the pictures painted by one great artist we have a more probable ground for the expectation that new examples of that class will possess not only excellence but excellence of the kind exemplified in the pictures we have examined.

Yet the adoption of too wide a class as our first working hypothesis may be prolific of result. Suppose, for example, that electricity has been observed to move freely in silver and copper. There are two obvious concepts under which 'silver' and 'copper' fall, viz. *metals* and *substances*. The direction of enquiry will be different according as one or the other of the two is adopted. If we take the first, our attempted generalisation, 'In all metals electricity moves freely,' will as a matter of fact be correct: if we take the second and assume that 'Electricity moves freely in all substances,' the assumption will be false, but the enquiry based on it will be more fruitful, for it will probably lead to the true concepts under which to think the phenomena —*conductors* in which electricity moves freely, *insulators* in which it does not. In this case the concepts that we really need are not fully defined until the end of the investigation, and as a rule the concepts available, because already named, merely serve to give a starting-point for the enquiry.

Now when from observed resemblances we tentatively proceed to a universal relation we have a syllogism in the third figure, which regarded as an attempt at demonstrative inference is invalid. We have observed that certain individuals a, b, c, d have the quality P, and we have classed a, b, c, d under the class-name S. So we get the premises

$$a, b, c, d, \textit{ are } P,$$
$$a, b, c, d, \textit{ are } S.$$

As S is undistributed we have no formal ground for asserting that *every* S is P; but it is suggested as a possible truth.

3. Analogy.

(i) **Nature.**—How shall we set out to verify it? Two ways lie open before us. The most obvious is to complete the enumeration of the contents of the class S. This we have already seen to be incapable of yielding any assurance that P is so connected with the nature of S that it is necessarily present with it. To reach this assurance is possible only through an analysis of that nature. By itself enumeration can never get beyond a probability; which, while it may be high enough for practical needs, has no theoretical certainty. If we had nothing but uncontradicted experience to go upon we could not establish as a universal truth that all men die, though the expectation would be safe enough to determine the action of Assurance Companies. And in most cases we have nothing like the amount of uncontradicted experience which is here available.

We must, then, take the other line and proceed to the analysis of the nature of the phenomena in question. This is the work of all modern science, and the one task of inductive logic is to analyse its methods. Its first step is *Analogy*.

Why do we class a, b, c, d under S when we are considering their relation to P? The only possible answer is that we see in the nature of S some relation to P which we believe to be pertinent to the matter in hand. If we denote this by X, we have the premises

$$P \text{ is } X,$$
$$S \text{ is } X,$$

from which there still follows nothing but a probability that S is *always* P, though this probability is higher than it was before, because it is now based on a first examination of the nature of the phenomena—the X which forms the middle term is a supposed causal relation.

Let us suppose, as an example, that certain schools—a, b, c, d—are shown by statistics to be abnormally successful at a certain examination. This success we will denote by P. When we try to account for this we seek to find something in common between a, b, c, d which will

explain P. In other words, we search for an S under which to class a, b, c, d. Now the first S suggested is schools—and that is obviously insufficient for our purpose. We may then seek among such specific divisions between schools as are given by local position, character of buildings, social class of pupils, number of pupils. If we find agreement in any one of these points between all the cases of abnormally successful schools we have a ground for trying to pass on to the analogical argument. For example, if a, b, c, d seem to agree in the social class of their pupils we should ask whether that involved any conditions especially favourable to study, as, *e.g.* opportunities in the way of books, of cultured help from parents, of a quiet room for work. These would form the X of our analogical argument, and we should infer a probable connexion between facilities for study and success in examinations.

But if a, b, c, d do not agree in any such points we must seek further for our S, and we shall naturally look at the teachers employed and see if we can find there anything differentiating a, b, c, d from those other schools which are less successful in the examination. If we cannot, we obviously have not a sufficient S; if we can we have an S in which we may find the bond of relation to P in superiority of skill, of industry, or of numbers.

Analogy then takes us an important step beyond mere enumeration. It is our first tentative effort towards explanation, and it rests on resemblance. Now as resemblances are of all degrees of importance to the relation we wish to establish it is evident that the probability of our conclusion may be of any degree of strength. If the X is superficial the argument carries little or no conviction. The inference that any other planet is inhabited by living beings like ourselves has no force if it is based only on the fact that they also are members of the solar system. To the extent to which conditions essential to human life can be shown to exist, say on Mars, to that extent the argument to the probability of the existence of 'Martians' is strengthened.

Arguments from analogy may thus be extremely super-

ficial—of rhetorical rather than of logical force. On the other hand, they may be of considerable cogency. Everything depends on the nature of the resemblance. If it is really the ground of the *P* we are inferring—then we have only to make sure of this to pass from analogy to proof. We are seeking that bond of identity between our *S* and our *P*. So long as it is only probable, we are on the level of analogy: the more we can strengthen that probability, the stronger becomes our inference. So that analogy points beyond itself to further inductive methods.

(ii) **Force.**—Before passing on to examine them we must briefly consider the question: What is the criterion of importance by which to judge the characteristics on which the inference is based? Now 'important' and 'essential' are relative terms, and point beyond themselves to something in relation to which the 'importance' and 'essentiality' is grounded. A characteristic is important for one purpose, but unimportant for another; and if its importance is so great that without it the former purpose cannot be achieved, we say it is essential. Essential, then, means 'essential for the end in view.' Thus, the criterion of the value of an analogical inference must be found in the conception of purpose or end.

This is easily seen when the cases with which an inference is concerned are purposive works of man. For example, by analogy we conclude that certain flints found in the earth are remains of weapons, because they bear marks of artificial shaping of such a kind as to adapt them to be cutting or piercing instruments, and corresponding, moreover, to those of flint weapons made and used by savages at the present day. Similarly we should infer by analogy that a pair of spiked shoes were intended for use at cricket, because the affixing spikes to cricket shoes has an obvious purpose. In organic life the end or purpose must be regarded as the perfection of nature reached through a long course of evolution. But inference here is often backwards, and then the nature is sought in the past. Thus, the biologist when his purpose is to attain knowledge of the common origin of divergent species of organisms seeks, as the ground of an inference from

analogy, those marks which the course of evolution is most likely to have left largely unmodified; and they are not generally those which are most striking as points of resemblance.

From what has just been said it is evident that the force of an argument from analogy depends upon the character of the identity, and not upon the apparent amount of similarity. As Mr. Sidgwick says: "Whenever 'degree' or 'amount' of resemblance or difference is spoken of, the student must remember that, for all purposes of reasoning, a resemblance or difference is great or small, not according either to its power of striking the observer's notice, or to the *number* of 'points' (or details) into which it may be analysed; but according to the importance of of its details in regard to the matter in hand."[1]

Nevertheless this mistake has often been made, even by writers on logic. Thus Mill says: "Since the value of an analogical argument inferring one resemblance from other resemblances without any antecedent evidence of a connexion between them, depends on the extent of ascertained resemblance, compared first with the amount of ascertained difference, and next with the extent of the unexplored region ofunascertained properties; it follows that where the resemblance is very great, the ascertained difference very small, and our knowledge of the subject-matter tolerably extensive, the argument from analogy may approach in strength very near to a valid induction. If, after much observation of *B*, we find that it agrees with *A* in nine out of ten of its known properties, we may conclude with a probability of nine to one, that it will possess any given derivative property of *A*."[2]

But we do not number but weigh our instances in analogy. And if we wished to number them, how could we do so? Who shall decide as to whether a given point of identity or of difference is one 'property' or a dozen? It is all a matter of arbitrary mental analysis. 'Properties' are not isolated and separate individualities, which we can

[1] *Process of Argument*, p. 194, *note*.
[2] *Logic*, III. xx. § 3.

count and enumerate as we can balls or books. It is, therefore, meaningless to speak of a 'ratio' in this connexion; for a ratio requires a unit, and a unit is here unattainable. Moreover, it is impossible to estimate the number of 'unknown' properties; such estimation assumes them known, and therefore to speak of "the extent of the unexplored region of unascertained properties" is to fall into a contradiction in terms.

Nor must it be assumed that every point of difference furnishes a probability against the connexion of *S* and *P*. If the points of resemblance are essential, the points of difference may be disregarded; and similarly, if the points of difference are essential, no amount of resemblance in other points will make the inference a safe one. The utter hopelessness of inferring from the relative "extent of ascertained resemblance" and "extent of ascertained difference" is strikingly shown in the following passage from Dr. Wallace's *Darwinism*, in which he is speaking of the variations of plants, and in which it is seen that an analogical argument from amount of resemblance not only may lead an enquirer wrong when he is comparatively unacquainted with the subject-matter, but actually did mislead a great number of specialists:—

"All the cucumbers and gourds vary immensely, but the melon (Cucumis melo) exceeds them all. A French botanist, M. Naudin, devoted six years to their study. He found that previous botanists had described thirty distinct species, as they thought, which were really only varieties of melons. They differ chiefly in their fruits, but also very much in foliage and mode of growth. Some melons are only as large as small plums, others weigh as much as sixty-six pounds. One variety has a scarlet fruit. Another is not more than an inch in diameter, but sometimes more than a yard in length, twisting about in all directions like a serpent. Some melons are exactly like cucumbers; and an Algerian variety, when ripe, cracks and falls to pieces, just as occurs in a wild gourd."[1]

Differences, then, cannot override essential identity.

[1] Pp. 87-88.

But, in so far as our instances differ, these differences call for explanation, and when this explanation shows that they are not essential, but merely accidental, in relation to the purpose in view, the argument from analogy is confirmed; though it must be borne in mind, that this confirmation can never amount to demonstration while the inference remains analogical. Thus, in the example just quoted, the striking differences of the different varieties were shown on investigation not to be essential; that is, they were proved to be due to the course of evolution and not to the original common nature which was the subject of investigation.

(iii) **Fallacies incident to Analogy.**—It is evident that the use of analogy is peculiarly liable to lead to fallacious inference. If the analogy is a false one, the hypothesis suggested by it will be wrong and the enquiry will be started on a false scent. This is continually happening in scientific investigations; further examination of phenomena constantly leads to modification or rejection of the hypothesis. In the former case the analogy on which the hypothesis was based was partly, and in the latter case wholly, erroneous.

When an analogy leads to a hypothesis which is afterwards found to be only partially true, and, therefore, to need modification, it is because the force of the analogy has been wrongly estimated. The analogy is really present, and really suggests a hypothesis of this general character. But the points of identity have been allowed too much weight relatively to the points of difference with which they are bound up, and, as a consequence, the hypothesis is found to break down in a detailed application. Here, it will be seen, we have really a fallacy of non-observation of conditions as the starting-point of our false analogy.

One special case of false estimation of the force of an analogy is when the exact point of the analogy is missed. In this case, points of identity which are important relatively to one purpose are made the basis of an analogy directed to another purpose. The importance of points of identity we have already seen to be relative to purpose.

To apply to one end an analogy which holds for another end may, therefore, yield a hypothesis which may be of any degree of inaccuracy—partial or total—according to the remoteness of the two purposes from each other. The argument, humorously put into the mouth of Socrates by Plato in the *Republic*, "that if justice consists in keeping property safe, the just man must be a kind of thief; for the same kind of skill which enables a man to defend property, will also enable him to steal it"[1] is an instance of this species of false analogy. The identity in skill is not important in reference to the question under consideration. For "justice is not a kind of skill, but a kind of activity. The just man is not merely one who *can*, but one who *does*, keep property safe. Now, though the *capacity* of preserving property may be identical with the *capacity* of appropriating it, the *act* of preserving is certainly very different from the act of appropriating."[2]

To mistake the exact bearing of an analogy may, then, lead to as much error as to assume the existence of an analogy where none exists; in fact, utter mistake of the bearing of an analogy will probably give rise to the application of it to cases on which it has no real bearing.

The use of metaphorical language is a frequent source of such errors; in those cases the false analogy may be said to be based on, and to grow out of, a fallacy of Figure of Speech. For instance, it is false analogy due to metaphorical language to condemn the metropolis because it is called the 'heart' or the 'head' of the body. As abnormal increase of these members points to disease in the natural body it has been urged that analogous diseases will follow in the body politic if the capital of the country becomes very large. Thus Smollett in *Humphry Clinker* says of London: "The capital is become an overgrown monster; which, like a dropsical head, will in time leave the body and extremities without nourishment and support. . . . What wonder that our villages are depopulated, and our farms in want of day-labourers?" Now, the abnormal influx of labour into the great towns from the country is,

[1] i. 334. [2] Mackenzie, *Manual of Ethics*, 3rd Edn., p. 15.

undoubtedly, a social evil of considerable magnitude. But the growth of the towns is the effect of this influx, not its cause, and the consequent evils are not essentially analogous to "leaving the body and extremities without nourishment and support."

Arguments of this character were common among the Greeks, who attached undue importance to words and the analogies they suggest. That the governor of a state is like a pilot, for instance, is true in some points of not first-rate importance; but that his functions are unlike those of a pilot in many other points which are of great importance is also true.

A very common false analogy is that between a community and an individual. As the latter goes through successive periods of growth, maturity, and decay, so the hypothesis is formed that every nation must do the same, and it is assumed that after a longer or shorter period of vigour and prosperity the nation must gradually lose its place in the world. The disastrous consequences of such a theory upon the energy of the nation in general and its leaders in particular are obvious and can scarcely be exaggerated. And history lends some colour to the theory in recounting the rise and fall of great empires.

Examination of the circumstances tends, however, to show that such decay is not a necessary consequence of the continued life of the state; it occurs at the end of periods more or less protracted, and may in every instance be explained by conditions independent of the mere duration of the community as an organised state. On the other hand, in the case of the individual organism, senile decay is a direct result of the constitution of the body, which is such that after a certain interval decay exceeds recuperation and at last necessarily ends in death. The fall of great empires is more analogous with death from disease than with death from old age. Hence, there is no reason to regard such decay of power and prosperity as the necessary end of a lengthened existence.

The falsity of the analogy is well shown in the following passage from Burke's *Letters on a Regicide Peace*: "I am not quite of the mind of those speculators who seem

assured that necessarily, and by the constitution of things, all states have the same periods of infancy, manhood, and decrepitude, that are found in the individuals who compose them. Parallels of this sort rather furnish similitudes to illustrate or to adorn, than supply analogies from whence to reason. The objects which are attempted to be forced into an analogy are not found in the same classes of existence. Individuals are physical beings, subject to laws universal and invariable. The immediate cause acting in these laws may be obscure: the general results are subjects of certain calculation. But commonwealths are not physical but moral essences. They are artificial combinations, and, in their proximate efficient cause, the arbitrary productions of the human mind. We are not yet acquainted with the laws which necessarily influence the stability of that kind of work made by that kind of agent."[1]

A favourite rhetorical form of the argument from analogy is to illustrate a great and complex relation by one familiar and simple. That by a ladder one may climb upwards is evident. So the enthusiastic platform 'educationist' has only to speak of "an educational ladder from the gutter to the university" to justify to most of those who hear or read his remarks the lavish expenditure of public money in scholarships. Few ask what is meant by such rising in the world, or whether it is a public duty to help certain individuals so to rise above their fellows. The metaphor with its play on words is taken as a reality with an identity in nature.

Such metaphors are the stock in trade of the popular politician. He assumes that general and abstract reasoning is "caviare to the vulgar," and so he dispenses both himself and them from the necessity of real thought, and, giving vein to his pictorial imagination, finds that he can 'prove' anything, so that when he changes his fundamental opinions he can support the new with equal sound and fury—and with equal want of conclusiveness—as he formerly upheld the old.

Doubtless at times such analogical illustrations are per-

[1] *Works*, vol. viii., pp. 78-79.

tinent and helpful. But always they are open to the dangers that they may be superficial and therefore misleading, or that even if valid they may be pressed further than their nature really justifies. That a wall is an obstacle to communication is a plain matter of fact. So the orator has only to speak of a tariff "wall" to suggest that the imposition of customs duties hinders trade in the same way as a wall hinders free passage, and that the higher they are the more effectively they do so. Neither he nor his hearers think it necessary to ask how far the analogy is a true one.

As an exercise, the reader may ask himself what conclusion he on rational grounds feels justified in drawing from the following argument of Mr. Deakin, at the time prime minister of the Commonwealth of Australia: "I should be very sorry to see any doctrine adopted which suggests that it is intended to wrap the British Empire in a napkin in case it should catch cold. To treat it as if it *possessed so tender a cuticle that it could not be touched* without permanent and fatal irritation is to brand it as a poor organism incapable of coping with the ordinary difficulties in its path or the necessary ailments which come from abuses or mistakes."[1]

Fallacy from analogy, then, always consists in regarding the analogy as of greater probative force than it really has in relation to the case in hand. Obviously it reaches its climax when an analogy by itself is set forth and accepted as really proving anything. The strongest analogy only adds to the probability of a hypothesis, the proof of which must be sought in further enquiry.

[1] *The Great Preference Debate*, pp. 187-188.

CHAPTER XXXI.

ESTABLISHMENT OF HYPOTHESES.

1. **Conditions of Establishment.**—We have seen that neither simple enumeration nor analogy can do more than suggest a tentative hypothesis which can only be transmuted into an established truth by other and more exact processes of investigation. The logical analysis of these is now to occupy us. As a necessary preliminary, however, to our enquiry we must see clearly what is its aim; that is, we must decide what conditions a hypothesis must fulfil to entitle it to be regarded as established and proved.

These requirements were succinctly stated by Clifford: "In order to make sure that your supposition is true, it is necessary to show not merely that that particular supposition will explain the facts, but also that no other one will."[1] To put it symbolically: If our suggested hypothesis is *If* **A**, *then* **X**, it cannot be held to be proved until we also establish *If* **X**, *then* **A**. This we have already seen is the aim of experiment.[2] Enumeration suggests **A** as the cause of **X**; analogy strengthens that suggestion by finding a ground for the relation. But if the hypothesis states a real causal law it is reciprocal. Not only does **A** secure **X**, but **X** can be secured in no other way.

This is always a task of much difficulty, and before the law or theory can be regarded as absolutely established it will have passed through many successive stages of increasing probability. The terms 'Law' and 'Theory' are not clearly differentiated, but the distinction, so far as it exists, is that a theory systematises a number of general relations or laws.

[1] *Lectures and Essays*, p. 137. [2] See pp. 333-336.

2. **Direct Development of Hypothesis.**—When we seek evidence to establish the truth of a suggested causal relation we may or may not find it open to direct observation, with or without the aid of experiment. If it is not we must work indirectly, and compare the facts we can observe, not with the hypothesis itself, but with consequences we can deduce from it. This process we shall consider later. If, however, the hypothesis itself can be directly brought to the test of comparison with fact, then the scientific worker adopts certain precautions in the selection and manipulation of the evidence; and these the logician is able to analyse. The first clear exposition of these practical methods of direct enquiry was given by Herschel[1]; the classical logical analysis was made by Mill.[2] Since then they have been generally accepted by logicians, though with the reservation that they are not nearly so conclusive and rigid as Mill appears to have believed.

As analysed logically the methods of course stand apart as separate and independent modes of enquiry. This separation, however, is wholly the work of the logician, made for simplicity of exposition of their logical character. In actual scientific work they no more exist apart than do the vital, physical, and chemical processes in the life of a plant or animal. Nor can they ever be employed with that formal rigidity which would be necessary to enable them to be by themselves absolute grounds of proof.

All the methods aim at elimination. Causal connexions are never found pure in nature: they are always associated with conditions which are irrelevant to them. To distinguish what is relevant from what is irrelevant, and so to isolate the real causal connexion sought, is the only way in which that connexion can be revealed. This is a process of logical elimination which may or may not be aided by a physical separation. It proceeds on two fundamental principles; first, that whatever can be absent when the phenomenon is present is not causally connected with it,

[1] *Preliminary Discourse on the Study of Natural Philosophy.*
[2] *Logic,* III. ix. 3.

and secondly, that whatever can be present in the absence of the phenomenon is no part of its cause. These principles are direct applications of the doctrine of causation: they assume that every event must have a cause and that the same cause produces the same effect.

The formal principle that if a definite set of conditions give rise to a definite set of phenomena, and if another set of conditions, partly like and partly unlike the former, give rise to another set of phenomena, also both like and unlike the former, then that a causal bond is indicated between the elements which remain stable in both sets, and negated between those that rest unchanged in one set and change in the other, may be symbolically expressed.

AB followed by xy
AC „ „ xz

∴ causal connexion probable between A and x, but not between A and y, A and z, B and x, B and z, C and x, C and y. The possibility of such a connexion between B and y, C and z is untouched.

Such a formal statement obscures the complexity of the real work of thought. It suggests that the letters symbolise elements which are so independent of each other that one can be added or removed without changing the other; so that whatever we do with B or C leaves A free to work, and to work always in the same way and with the same measurable result. But in reality B or C may change the operation of A in any way and to any degree including complete neutralisation; as for example increase of pressure (B) may within limits prevent the expansion of water under heat (A). Nature does not consist of elements thus separable, but of constituents each of which enters into every process in a way determined by the total nature of that process.

Further, the symbolic statement suggests that the preliminary work of connecting conditions and result in thought has already been done. They assume that we know that it is in AB we are to look for the conditions of xy, and in xy for the consequences of AB. Rarely, indeed, is this the case in actual scientific enquiry. That selection

is the first work to be done and is liable to error. To make an exhaustive enumeration of conditions would be impossible. As has just been said, the 'conditions' are not independent things but constituents of one process. We give them a kind of spurious and artificial separate existence according to our own convenience. Largely at first we think thus separately constituents which have received separate names. But this takes us but a little way. Soon it is found that these obvious distinctions due to more or less superficial observation are insufficient, and others have to be made. But always the independent existence of what we symbolise by our letters is solely constituted by our own thought.

From the whole mass of existence we select a little piece for examination, and in so selecting we isolate it in thought from the whole of which it is, and must always remain, really a part. In changing *it* we of necessity change the whole. Whether, then, what we attend to is what we ought to attend to for the purpose in hand is not a matter that can be decided in the easy way the symbolic formula suggests. The knowledge, insight, and tact of the enquirer are needed in their fullest power, and even so mistakes are frequent. It may easily happen that relevant conditions are ignored. Often it is only in the actual course of an enquiry that it is recognised that some condition is really operative. Of course it *existed* at the beginning, but it was not thought worthy of consideration. So it is seldom indeed that a hypothesis emerges from an enquiry in the form in which it entered that enquiry. As the work proceeds the conditions really operative are more and more recognised, and the part they play more and more clearly apprehended. It is only gradually that we reach anything like the simple connexion A followed by x, and when we do A and x have always ceased to be the coarsely apprehended 'things' and 'events' which lie open to perception by the senses. So it is that these methods of enquiry which deal with just such perceptible sequences can never do more than make a causal connexion extremely probable. They can never lay bare its essence.

From this it follows, too, that they speak of 'cause' and

'effect'—especially the latter—in a way intermediate between the rigid scientific conception and the loose use of popular speech. Mill granted that their results could not escape from the possibility that an event might be produced by various alternative 'causes.'

Such a doctrine of "plurality of causes" if applied to the ultimate nature of things makes knowledge impossible; for it reduces to chaos any system which thought attempts to establish. But while we deal only with events and changes open to perception we cannot get to that inner bond of union which is only appreciable by thought. On the level of these direct methods, then, it must be granted that we are very unlikely to establish undeniably the reciprocal relation we seek.

It is well to grasp these limitations of the methods, because the formal certainty suggested by their symbolic expression may otherwise lead us to regard as conclusively proved that which has only a presumption of greater or less strength in its favour. For the ordinary purposes of life such presumption is often enough, but it never satisfies the demand for theoretical certainty.

We will now examine the methods in detail, but it is well to point out that the examples by which we illustrate each of them are bound to wear an air of artificiality, since in no case are they more than fragments torn from their true context in real investigations, and in some cases they have no importance beyond their illustrative value. In the next chapter an endeavour will be made to give specimens of actual scientific work directed to the discovery of truth, and it will then be seen that the methods are used to supplement one another in one and the same enquiry in a way which their separation for the purposes of exposition tends to obscure.

(i) **Method of Agreement.**—When we attempt to make enumeration more precise the obvious course to pursue is to seek the observed sequence in as varying surroundings as possible. The logical generalisation of this is known as the *Method of Agreement*. If only the sequence $A - x$ is found constant while the concomitant circumstances change in many ways the probability that

the relation of A to x is causal is enhanced by its persistency. Repeating the general formula already given:

$$\begin{array}{lll} A\,B & \text{followed by} & x\,y, \\ A\,C & \quad\text{,,} \quad \text{,,} & x\,z: \end{array}$$

the greater the divergence in the mass of concomitant circumstances summed up symbolically in B, C, y, z, the greater the probability that A necessitates x. For example, "A person might suppose that the peculiar colours of mother-of-pearl were due to the chemical qualities of the substance. Much trouble might have been spent in following out that notion by comparing the chemical qualities of various iridescent substances. But Brewster accidentally took an impression from a piece of mother-of-pearl in a cement of resin and bees'-wax, and finding the colours repeated upon the surface of the wax, he proceeded to take other impressions in balsam, fusible metal, lead, gum-arabic, isinglass, etc., and always found the iridescent colours the same. He thus proved that the chemical nature of the substance is a matter of indifference and that the form of the surface is the real condition of such colours."[1]

The method of agreement resembles simple enumeration in its reliance on number of instances, but it differs from it in the stress laid on variety in the accompanying circumstances. As far as possible the examples taken should be typical, and a mere repetition of instances of the same kind is avoided. Speaking of chlorophyll, the green colouring matter in plants, Hobhouse says, "All buttercups are plants, and all buttercups contain chlorophyll; but I might examine a million buttercups, and on the question whether all plants contain chlorophyll, I should remain precisely where I was when I had examined half-a-dozen. But if I examine a buttercup leaf, a blade of grass, a fern, a moss, a volvox, and a protococcus, my six observations will give me a very fair right to generalise. Normally, plants contain chlorophyll."[2]

[1] Jevons, *The Principles of Science*, p. 419.
[2] *Theory of Knowledge*, p. 368.

The strength of the generalisation depends on the agreement in one particular amid very wide divergencies in other respects. To be formally cogent the method demands that there should be agreement in only one particular, and of that we can never be sure. The uncertainty may be lessened by the knowledge that some circumstances which have not been excluded are irrelevant from the point of view of the enquiry which is being undertaken. Moreover, certainty can only be assured if the instances taken are exhaustively representative. This, again, is a demand which in practice cannot be absolutely fulfilled.

As a matter of fact there are exceptions to the rule that plants contain chlorophyll, though probably the majority do. However, if the differences in the instances are very marked, a few examples are sufficient to suggest a connexion. Darwin observes in the *Voyage of the Beagle*: "There is one vegetable production deserving notice from its importance as an article of food to the Fuegians. It is a globular, bright-yellow fungus, which grows in vast numbers on the beech-trees. When young it is elastic and turgid with a smooth surface; but when mature it shrinks, becomes tougher, and has its entire surface deeply pitted or honeycombed. This fungus belongs to a new and curious genus; I found a second species on another species of beech in Chile; and Dr. Hooker informs me, that just lately a third species has been discovered on a third species of beeches in Van Dieman's Land. How singular is this relationship between parasitical fungi and the trees on which they grow in distant parts of the world."[1] The expression of surprise is prompted by the thought that there is probably some reason for the growth of the fungus on one class of tree, but Darwin is far too cautious to state the connexion more than tentatively. All that the method of agreement can do to confirm a suggested relation is by varying the circumstances to make less and less probable the irrelevance of any element which remains. When so much else is changing we can in most

[1] *The Voyage of the Beagle*, Ch. XI.

cases suggest no sufficient reason for the continued presence of the element except that it is causally connected with the phenomenon.

There are other reasons why the method of agreement does no more than strengthen a hypothesis of causal connexion. It is mainly a method of observation, and is most appropriate in those cases where the phenomenon is not amenable to manipulation, but occurs in nature in a variety of forms. Occasionally the variety may be partly produced by experiment. None the less, analysis cannot on this method be pushed very far. We may find, for example, that many bodies agree in the one fact that they are transparent to light. But obviously this is merely a suggestion for enquiry: it does not explain in any way the connexion. And if an examination be conducted into the relation of light-vibration and the molecular structure of these transparent bodies, the indirect method must be resorted to. So the probable connexion between the iridescent colours of mother-of-pearl and the form of the markings in each substance is an indication that the explanation probably depends on some relation of the light to this special form, but no direct method is adequate to determine what that relation is.

Moreover, the method of agreement is peculiarly liable to be vitiated by that "plurality of causes" which no direct method can altogether escape. To revert to our symbolism, $AB...xy$; $AC...xz$; $AD...xt$: $AE...xs$...etc. No doubt with each elimination the probability that x is due to anything present in the instances except A is decreased. But it must be remembered that B, C, D, etc., symbolise the whole mass of possibly relevant concomitants and not merely such isolated elements as we attempt to secure under A. Further, that all the constituents of a phenomenon are so inter-related that it is quite conceivable that if we could really isolate A we should find it inadequate to secure x. Industry may be essential to success in life, but if we could conceive a person who had none of the auxiliary quality of intelligence and no favourable circumstances of life, we should not expect him to be successful. We never get a real isolation of the working of A, and it may well be

that by itself it is powerless; that is, that our isolation of A in thought as the cause of x has been only partially correct. So that while the method of agreement gives a presumption in favour of a bond of causality between A and x it does little to support a hypothesis that in A we have the full and sufficient cause of x.

The method is powerless, too, to discriminate between a causal connexion and a case of co-existence. Thus we may have at the end of our elimination the apparent sequence $A - x$, but x may be a permanent fact in nature which remains as a residuum because of its permanency. The true test would be made if A were introduced: the consequent appearance of x would disprove the suggestion that x is permanent. "Wherever there is sea we find sky: but we do not make sea the cause of sky, because we do not find the sky coming into being when or where the sea appears."[1] The fact that wherever animals are ruminants they are cloven-hoofed does not make ruminancy a cause of cloven feet. Both may be directly connected with some further condition, or the connexion may be so remote, as in the case of sea and sky, as to be considered purely casual. An example already given as an illustration of a non-causal sequence is also in point here. However different the nights we experience may be in other respects they all agree in being preceded by day. Here day and night are joint effects of a cause not open to sense-perception: and theoretically there is always the chance that this may be the case in any conjunction of elements arrived at by the method of agreement.

Enough has been said to show that the method of agreement cannot give certainty. There is no need to exaggerate its defects. It is frequently sufficiently cogent for practical purposes; and sometimes it is the only method of investigation which the nature of the material under consideration allows. Its main function, however, is to suggest hypotheses which require further examination. When Pasteur found in a large number of diseased silkworms placed in turn beneath the microscope in various

[1] Hobhouse, *Theory of Knowledge*, p. 367.

stages of growth that certain corpuscles were always present, the connexion between the corpuscles and the disease was a hypothesis worthy of closer examination: when in many healthy worms no trace of corpuscles could be found the hypothesis was greatly strengthened.

(ii) **Method of Exclusions.**—If, then, mere positive agreement in the presence of x with A is so inadequate as a means of theoretical proof we naturally turn next to seek negative confirmation by examining instances of the absence of A to see if they are all marked also by the absence of x. This gives rise to the *Method of Exclusions* or *Joint Method of Agreement and Difference,* as Mill named it.

Evidently our negative instances should resemble the positive ones as nearly as possible in every particular except the absence of A. They must, therefore, be the same kind of facts. It would not do to take *any* example of the absence of the phenomenon. Indeed, if the cases in which it occurs possess besides the supposed cause A, a number of other elements, say B, C, D, E, F, G, the negative instances, to be at all conclusive, should contain among them all these other circumstances, and thus show that each of them can be present when the phenomenon is absent. This is in practice too stringent a requirement: but the method adds little to the probability unless the resemblances between the two sets of phenomena are so marked that they are recognised as instances of the same type. The only important difference should be the presence in one case, and the absence in the other of the suggested cause and effect.

The force of the argument depends on the degree of probability with which we are assured that all the relevant conditions are under consideration. The method does not require prepared instances: it takes such as are open to observation. We search for examples as nearly alike as possible except for the one important difference of presence and absence of $A - x$, but it is rarely that nature presents instances which do not exhibit other differences as well, or which allow in their totality of a complete elimination of such differences. The only way to minimise this objection

is to make both positive and negative instances as varied and numerous as the given investigation seems to demand. Then the probability that an adequate exclusion has been effected will be considerably increased.

The consideration of negative instances makes the liability to error through the existence of "plurality of causes" practically negligeable. If we take a number of cases of agreement, *e.g.* $A\ B\ C - p\,q\,r$, $A\ D\ E - p\,s\,t$, we have to allow that B, C, D, or E may be the cause of p, but when we find that these elements can be present when the phenomenon does not occur, there is no motive for entertaining them as possible alternative causes unless we have reason to believe that any one of them is counteracted in its tendency to produce the result by an element with a tendency of opposite quality. If we were sure that each relevant circumstance had been included in the negative instances the argument from plurality of causes would have no weight in the absence of such a suggested counteraction.

The method is frequently used in common life and in social investigations. Suppose a man to suffer from sleeplessness and to seek its cause. The range within which the search must be made is wide, and the possible causes somewhat indefinite in character. He will proceed to compare carefully the circumstances preceding his sleepless nights with those preceding nights of satisfactory rest. He may find that among such conditions as late hours, prolonged study, varied drink and diet, worry and so forth, followed by sleeplessness, all may be present in cases of normal slumber except say the nightly cup of strong coffee. The latter then is suggested as the recalcitrant habit which must be broken if insomnia is to be cured.

The application of the method is here, as in most of the enquiries of ordinary life, of a loose description. For example, there are many factors of temperament, constitution, and general habits of life, which are incapable of elimination. An investigation into the effect of half-time employment on physical development avails itself of the same procedure. Height and weight are taken as trustworthy indications of physical condition.

Statistics are compiled in relation to children of the same average age and general home surroundings (1) in half-time attendance, (2) in full attendance at school. In the first cases a physical deterioration is present which in the second series—that is, the negative instances—is absent. Here half-time attendance is a compendious description of a group of elements such as heavy physical strain, dust-laden atmosphere during toil, noise of machinery, excessive heat, whose separate effects in the whole loosely termed physical deterioration can be recognised, if not precisely determined, by a continued application of the same method.

An example from a scientific investigation is worked out in the next chapter. Darwin determined that the formation of vegetable mould was the result of the action of earthworms by an examination of positive and negative instances. Where the vegetable mould was formed earthworms were active: where it was not formed the earthworms were absent from the soil.[1]

The use of the negative instance enables us to set aside definitely all the elements which are present in the absence of the phenomenon. It thus limits the field of possible causes, and if made precise enough, would indicate by a complete exclusion the true cause.

(iii) **Method of Difference.**—The desire to secure two instances differing only in the presence in the one, and the absence in the other, of the suspected cause naturally leads to experiment. Nature does not present us with such instances. But in experiment we can sometimes introduce our A into a set of conditions from which it was absent. We can, for example, put some ground coffee into boiling water, and we are justified in attributing the change in colour, odour, taste, and effect as a beverage, to that infusion. Such an instance brings out an important point, which we have already considered under causation, that it is not the coffee by itself—*i.e.* as a separate element—but only in its union with the boiling water which produces the result.[2] So when into a set of conditions B which have as a result y we introduce A, and find y becomes $x\,y$,

[1] See pp. 416-420. [2] See p. 310.

we can only say that the introduction of A caused the change, not that A by itself would ever give x.

To make one change at a time and to watch the result is the object of the *Method of Difference*, which is essentially the *Method of Experiment.* How complex or how simple is the change made depends upon the minuteness of our mental analysis of the phenomenon in question and upon our power to isolate in reality elements we have already segregated in thought. Obviously the more simple it is the nearer we approach to the certainty that the hypothesis of causal connexion between A and x is justified.

Formally the method may equally well start with $A\ B—x\ y$ and proceed by removing A in the hopes that x will then disappear, or with $B—y$ and proceed to add A in the expectation of securing x. But practically the latter is generally the more convenient—it is easier to add coffee to hot water than to start with the coffee infusion and remove the coffee.

Examples abound in common life and in science. A man attributes a bruised elbow to a fall, a sudden draught to the just opened door, the blotting of a landscape from view to an uprolling mist. When a current of electricity is passed through a galvanometer the resulting deflection of the needle is ascribed to the current; when sodium is added to dilute sulphuric acid it is regarded as causing the release of hydrogen; when a lesion is made in the cerebrum of a frog there is no hesitation in connecting it with any consequent paralysis of movement.

Now experiment involves the control and manipulation of some at least of the elements which are present. Since the number of elements is indefinite, and we wish to add or subtract one only, we must ask on what grounds one is selected in preference to another. Occasionally the elements may be chosen without any definite reason in order to see what happens when they are added or taken away. But such random experiment is suited rather to complete ignorance than to any stage of knowledge. Analogy will probably indicate what variations are promising, if no more definite suggestion is at hand. As a rule, however, the investigator by one or other of the methods is in posses-

sion of a suggested causal relation which he desires to test. There is some definite A which is thought to be the cause of the phenomenon, and it is his business by introducing or withdrawing A to watch for any corresponding change in the facts.

The method, it will be noted, directly investigates the effects of a given cause. It works forwards, from cause to effect. Regarded as a mode of discovering the cause of a given effect, it is an application of the indirect method. We assume the possibility of a given cause as sufficient to account for the phenomenon in question. We then try to introduce that cause into a suitable set of conditions so as to see if it fulfils our expectation by securing the change which we believe it capable of producing.

The method formally demands the inclusion or exclusion from a given set of conditions of one element at a time. This is necessary on the ground that if two or more elements were involved it would be impossible to tell in what way the effect was related to them. It might be due to them in any conceivable combination, and the method would lose the precision it claims. To obviate this and to make the experiment decisive there should be *one* circumstance added to or withdrawn from known conditions.

But to lay down this requirement is to demand a much greater knowledge and manipulative skill than is usually available. It may need many experiments extending over a longer or shorter period, it may even need a new discovery, before the desired single difference is obtained. The conditions into which a new element is introduced may be imperfectly known, and thus give rise to an erroneous inference. It is well known that a current of electricity passed through water decomposes that substance into its component gases, hydrogen and oxygen. In early experiments of this kind it was noticed that an acid and an alkali formed at the two opposite poles where the current entered the water. Since the composition of water was known it was thought by some that the introduction into it of the electric current was the sole cause of the production of the new substances. But unknown to them other conditions were present which accounted for the

change. It was left to Davy to demonstrate their presence. He "conjectured that there might be some hidden cause of this portion of the effect: the glass containing the water might suffer partial decomposition, or some foreign matter might be mingled with the water, and the acid and alkali be disengaged from it, so that the water would have no share in their production. Assuming this, he proceeded to try whether the total removal of the cause would destroy the effect, or at least the diminution of it cause a corresponding change in the amount of the effect. By the substitution of gold vessels for the glass, without any change in the effect, he at once determined that the glass was not the cause. Employing distilled water, he found a marked diminution of the quantity of acid and alkali evolved; yet there was enough to show that the cause, whatever it was, was still in operation. The impurity of the water, then, was not the sole but a concurrent cause. He now conceived that the perspiration from the hands touching the instruments might affect the case, as it would contain common salt, and an acid and an alkali would result from its decomposition under the agency of electricity. By carefully avoiding such contact, he reduced the quantity of the products still further until no more than slight traces of them were perceptible. What remained of the effect might be traceable to impurities of the atmosphere decomposed by contact with the electrical apparatus. An experiment determined this: the machine was put under an exhausted receiver, and when thus secured from atmospheric influence, it no longer evolved the acid and the alkali."[1] Thus by successive experiments the passage of the current through the water was seen to be the sufficient cause of its decomposition into hydrogen and oxygen.

It is equally easy to be under the impression that only one element is being added when in reality more than one are introduced. This was well illustrated in the controversy between Pouchet and Pasteur as to the existence of spontaneous generation. Pouchet submitted that his

[1] Gore, *The Art of Scientific Discovery*, pp. 432-433.

experiments, conducted with extreme precautions, proved that "animals and plants could be generated in a medium absolutely free from atmospheric air, and in which, therefore, no germ of organic bodies could have been brought by air."[1] All germs in the material used were destroyed by heat, and chemically prepared oxygen was substituted for ordinary air in order to sustain the life generated. Pasteur showed that in spite of his precautions Pouchet had deceived himself in thinking that germs from the air had been excluded. He then went on to demonstrate by ingenious tests of his own devising that when air was purified from germs by being passed through cotton wool or asbestos it was powerless to generate life in an alterable liquid, itself deprived of germs by ebullition; while the same liquid abounded in life when exposed to unfiltered air. Another experiment in which the germs in the air were destroyed by heat was objected to on the ground that the heat might conceivably affect other elements in the air which it was said might be necessary for the spontaneous generation, *e.g.* 'electricity, magnetism, ozone, unknown forces even.' To change one circumstance and one only is a matter for great practical sagacity.

We cannot assume that the introduction of a new element is the sole cause of any change which may ensue. It is always necessary to take account of the conditions on which it supervenes. In another context no such change might appear, and in such cases the new element is the occasion rather than the cause of the result. The lighting of a fuse attached to a charge of blasting powder may be followed by the shattering of a rock. Although the only difference between the elements before and after is the lighting of the fuse we do not think of it as the cause of the explosion.

Again, collateral but unobserved changes may be set up which are the real conditions of the observed effect. This is often so in physiology. A certain brain area is stimulated by pressure, electricity, or some other means, and a reaction say in the shape of a specific movement follows.

[1] Vallery-Radot, *Life of Pasteur*, Eng. Trans., p. 93.

That reaction may be due, however, not to the excitement of the first area, but to the collateral and unobserved excitement of a second area with which the movement is directly connected.

Too great an interval must not elapse between the introduction of the supposed cause and the noting of the effect. For in the meantime other elements may creep in which may modify or be the true constituents of the cause. Quinine taken one day may or may not have allayed the fever which disappears a few days later. It is not the mere interval but the difficulty of maintaining continued control of the conditions which gives importance to the element of time. Where the conditions throughout are not liable to outside interference a great length of time may even be indispensable. Certain experiments of Sir William Thomson are of this type. He placed "small quantities of blue vitriol and other coloured soluble substances in the bottom of very tall vertical glass tubes, filled with water and hermetically sealed, in order to ascertain the amounts of diffusion, etc., and other effects after a great length of time. . . . It is calculated that several hundreds of years will be required to complete the diffusive process."[1]

It sometimes happens that certain forces are kept in equilibrium by the counteraction of their several tendencies to produce given effects. A new factor introduced may simply liberate these forces so that they may have free play. A train at rest on a steep incline is set in motion by releasing the brakes: but no one would assign such release as the true cause of the motion. A closer analysis would reveal weight, incline, and comparative absence of friction, as elements in producing the acquired momentum, and that each is so operative could be shown by further applications of the method of difference.

In these various ways the apparent inference to a pure causal connexion through the method of difference is rendered precarious. Its probability is increased, if there is other evidence, such as that gathered by the method of agreement, to support it. By varying the contexts to which

[1] Gore, *Art of Scientific Discovery*, p. 558.

a given cause is added it may be possible to determine just what other factors are necessary to the change, and a knowledge of their properties may enable us to decide more precisely the contribution to the effect made by the supposed cause.

When it is said that one experiment is sufficient to establish a causal relation it is implied that only one known element is added to, or withdrawn from, a set of conditions whose results are known. The difference that ensues can then be assigned to the new factor of the introduction or withdrawal of the element. But the causal bond is between the two changes, not between two 'things' or 'elements.'

(iv) **Method of Concomitant Variations.**—The methods we have considered are lacking in quantitative precision, and in some cases cannot be applied. They are usefully supplemented by the Method of Concomitant Variations, which does not differ essentially from them in principle. Instead of an examination of instances in which cause and effect are present or absent as a whole, cases are considered in which there is a corresponding variation in the elements suspected of being causally connected. Any kind of variation is not sufficient. The argument rests on the assumption that cause and effect are equal in energy: increase or decrease in the one must be followed by a proportionate change in the other. For direct causal connexion then we require a variation in simple proportion. Symbolically

$$\begin{array}{cccc} A & 2A & 3A & \\ | & | & | & \cdot\ \cdot\ \cdot\ \cdot \\ x & 2x & 3x & \end{array}$$

Evidently if A is the whole cause of x each additional A must be followed by an additional x. If this is secured the probability of causation is strong. But stated thus abstractly no account is taken of the other elements present. It is assumed that these remain the same, and that the sole difference in each case is the quantitative change in A. The assumption is justified if A can be varied at will, so that we may try the effect of the introduction or withdrawal of successive amounts of A. We are then

adopting a modified form of the method of difference: the change in this case being purely quantitative and stopping short of the complete removal of the suspected cause.

But experiment is not always possible, and then it is not likely that instances exhibiting change in simple proportion will be found open to observation. Whether they are, or are not, a new difficulty arises. Both changes may be due to a third element which remains undetected, because hidden from observation. If the connexion is of the simple type that we have considered there may be no reason to suspect that it is dependent on unknown conditions. If, however, the variation is in any other than simple proportion we know that we have not the whole cause of the phenomenon before us. In such a case as $A - x$, $2A - 4x$, $3A - 9x$, etc., were A the sufficient cause of x we cannot see why the addition of A should be followed by a more than proportionate increase in x. The evidence for some connexion between them is strong, and since it is not directly that of cause and effect, we are led to assume that they are probably joint effects of some unknown cause. It is not necessary that the changes in amount should be measured in the same direction. An increase in the cause may be followed by a numerical decrease in the effect; for example, an increase in pressure will at a constant temperature be followed by a diminution in the volume of a gas. But the energy before and after remains the same.

We cannot by the method of concomitant variations, any more than by the other methods taken singly, determine with certainty a causal connexion. This is especially so when the variation cannot be exactly measured. Then we cannot tell whether the change is or is not fortuitous. In combination with other methods we may, however, determine the nature of the connexion if any exists. To take a simple illustration from Darwin. He found that on Keeling Island "the wells are situated from which ships obtain water. At first sight it appears not a little remarkable that the fresh water should regularly ebb and flow with the tides; and it has even been imagined that sand has the power of filtering the salt from the sea-water. These ebbing wells are common on some of the low islands in the

West Indies. The compressed sand or porous coral rock is permeated like a sponge with the salt water; but the rain which falls on the surface must sink to the level of the surrounding sea, and must accumulate there, displacing an equal bulk of the salt water. As the water in the lower part of the great sponge-like coral mass rises and falls with the tides, so will the water near the surface; and this will keep fresh, if the mass be sufficiently compact to prevent much mechanical admixture; but where the land consists of great loose blocks of coral with open interstices, if a well be dug, the water, as I have seen, is brackish."[1] Here we see the method of exclusions resorted to in order to explain a concomitant variation, which in itself only suggested some kind of connexion between wells and tides, but in that very suggestion presented the difficulty that the water in the former case differed from that in the latter. To establish a causal connexion it was essential to account for this difference by showing how the salt of the sea-water was eliminated.[2]

The method of concomitant variations is of greatest value when the variations in amount or intensity can be exactly measured. Indeed in this respect it has the advantage over all the methods we have already considered. A relation which has been broadly established by the method of difference or the method of exclusions is at once rendered precise when it is determined as a relation of quantity. That heat causes the expansion of metals is a sound enough conclusion from the method of difference, but it is vague. When we know that a certain quantity of heat will produce in each case an increase in bulk exactly represented by the coefficient of expansion, we not only increase our power of effective manipulation in cases where the force is under control, but the understanding has a better grasp of the facts. Mathematics is the best means that science has at its command for attaining precision, and a law of nature is held to be most completely determined when it can be mathematically expressed.

[1] *The Voyage of the Beagle*, Ch. XX.

[2] The complete logical analysis is left as an exercise to the student.

The corresponding changes in quantity between two elements which are supposed to be directly or indirectly in causal relation to each other may be graphically represented by means of a curve. A horizontal line is drawn from a fixed point, and a series of points at equal distances from one another are taken to represent equal amounts of the one element, while on a vertical line drawn from the same fixed point are marked off in a similar way equal spaces to represent equal amounts of the other element. From each point on the horizontal line vertical lines are drawn to meet horizontal lines drawn from each of the corresponding points on the original vertical line. The intersection of each pair of lines gives a series of points which when joined give the curve representing the relation. This curve shows at a glance the degree of the variation and whether it takes place in a regular manner. The newspaper weather charts represent in this manner the variation in the height of the barometer from day to day. Here, of course, there is no direct causal relation. Economists often use the same device. For example, it is usual to draw a curve representing the influence of successive doses of capital on the return got from land under cultivation. The horizontal divisions, or abscissae, represent the capital: the vertical divisions, or ordinates, the yield in crops. The curve within certain limits is regular in form, and in the direction of a more than proportionate increase in the return. One might be tempted to expect that since the general direction of the curve indicates an increase it might be indefinitely continued, and so the conclusion be drawn that every increase in the amount of capital will be followed by a more than proportionate return. But when the increase goes beyond a certain point it is found that the yield diminishes in value, and the curve consequently undergoes a change in direction.

What the curve represents to the eye as a break in the continuity of the relation is merely a graphic representation of the truth that a variation which holds good within a definite range of experience may not hold when the range is extended. If then we extend the range of such variation

simply as an inference without testing its truth by an appeal to fact we may be exposed to contradiction. "Water contracts as it is cooling. Suppose we begin to note this continued contracting of water from 100° F. to 90°; we naturally expect to find it continuing through 90° to 80°. And as we observe, we find our expectations confirmed. And so on through to 40°, we find that water continues to contract. It is, therefore, most natural for us to expect to find water contracting at 39°. But just at this point in the series, there is a break in the continuity of variation; at 39° water begins to expand and so continues until it passes into the solid form at the freezing-point."[1] We must, then, be cautious in making or accepting such inferences. The only safeguard is to submit them to experimental tests over a wide range of instances until the critical points at which the law is modified, if such exist, are discovered.

The complete exclusion of a cause required by the method of difference is not always practicable even where the cause allows to some extent of experimental control. There are permanent elements in nature such as the force of gravitation, heat, friction, of which we cannot get rid. But though we cannot remove them we may by various devices make their action greater or less in amount, and note the changes which ensue. In this way we can learn much of their nature, and often formulate the law of their action even though we only find it exemplified as a tendency which is always to some degree counteracted. This is a most useful application of the method of Concomitant Variations. Mill instances the first law of motion—that all bodies in motion continue to move in a straight line with uniform velocity until acted on by some new force. No example of perpetual motion occurs in nature; but various obstacles to it such as friction and retardation of the atmosphere may account for this. If that be so a diminution in the resistance due to them should be followed by an increase in the time during which a given velocity is maintained. Experiments showed that this was the case. "The simple oscillation of a weight

[1] Hibben. *Inductive Logic*, pp. 142-143.

suspended from a fixed point, and moved a little out of the perpendicular, which in ordinary circumstances lasts but a few minutes, was prolonged in Borda's experiments to more than thirty hours, by diminishing as much as possible the friction at the point of suspension, and by making the body oscillate in a space exhausted as nearly as possible of its air. There could, therefore, be no hesitation in assigning the whole of the retardation of motion to the influence of the obstacles; and since, after subducting this retardation from the total phenomenon, the remainder was an uniform velocity, the result was the proposition known as the first Law of Motion."[1]

Perhaps the most important examples of variations due to a common but unobserved cause are those which occur in fixed periods. The movements of the tides which change regularly in character according to the position in the heavens of the sun and moon are an often quoted example. Another remarkable periodic coincidence is that between the occurrence of the Aurora Borealis, magnetic storms and changes in the spots on the sun. In the middle of the nineteenth century it was established that once in about ten years magnetic disturbances reach a maximum of violence and frequency, and that at about the same interval the activity of the sun spots is at its greatest. Further, it was discovered that the Aurora Borealis is associated with a disturbed condition of the sun. "Subsequent detailed observation has exhibited the curve of auroral frequency as following with such fidelity the jagged lines figuring to the eye the fluctuations of solar and magnetic activity as to leave no reasonable doubt that all three rise and sink together under the influence of a common cause."[2] What that cause is has not yet been discovered.

(v) **Method of Residues.**—We have considered the preceding methods in the simplest form by assuming that each investigation attempts to establish one causal connexion in a series of complex phenomena. Of course, the

[1] Mill, *System of Logic*, Bk. III., Ch. viii., § 7.

[2] Clerke, *History of Astronomy during the XIXth Century*, p. 161, cf. pp. 156-161.

advance of knowledge demands that such investigations should be made again and again with the same kind of sequence, in each case considering a fresh set of relations. At first the residue denoted by $B-y$ in our general formula $A\ B\ \ldots\ldots\ x\ y$ is a great complex as yet unanalysed. But as enquiry goes on this is more and more known and we are able to describe the sequence as

$$A\ C\ D\ E\ \ldots\ldots\ Z \text{———} x\ o\ p\ q\ \ \ldots\ m$$

where

$$A-x,\ C-o,\ D-p,\ E-q\ \ldots\ldots$$

are all known connexions, so that the remainder, $Z-m$, is itself of a very simple nature. There is then a strong presumption that $Z-m$ is a causal connexion. This is known as the *Method of Residues*, and it is evident that it is only applicable to matter which has already been the subject of much investigation.

Now either Z can be withdrawn, in which case the hypothesis of its relation to m can be tested by the method of Difference; or it can be varied in amount, when the method of Concomitant Variations is available; or parallel instances can be found of its presence and absence, when the method of Exclusions comes into play; or none of these are possible, in which case it is necessary to resort to the indirect method of deducing the various forms in which the consequences of Z would appear in different sets of co-operative circumstances, and seeking verification by comparison of these expectations with actual facts.

Obviously, the mere fact that $Z-m$ remains unexplained is only a negative kind of presumption that they are causally connected, and before it can be accepted as established it must have positive confirmation. Certainly the more the whole phenomena have been analysed the greater presumption of truth this hypothesis has at the beginning—the simpler is the positive demonstration needed.

In other cases the hypothesis may start, as it were, much further back. Only one side of the residual sequence is known to exist: the other is supposed as a hypothesis. Such supposition, if it can be made at all, cannot be made

with much likelihood of success except by a mind well stored with pertinent knowledge, which in framing it applies at once the indirect method of seeking for a relation which on known laws would yield this unexplained result. Put symbolically there is known

$$A\ C\ D\ E \ldots. \text{———} \ x\ o\ p\ q \ldots. m.$$

The whole of $A\ C\ D\ E$ .... as known is required to account for $x\ o\ p\ q$ ..... Nothing in the known conditions remains as an explanation for the m which has just been discovered to exist in the result. There is evidently a call for enquiry. There must be assumed some Z which on the analogy of all pertinent knowledge will account for just this m in just this form.

The history of astronomy offers many pertinent examples. Before it was observed through the telescope the planet Neptune was assumed as a hypothesis to account for deviations of the planet Uranus from the path which exact calculation of the effects of the known heavenly bodies laid down for it. The mathematical nature of the deductions involved from this assumption made it possible to be sure that if such a new planet was in existence it would be in a certain place in the heavens at a certain time. The verification was then only dependent on the telescopes used being of sufficient power. Had the planet not been found increased power of lens would have been sought before the hypothesis was given up. On all known analogy the existence of the new planet was the *only* way of explaining the residual phenomena consistently with the theory of gravitation, so the hypothesis had an extremely high probability. Yet astronomy could not be satisfied without positive verification.

As another example we will take Römer's determination of the velocity of light from observations of the satellites of Jupiter. These are five in number, and their motions are of such a nature that each satellite, once in every revolution, enters the shadow-cone thrown by Jupiter, and so becomes eclipsed. The time when any particular satellite should be eclipsed could be calculated exactly, but there was found to be a discrepancy between this calcu-

lated time and the observed time. Here was a residual phenomenon which needed investigation. Now the period of revolution for each satellite is comparatively short, and consequently the eclipses are frequent. In each case the eclipse is calculated to take place at regular intervals so that the time between two eclipses remains the same. But "astronomical observations of Jupiter's satellites show that while the earth in its orbital motion is receding from Jupiter the mean period between two successive eclipses of a particular satellite is longer than that which elapses when the earth in its orbital motion is approaching Jupiter."[1] The hypothesis was, therefore, advanced that the discrepancy between calculation and observation was due to the velocity of light. Thus "when the earth is receding from Jupiter the light from a disappearing satellite has to travel a greater distance at each successive disappearance."[2] The interval, therefore, appears to lengthen. On this supposition Römer obtained the first trustworthy value for the velocity of light.

The residual phenomenon is usually of small dimensions. It was a small discrepancy in weight which led to the discovery of argon. So in the example already quoted of the irregularities of the motion of Uranus, "It may be stated as illustrative of the perfection to which astronomy had been brought that divergencies regarded as menacing the very foundation of its theories never entered the range of unaided vision. In other words, if the theoretical and the real Uranus had been placed side by side in the sky, they would have seemed to the sharpest eye to form a single body."[3]

3. **Indirect Establishment of Hypotheses.**—The work of each of the direct methods singly, and of all of them combined, is then to suggest hypotheses and to strengthen those already suggested and, it may be, to mould their form. But as these methods deal only with phenomena open to observation and manipulation they can

[1] Edser, *Light for Students*, p. 219 [2] *Ibid.*, pp. 219-220.
[3] Clerke, *op. cit.*, p. 96.

never directly reach those hidden connexions which physical science more and more seeks. They cannot deal directly with atoms or ether, or any natural force. They describe the visible results of a causal connexion, they even go far to establish its existence, but they cannot explain its nature. Even when most successful, therefore, they only give a very strong probability to an empirical law—that is, a statement that such and such conditions *do* secure such and such a phenomenon.

Such results stand apart in a kind of fictitious independence till they are shown to be expressions of a much smaller number of more fundamental relations. "Friction, combustion, the liquefaction of a vapour, freezing, pressure, all produce heat. What could be more apparently disparate than these agencies? Yet all of them alike involve the liberation of molecular motion in accordance with mechanical laws common to all the cases."[1] The direct methods can establish that heat does result from each of these agencies. To show them all as variants of a deeper law of molecular motion can only be done indirectly. For molecular motion does not lie open to direct observation. The whole of the ultimate construction of the physical world is for science a set of relations which cannot be perceived by the senses, and which are held to be established because the results which can be shown with mathematical certainty to follow from them in various combinations are found fulfilled in nature. This unifying of the empirical laws as a necessary consequence of a very small number of theories as to the ultimate nature of the world is the only form of explanation science can reach. Why that ultimate nature is what it is science cannot say. Its task is to know that nature, and there can be no other proof that it has attained that knowledge than that its assumptions yield a system consistent both with itself and with the results of continued careful and accurate observation.

The direct methods, therefore, lead up to the formation of these wider hypotheses which can only be tested indirectly, and they are the handmaids of this testing.

[1] Hobhouse, *op. cit.*, p. 366.

In other cases only indirect enquiry is possible. This is so in all cases in which the facts cannot be got at directly, but in which our reasoning has to start from some record of them.

The causes of historical and social phenomena are largely hidden, and can only be recovered by a hypothetical reconstruction. Geology and Biology are also in a broad sense historical. No record remains of the causal sequences by which the earth has reached its present state, and living species of plants and animals their present characteristics. The history of development in either sphere is reconstructed by framing hypotheses in the light of analogies drawn from experience, and by testing whether the consequences they entail would result in the facts which prior and subsequent enquiry make known. For a small range of fact is sufficient to suggest a supposition which, through the inferences it logically supports, becomes the revealer of facts which otherwise had remained unobserved.

The Wave-Theory of Light[1] and the Theory of Gravitation both show the use of the indirect method in establishing the most comprehensive truths of science. The story of Newton's development of the theory of gravitation is well known. Observation of the motion of falling bodies by Galileo had led to the empirical law that bodies, irrespective of size or material, fall through the same distances in equal times. Newton by experiments with the pendulum established the same law with greater exactness. Moreover Kepler, improving on the earlier labours of Tycho Brahé, had by incessant observation of the heavens and much conjecture reached the conclusion that the motion of the planets can be described in three laws—that each planet revolves round the sun in an elliptical path with the sun in one of the foci; that it revolves with such a velocity that a straight line drawn from it to the sun passes over equal areas in equal times; and that the squares of the number of days taken by the planets to complete a revolution are proportional to the cubes of their

[1] See pp. 440-446.

mean distances from the sun. But so far no reason had been given why the empirical laws of falling bodies and of planetary motion should be as they had been determined. Newton supplied a reason by his hypothesis that they were due to the attraction of bodies by one another directly in proportion to their mass and inversely as the square of the distance between them. Taking the moon as his first example he showed by mathematical reasoning that if the hypothesis were true its motion would be in a certain path. After some years this was confirmed by observation. He then extended his reasoning to the case of other planets, and ultimately to all particles of matter. His deductions were verified by an examination of the actual motion of the planets, and in astronomy his principles have been verified many times by subsequent observations and discoveries.

As an example in Geology we may indicate the kind of reasoning by which the transport of boulders often of great size has been attributed to glacial action. The existence of these rocks in valleys and on hills of a different geological structure from that of the rocks themselves gives rise to the problem as to how they have arrived at their present position. For instance, "such 'erratics' not only abound in the Swiss valleys, but cross the great plain of Switzerland, and appear in numbers high upon the flanks of the Jura. Since the latter mountains consist chiefly of limestone, and the blocks are of various crystalline rocks belonging to the higher parts of the Alps, the proof of transport is irrefragable."[1] But how has the transport been effected? There is no visible agency at work with which the movement of the rocks is obviously connected. Recourse must be had to hypothesis. Various agencies have been suggested, but it is now generally accepted that they have been transported to their present position by means of ice. It follows at once that ice (1) must be capable of doing the work, and (2) must have extended at one time over areas from which in many cases it is now remote.

[1] Geikie, *Text-Book of Geology*, 4th edn., Vol. I., p. 554.

Glaciers are the most conspicuous examples of the action of ice on land. It is found that though to a casual glance they seem to be at rest they are in reality moving. On their surface they bear earth, stones, and rubbish, weathered from the cliffs and slopes of the valley down which they are descending. The masses of rock are sometimes "as big as a large cottage." The detritus is liable to slip into crevasses, and may descend to the bottom of the ice to be moved with it along the rocky floor, so that "whether on the ice, in the ice, or under the ice, a vast quantity of detritus is continually travelling with a glacier down towards lower ground."[1] If the action of ice, then, be admitted as a cause it would be sufficient to explain the transport of rocks as far as the movement of the ice can be traced. By what signs can we support the hypothesis that masses of ice have at one time passed over any given surface?

"Beneath the ice of the Swiss glaciers lies a thin inconstant layer of fine wet mud, sand, and stones, derived partly from the descent of materials from the surface down the crevasses, partly from the rocks of the sides and bottom of the glacier bed. These materials may be seen fixed sometimes in the ice itself. Though it may locally accumulate, this layer is apt to be removed by the ice or by the water that flows under the glacier."[2] This deposit constitutes the so-called boulder clay. If, then, we find boulder clay, that will be one indication of glacial action. Again, the erosion caused by glaciers is of an easily distinguishable kind. It is effected not so much by the contact and pressure of the ice on the rocks as by means of the fine sand, stones, and blocks of rock that fall between the ice and the rocks on which it moves, or between the ice and the side of the valley in which it lies. "Under the slow, continuous, and enormously erosive power of a glacier, the most compact resisting rocks are ground down, smoothed, polished, and striated. The striae vary from such fine lines as may be made by the smallest grains of quartz up to deep ruts and grooves."[3] The stones effecting the erosion are polished and marked in the same way, the

[1] *Ibid.*, p. 546. [2] *Ibid.* [3] *Ibid.*, p. 550.

striae or parallel lines running generally in a longitudinal direction. If the hypothesis of ice action be true such marks will probably be found on the erratic boulders and in their neighbourhood. We must search for them and so apply the test of fact.

Various other indications may be present, such as the smooth undulating "hummocky bosses of rock" left behind on the retirement of a glacier or the huge hollows worn away where the rock was softer and which may now be filled with water, forming tarns or lakes. The process of investigation and reasoning which led to the theory of an ice age extended over many years, required many workers, and presented many difficulties. We have merely given in bare outline illustrations of the type of inference involved.

Evidently the conclusions of history can only be reached by the indirect method. The phenomena which the historian seeks to explain are extremely complex. There is no clear sequence open to observation, nor is it possible to isolate a causal connexion by any of the direct methods. The explanation of the state of a nation at any given period involves the consideration of its knowledge, morals, industry, institutions, race characteristics, geographical conditions, and numerous other factors each complex in itself. And if we single out one aspect for study and endeavour to trace, say, political, religious, or economic progress there is the same complexity. The conditions modify one another in countless ways, and it is difficult to determine their relative importance. Even a single event often presents a similar difficulty of interpretation. The play of human motive and of circumstances but imperfectly realised produces effects for which it is difficult to find grounds at once adequate and indisputable. Indeed, motives can only be conjectured from words and actions: the influence of dominant interests on men can at best be assigned with some reserve. When we add to this the necessary scantiness and imperfection of the data in many instances, and that such facts as we have are based on testimony with its many liabilities to error, we see that the work of the historian is one both of criticism and of construction.

Having determined the facts by criticism, he desires to weld them together so far as may be in causal sequence. The only procedure open to him is to choose one of the known conditions, or it may be one conjectured to be present, which on analogy with what is known of human nature in similar circumstances is likely to have produced or to have helped in producing the given effect. Taking this hypothetical condition as true he will then seek to establish that, in accordance with recognised principles of human nature, it would lead to consequences of a certain kind under which all the relevant facts may be embraced. At the same time he will try to prove that rival suggestions involve principles leading to results incompatible with the established data. In actual investigation it will be found that, as a rule, not one but many conditions contribute to produce an event or series of events, and the emphasis to be laid on the different factors will vary.

It may be possible to show that the different conditions flow from one root cause of wider generality. For example, let us take the causes of the decline of the American Confederation. As usually given they are as follows: "1. The Confederation had no executive or judicial department. 2. Congress could not raise an army. 3. No power of direct or indirect taxation was given to the Confederation. 4. Congress had no control over domestic commerce. 5. Congress could not enforce treaties with other nations. 6. The Confederation operated on states and not on individuals. 7. The Articles of Confederation recognised the sovereignty of the state. 8. Voting in Congress was by states. 9. The people owed allegiance to the state only."[1] The first eight can be shown on examination to be closely related to the last, which embodies the fundamental principle that "wherever primary allegiance is placed there sovereignty will reside."[2]

Such principles should be used with caution. It is so easy to support them 'from the nature of the case' without bringing them to the touchstone of facts. History may err by reaching out to either of two extremes. It may

[1] Mace, *Method in History*, p. 30. [2] *Ibid.*, p. 31.

content itself with a collection of facts, or it may develop principles without sufficient regard to the results of experience. Even when the facts are carefully considered it often happens that they are well explained on one hypothesis without being so exactly determined or so decisive as to exclude another. In the last resort there is frequently a balance of probability, which men of different temperaments and training will strike in different ways.

A similar uncertainty attaches itself to the use of the indirect method in ordinary life. The teacher perplexed by an epidemic of disorder, or the merchant anxious to gauge the stability of his markets, casts about for probable causes which in their combination would produce results apparent or to be predicted. The certainty they reach is problematical in theory but sufficient for practical action. The circumstances of the case point sufficiently clearly in a given direction.

The circumstantial evidence in our law courts, upon which criminals are often convicted, is of this kind. It is to the interest of the criminal to commit his crime unobserved: it is the duty of the prosecution to show, in the absence of the evidence of eye-witnesses, that no one but the prisoner could have committed it. Many circumstances may point to guilt: there may be obvious motives to the crime, apparent preparations to commit it, possession of its fruits, unsatisfactory explanations of suspicious appearances, suppression, destruction, or fabrication of evidence, failure to account for movements at the time the crime was committed in the face of testimony to presence in the neighbourhood.[1] Not one of these is sufficient in itself to prove guilt; each is susceptible of an innocent explanation. It is the combination and convergence of the separate strands of evidence which makes innocence improbable. On the hypothesis that the crime has been committed by the accused the facts fall into their place and form a conclusive whole, though each part may be inconclusive in itself. If no other hypothesis can be devised which explains the circumstances as well or

[1] Cf. *Logic for the Million*, p. 106.

better, the prisoner will probably be condemned. "A man is found dead with his throat cut. A knife is found in a ditch close by. There are footprints in the mud. X was known to be in the neighbourhood on the day; evidence is given that he purchased the knife a week before; his boots fit the footprints. All these facts might be due to a collocation of separate causes, but all are explicable by a single cause, namely, that X planned and carried out the murder. The single assumption is so much more probable than the multiple combination of circumstances that it is likely to go hard with X, and his business is to produce some fact incompatible with the above explanation. Failing this, one or two more such combinations of circumstances, and our conviction of the strength of the hypothetical argument will be evinced in a very practical manner."[1]

[1] Hobhouse, *The Theory of Knowledge*, p. 422.

CHAPTER XXXII.

EXAMPLES OF INDUCTION.

We will in this chapter examine from a logical standpoint some actual pieces of scientific enquiry. It is only by careful study and analysis of such examples that one appreciates how intensely difficult is the search for knowledge, how many opportunities there are for error, how far removed from the simplicity of a formal statement of the methods employed is the actual employment of those methods. A process whose abstract nature can be described by the logician in a few minutes may take months or years to carry to a successful issue in the actual work of the discovery of truth. The successful questioner of nature needs infinite patience, infinite resource, as well as great imagination and constructive power, a keen critical faculty, and a sound well-balanced judgment.

1. **Formation of Vegetable Mould.**—As an example of a very careful and thorough enquiry conducted nearly entirely by simple observation, and yet reaching a very high degree of probability in its conclusion, we will take Darwin's investigations into the formation of vegetable mould. The hypothesis suggested was that this is due to the action of earth-worms.

If this is true, that is, if earth "is brought up by worms from beneath the surface, and is afterwards spread out more or less completely by the rain and wind," then small objects left on the surface will gradually become buried. Mr. Darwin records many observations to establish this proposition. For instance: "In the spring of 1835, a field, which had long existed as poor pasture, and was so swampy that it trembled slightly when stamped on, was

thickly covered with red sand so that the whole surface appeared at first bright red. When holes were dug in this field after an interval of about two and a half years, the sand formed a layer at a depth of $\frac{3}{4}$ in. beneath the surface. In 1842 (that is, seven years after the sand had been laid on) fresh holes were dug, and now the red sand formed a distinct layer, 2 inches beneath the surface. . . . Immediately beneath the layer of red sand, the original substratum of black sandy peat extended."[1]

But may the character of the land be disregarded, or is that an essential element? This point was settled by observing that in cases of land differing widely from that on which the first observations were made, whenever worms were present the result was the same. This, of course, was an application of the Method of Agreement. To take an unpromising instance: Darwin records that "The chalk formation extends all round my house in Kent; and its surface, from having been exposed during an immense period to the dissolving action of rain-water, is extremely irregular, being abruptly festooned and penetrated by many deep well-like cavities. During the dissolution of the chalk, the insoluble matter, including a vast number of unrolled flints of all sizes, has been left on the surface, and forms a bed of stiff red clay, full of flints, and generally from 6 to 14 feet in thickness. Over the red clay, wherever the land has long remained as pasture, there is a layer a few inches in thickness, of dark-coloured vegetable mould."[2] In some instances, the observations partook of the nature of experiment, as when chalk was spread on a pasture "for the sake of observing at some future period to what depth it would become buried," and it was found that after twenty-nine years, the chalk was 7 inches below the surface. Even a steeply sloping field "thickly covered with small and large flints," was after thirty years so completely covered that "a horse could gallop over the compact turf from one end of the field to the other, and not strike a single stone with his

[1] Darwin, *Vegetable Mould and Earth Worms*, pp. 134-135.
[2] *Ibid.*, pp. 137-139.

shoes. . . . This was certainly the work of the worms, for though castings were not frequent for several years, yet some were thrown up month after month, and these gradually increased in numbers as the pasture improved."[1] Moreover, "the specific gravity of the objects does not affect their rate of sinking, as could be seen by porous cinders, burnt marl, chalk and quartz pebbles, having all sunk to the same depth within the same time."[2]

We have, necessarily, only cited a very few of the observations made, but the total evidence goes to support a connexion between the existence of earth-worms and the formation of mould.

However, there are apparent exceptions—large boulders do not sink. All the hollow spaces between such a boulder and the earth will be filled up, and the surface of the ground will be raised to a height of several inches all round the edge of the stone. But further examination shows that the exceptions are only apparent, and really prove the rule. "If . . . a boulder is of such huge dimensions, that the earth beneath is kept dry, such earth will not be inhabited by worms, and the boulder will not sink into the ground."[3] In one case in which a large stone "67 inches in length, 39 in breadth, and 15 in thickness" . . . had only "sunk about two inches into the ground" in thirty-five years, "on digging a large hole to a depth of 18 inches where the stone had lain, only two worms and a few burrows were seen, although the soil was damp and seemed favourable for worms. There were some large colonies of ants beneath the stone, and possibly since their establishment the worms had decreased in number."[4]

Thus, the apparent exceptions turn out upon closer examination to be negative instances—where there are no worms, there is no sinking of objects. A negative instance on a larger scale is recorded. "I examined in Knole Park a dense forest of lofty beech-trees beneath which nothing grew. Here the ground was thickly strewed

[1] *Ibid.*, pp. 143-144. [2] *Ibid.*, p. 157. [3] *Ibid.*, p. 149.
[4] *Ibid.*, pp. 152-153.

with large naked stones, and worm-castings were almost wholly absent. Obscure lines and irregularities on the surface indicated that the land had been cultivated some centuries ago. It is probable that a thick wood of young beech-trees sprung up so quickly, that time enough was not allowed for worms to cover up the stones with their castings, before the site became unfitted for their existence."[1] Both positive and negative instances, then, go to support the universal propositions "If there are worms, there is vegetable mould" and "If there is vegetable mould, there are worms." The method applied is mainly that of Exclusions, but Concomitant Variation adds its support.

But certain objections must be met as to the inadequacy of the suggested agency to produce the results. Investigations were, therefore, made as to the number of worms existing in a measured space. Hensen, from counting those found in a piece of garden, calculated the number at 53,767 in an acre, though he "believes that worms are here twice as numerous as in cornfields."[2] But more definite results were obtained in estimating the weight of the castings. The results of four carefully examined cases give that in a year castings calculated to yield from seven and a half to over eighteen tons per acre are ejected.[3] By carefully breaking up the dried castings, and pressing them down in a measure, the cubical content was found, and allowance being made for the lesser degree of compactness of the triturated castings as compared with mould, it was calculated that the above mentioned results would yield a layer of mould from an inch to an inch and a half in thickness in ten years. This was found to be rather less than the observed depth to which objects had sunk in that time. But allowance must be made for "the loss which the weighed castings had previously undergone through being washed by rain, by the adhesion of particles to the blades of the surrounding grass, and by their crumbling when dry," and also for the lesser agency of burrowing larvae and insects, especially ants, and of moles. "But," says Darwin, "in our county these latter several agencies

[1] *Ibid.*, pp. 144-145. [2] *Ibid.*, pp. 158-159. [3] Cf. *ibid.*, pp. 168-169.

appear to be of quite subordinate importance in comparison with the action of worms."[1] Here we see the force added to a probability even by a somewhat rough application of measurement and mathematical calculation.

2. **The Silkworm Disease.**[2]—The investigation into the nature and origin of the disease which in the middle of the last century threatened to destroy the silkworm industry is an admirable example of actual inductive work, and of the value of the trained mind to the general community. It brings out clearly how only to such a mind does the fruitful hypothesis occur; it only has the pertinent knowledge without which the true analogy is unseen. The hypothesis once formed the consequences which must follow from it are inferred, and then tested. It is in this testing that the direct methods are available. Here appeal was mainly to that of Exclusions, though in the experiments the greater rigidity of the Method of Difference was approached. We see, too, how one enquiry grows out of another till there is established a system sufficient to explain all the phenomena under examination.

Louis Pasteur was induced to take up the enquiry in 1865. "Amidst the labyrinth of facts and opinions, it was not hypotheses which were wanting. For seventeen years they had been rising up on all sides"—atmospheric conditions, degeneration of the race of silkworms, disease of the mulberry tree, etc., were suggested. But none so far had proved fruitful.

The disease was called pébrine.[3] It appeared in all stages. "Some worms languished on the frames in their earliest days, others in the second stage only, some passed through the third and fourth moultings, climbed the twig and spun the cocoon. The chrysalis become a moth, but that diseased moth had deformed antennae and withered legs, the wings seemed singed. Eggs (technically called seed) from those moths were inevitably unsuccessful the following year. Thus, in the same nursery, in the course

[1] *Ibid.*, pp. 172-173. [2] *Louis Pasteur*, by his Son-in-law, pp. 127-163.
[3] From the patois word *pébré* (pepper).

of the two months that a larva takes to become a moth, the pébrine disease was alternately sudden or insidious: it burst out or disappeared, it hid itself within the chrysalis and reappeared in the moth or the eggs of a moth which had seemed sound."[1] This was the problem Pasteur had to solve.

In a memoir on the epidemic M. de Quatrefages related that certain naturalists "had discovered in the worms and moths of the silkworm minute corpuscles visible only with the microscope"; that another declared these corpuscles to be present in diseased silkworms, and that they had been perceived in silkworms' eggs. He "only mentioned this matter of the corpuscles as a passing remark, being doubtful of its importance and perhaps of its accuracy." But it was sufficient to suggest to Pasteur the hypothesis that these corpuscles, if existent, were causally connected with the disease, and it was this hypothesis which he decided to test.

He conducted his investigation at Alais, where the disease was rife. By means of the microscope he soon detected the presence of corpuscles in the worms. Their existence was, then, a fact.

He resolved to submit them to careful microscopic study. "Two series of worms were being cultivated. The first set was full grown; it came from some Japanese seed guaranteed as sound, and had produced very fine cocoons. . . . The worms of [the] second series were sickly and did not feed properly. And yet these worms, seen through the microscope, only exceptionally presented corpuscles: whilst Pasteur was surprised to find some in almost every moth or chrysalis from the prosperous nursery."[2]

"Faced by the contradictory facts that one successful set of cocoons had produced corpuscled moths, while an apparently unsuccessful set of worms showed neither corpuscles nor spots, he had awaited the last period of these worms with an impatient curiosity. He saw amongst those which had started spinning, some which as yet showed no spots and no corpuscles. But corpuscles were abundant in the chrysalides, those especially which were

[1] Vallery-Radot, *The Life of Pasteur*, Eng. Trans., p. 117.
[2] *Ibid.*, p. 118.

in full maturity, on the eve of becoming moths; and none of the moths were free from them."[1] The remains of many other backward cultures were examined, and these results were confirmed. He tentatively formulated his conclusions at this stage. "It was a mistake," he wrote, "to look for the symptom, the corpuscle, exclusively in the eggs or worms; either might carry in themselves the germ of the disease, without presenting distinct and microscopically visible corpuscles. The evil developed itself chiefly in the chrysalides and the moths; it was there that it should chiefly be sought."[2]

So far he had not proved that the corpuscles were the cause of the disease. He came to the conclusion that if his hypothesis were true "the only infallible method of procuring healthy eggs must be by having recourse to moths free from corpuscles."[3] To test this he selected certain eggs of moths from which corpuscles were absent, and others from moths in which corpuscles were numerous. These were hatched in the following year with the result anticipated, that the culture in the one case was healthy and in the other diseased. The experiment was repeated subsequently, and it was found that the pébrine disease did not appear in cultures from the eggs of healthy moths.

In the meantime the question was raised whether the disease was contagious. "Some considered that contagion was certain; the majority, however, either doubted or denied its existence; some considered it as accidental."[4] Pasteur resorted to experiment. "One of the first experiments was as follows: After their first moulting, he took some very sound worms free from corpuscles, and fed them with corpusculous matter, which he prepared in the following simple manner. He pounded up a silkworm in a little water, and passed a paint-brush dipped in this liquid over the whole surface of the leaves. During several days there was not the least appearance of disease in the worms fed on those leaves; they reached their second moulting at the same time as the standard worms which

[1] *Ibid.*, p. 120.
[2] *Ibid.*, p. 120.
[3] *Louis Pasteur*, Eng. Trans., p. 138.
[4] *Ibid.*, p. 141.

had not been infected. The second moulting was accomplished without any drawback. This was a proof that all the worms, those infected as well as the standard lot, had taken the same amount of nourishment. The parasite was apparently not present. Matters remained in this state for some days longer. Even the third moulting was got through without any marked difference between the two groups of worms. But soon important changes set in. The corpuscles which had hitherto only showed themselves in the integuments of the intestines began to appear in the other organs. From the second day following the third moulting—that is to say, the twelfth after the infection—a visible inequality distinguished the infected from the non-infected worms. Those of the standard lot were clearly in much the best health. On examining the infected worms through a magnifying glass, a multitude of little spots were discovered on their heads, and on the rings of their bodies which had not before shown themselves. These spots appeared on the exterior skin when the interior skin of the intestinal canal contained a considerable number of corpuscles. It was these corpuscles that impeded digestive functions and interfered with the assimilation of the food. Hence arose the inequality of size of the worms. After the fourth moulting, the same type of disease was noticed as that which was breaking out everywhere in the silkworm nurseries, especially the symptom of spots on the skin, which had led to the disease being called *pébrine.* The peasants said that the worms were peppered. The majority of the worms were full of corpuscles. Those which spun their cocoons produced chrysalides which were nothing but corpusculous pulp, if such a term be allowed."[1]

This comparison of cases in which the disease was introduced with those in which it was not seemed decisive in favour of the hypothesis that pébrine was contagious. But to allow for the influence of conditions in which the two cases differed, and which had been disregarded as unimportant or because they were unknown, "Pasteur was never

[1] *Ibid.*, pp. 142-143.

tired of repeating this curious experiment, or of varying its conditions. Sometimes he introduced the corpusculous food into healthy worms at their birth, sometimes at the second or third moulting. Occasionally, when the worms were about to spin their cocoons, the corpusculous food was given them. All the disasters that were known to have happened in the silkworm nurseries, their extent and their varied forms, were faithfully reproduced. Pasteur created at will any required manifestation of pébrine."[1] In each case "Pasteur was careful to reproduce these same experiments with the worms of the standard lot, from which all infected worms had been selected," but taking care that the matter introduced should be free from corpuscles. The worms, unlike the others, continued healthy.

A difficulty now arose. Contagion by experiment was proved, but it remained to show how contagion was possible under industrial conditions. Pasteur established that there were three causes of natural contagion. (1) The excreta of the worms may be more or less filled with corpuscles: the worms by the weight of their bodies press this against the leaves: and the leaves when eaten introduce the disease. "By the excreta of corpusculous worms which he crushed, mixed with water and spread with a paint-brush over the mulberry leaves intended for a single meal, Pasteur was able to communicate the contagion to as many worms as he liked."[2] (2) "The six fore-feet of the worm have sharp hooks at their ends, by means of which the worms prick each other's skins":[3] in doing so they may be soiled with corpuscles which they in turn introduce into another body by pricking it. Disease was induced in this way by experiment. (3) "Infection at a distance, through the medium of the air and dust it carries is a fact equally well established."[3] Sweeping the breeding-houses may be sufficient. "When very healthy worms were placed in a breeding nursery at a considerable distance from unhealthy worms, they, in their turn, became infected."[4]

Lastly, an apparent exception had to be explained. The above experiments seemed decisive. "Nevertheless,

[1] *Ibid.*, p. 144. [2] *Ibid.*, p. 147. [3] *Ibid.* [4] *Ibid.*, p. 148.

among facts invoked in favour of non-contagion there was one which it was difficult to explain. There existed several examples of successful cultivations conducted in nurseries which had totally failed from the effects of *pébrine* the year before. The explanation is, as shown by Pasteur, that the dust can only act as a contagion when it is fresh. Corpusculous matter, when thoroughly dried, loses its virulence. A few weeks suffice to render such matter inoffensive: hence the dust of one year is not injurious to the cultivations of the next year. The corpuscles contained in the eggs intended for future cultivation alone cause the transmission of the disease to future generations."[1]

3. **Source of Power in Voltaic Pile.**—As an example of essentially experimental investigation, we will take Faraday's experimental proof of the theory that the source of power in a voltaic pile is due to chemical action. At the time he entered upon this investigation there were several hypotheses, "but by far the most important are the two which respectively find the source of power in contact, and in chemical force."[2] It is impossible to enumerate the enormous number of experiments Faraday made to settle this question. But the logical ground of them all was the same—to establish the hypothetical: 'If chemical action, then an electric current,' and its reciprocal 'If a current, then chemical action,' that is, 'If there is no chemical action, there is no current.' This he did by examining both positive and negative instances, that is, by applying the Method of Exclusions supported by Concomitant Variations. By numerous experiments he established the positive connexion, and showed that chemical action is both efficacious in producing a current, and sufficient by itself, and without any contact, to do so. For example: "When tin was associated with platinum, gold, or, I may say, any other metal which is chemically inactive in the solution of the sulphuret [of potassium], a strong electric current was

[1] *Ibid.*, p. 148.
[2] *Experimental Researches in Electricity*, Vol. ii., § 1796.

produced," and as the chemical action decreased and finally ceased in consequence of the formation on the tin of an "insoluble, investing, non-conducting sulphuret of that metal," the electric current diminished and finally ceased also.[1] Further, "when the chemical action changes, the current changes also.—This is shown by the cases of two pieces of the same active metal in the same fluid. Thus if two pieces of silver be associated in strong muriatic acid, first the one will be positive and then the other; and the changes in the direction of the current will not be slow as if by a gradual action, but exceedingly sharp and sudden."[2]

The negative experiments establishing the reciprocal proposition were also numerous—"Where no chemical action occurs no current is produced.—This in regard to ordinary solid conductors, is well known to be the case, as with metals and other bodies. It has also been shown to be true when fluid conductors (electrolytes) are used, in every case where they exert no chemical action, though such different substances as acids, alkalies and sulphurets have been employed. . . . But a current will occur the moment chemical action commences.—This proposition may be well established by the following experiment. . . . Two plates of iron and platinum are placed parallel, but separated by a drop of strong nitric acid at each extremity. Whilst in this state no current is produced . . .; but if a drop of water be added . . . chemical action commences, and a powerful current is produced, though without metallic or other additional contact."[3] Here we have appeal to the Method of Difference.

Faraday thus sums up: "With such a mass of evidence for the efficacy and sufficiency of chemical action . . .; with so many current circuits without metallic contact and so many non-current circuits with; what reason can there be for referring the effect in the joint cases where both chemical action and contact occur, to contact, or to anything but the chemical force alone? Such a reference appears to me most unphilosophical; it is dismissing a

[1] *Ibid.*, § 1882, cf. § 2031. [2] *Ibid.*, § 2036.
[3] *Ibid.*, §§ 2038-2039.

proved and active cause to receive in its place one which is merely hypothetical."[1]

Such a brief sketch gives but a faint idea of the thoroughness with which Faraday tested the two rival hypotheses, or of the wealth of evidence in favour of the theory of chemical activity which he brought forward; but it is, perhaps, sufficient to illustrate the fundamental point that *the logical character of the method is throughout an appeal to both positive and negative instances.*

4. **Argon.**—We will next examine the researches of Lord Rayleigh and Sir William Ramsay into argon, the recently discovered constituent of the atmosphere, as recorded in the Abstract of their Paper read before the Royal Society on January 31st, 1895, published in *Nature*,[2] from which our quotations are taken. This furnishes us with a very complete and beautiful illustration of the logical inductive method.

The investigation started from the detection of an unexplained residual phenomenon. Careful determination of density had shown that nitrogen obtained from various chemical compounds is of a uniform density, but that 'atmospheric' nitrogen is about $\frac{1}{2}$ per cent. heavier. Two hypotheses—each of which placed the explanation in the character of the lighter 'chemical' nitrogen, and were suggested by analogy drawn from experience in the chemical laboratory—were successively conceived to account for this phenomenon, and were rejected after being tested. "When the discrepancy of weights was first encountered, attempts were naturally made to explain it by contamination with known impurities. Of these the most likely appeared to be hydrogen, present in the lighter gas in spite of the passage over red-hot cupric oxide. But inasmuch as the intentional introduction of hydrogen into the heavier gas, afterwards treated in the same way with cupric oxide, had no effect upon its weight, this explanation had to be abandoned, and finally it became clear that the difference could not be accounted for by the presence of any known impurity. At this stage

[1] *Ibid.*, § 2053. [2] Vol. li., pp. 347-356.

it seemed not improbable that the lightness of the gas extracted from chemical compounds was to be explained by partial dissociation of nitrogen molecules N_2 into detached atoms. In order to test this suggestion both kinds of gas were submitted to the action of the silent electric discharge, with the result that both retained their weights unaltered. This was discouraging, and a further experiment pointed still more markedly in the negative direction. The chemical behaviour of nitrogen is such as to suggest that dissociated atoms would possess a high degree of activity, and that even though they might be formed in the first instance their life would probably be short. On standing they might be expected to disappear, in partial analogy with the known behaviour of ozone. With this idea in view, a sample of chemically prepared nitrogen was stored for eight months. But at the end of this time the density showed no sign of increase, remaining exactly as at first."[1]

Another hypothesis had, therefore, to be sought on the established ground that "one or other of the gases must be a mixture." But as, "except upon the already discredited hypothesis of dissociation, it was difficult to see how the gas of chemical origin could be a mixture . . . the simplest explanation in many respects was to admit the existence of a second ingredient in air from which oxygen, moisture, and carbonic anhydride had already been removed."[2]

This hypothesis, however, is immediately met by an objection which partially takes the form of a logical exception. "In accepting this explanation, even provisionally, we had to face the improbability that a gas surrounding us on all sides, and present in enormous quantities, could have remained so long unsuspected."[3] The answer to this objection was only fully found at a much later period of the investigation, when it was shown to be only an apparent objection, as the inert character of the newly discovered gas sufficiently accounts for its having so long escaped observation. It was necessary, however, to examine at once "the evidence in favour of the prevalent doctrine that the inert residue from air after withdrawal

[1] P. 348 *b*. [2] P. 349 *a*. [3] *Ibid.*

of oxygen, water, and carbonic anhydride, is all of one kind," as such evidence was logically an exception to the new hypothesis, and, therefore, if sustained, must prove fatal to it. This evidence rested upon the experiments of Cavendish, "whose method consisted in operating with electric sparks upon a short column of gas confined with potash over mercury at the upper end of an inverted U tube."[1] But it appears that these very experiments are evidence in favour of, instead of in opposition to, the hypothesis of an additional constituent in the atmosphere, for Cavendish records that "a small bubble of air remained unabsorbed" at the end of his experiments. The exception is, therefore, only apparent, and when exactly stated, helps to 'prove the rule.'

The ground being thus cleared, experiments were instituted to establish the new hypothesis. These were, of course, both positive and negative, the former being directed to establish the existence of a new constituent in 'atmospheric' nitrogen, and the latter to prove that this constituent is absent from 'chemical' nitrogen. The positive experiments, in other words, were directed to establishing the proposition 'If the excess of density of 'atmospheric' over 'chemical' nitrogen is due to the presence in the former of another gas besides nitrogen, then this other gas will remain after the nitrogen is withdrawn,' and the negative experiments to showing that this residue cannot be nitrogen, as it is not found when 'chemical' nitrogen is the gas operated upon.

The first mode of positive experiment was sparking air to which oxygen was gradually added in the presence of an alkali—that is, the method of Cavendish mentioned above. In every case there was a small residue, and "that this small residue could not be nitrogen was argued from the fact that it had withstood the prolonged action of the spark, although mixed with oxygen in nearly the most favourable proportion."[2]

The next step illustrates the function of number of instances in experiment—to ensure that extraneous elements

[1] *Ibid.* [2] P. 349 *b*.

have not crept in, that is, that only known conditions are present. It is thus described: "Although it seemed almost impossible that these residues could be either nitrogen or hydrogen, some anxiety was not unnatural, seeing that the final sparking took place under somewhat abnormal conditions. The space was very restricted, and the temperature (and with it the proportion of aqueous vapour) was unduly high. But any doubts that were felt upon this score were removed by comparison experiments in which the whole quantity of gas operated on was very small. . . . Several repetitions having given similar results, it became clear that the final residue did not depend upon anything that might happen when sparks passed through a greatly reduced volume, *but was in proportion to the amount of air operated upon.*"[1]

The second mode of positive experiment was to pass 'atmospheric' nitrogen over red-hot magnesium. A residue was left, and "on passing sparks for several hours through a mixture of a small quantity of this gas with oxygen, its volume was still further reduced. Assuming that this reduction was due to the further elimination of nitrogen, the density of the remaining gas was calculated to be 20·0."[2]

The third mode of positive experiment was by atmolysis, or the transmission of 'atmospheric' nitrogen through a porous substance in order to determine whether it is a mixture of gases of different densities. When such a mixture is thus treated the lighter gas passes through the porous substance first. When air had, therefore, undergone atmolysis, and the oxygen, ammonia, moisture and carbonic anhydride had been removed from that portion which had not passed through the porous substance, the remaining 'atmospheric' nitrogen should—on the hypothesis adopted for investigation—be of a greater density than ordinary 'atmospheric' nitrogen, as it should contain a larger proportion of the denser gas, and this was found to be the case in each of three such experiments. The positive experiments, therefore, appear to

[1] Pp. 349 *b*-350 *a*. [2] P. 350 *a*.

yield overwhelming evidence that 'atmospheric' nitrogen is a mixture of 'chemical' nitrogen and a gas of greater density, hitherto unknown, and to which the discoverers gave the name *Argon*.

A series of negative experiments was then entered upon, in which each of the first two series of positive experiments was performed upon 'chemical' instead of 'atmospheric' nitrogen. In each case a very small residue of argon was found, from one-ninth to one-fortieth of the amount that would have remained had 'atmospheric' nitrogen been used. Even this small residue could be accounted for. In the cases in which it was largest there had been leakage and air had entered, and in the others "the source of the residual argon is to be sought in the water used for the manipulation of the large quantities of gas . . . employed." This is supported by analogy: "When carbonic acid was collected in a similar manner, and subsequently absorbed by potash, it was found to have acquired a contamination consistent with this explanation."[1]

The negative experiments thus corroborate the positive ones, and the existence of argon is proved; thus the hypothesis is established and the first induction is completed.

The next object was to examine the nature of argon, and such an examination yielded many confirmations of the results already obtained, by showing that argon differs radically from nitrogen.

The first problem was to determine its density. This was first reached by deductive reasoning "on the assumption that the accurately known densities of atmospheric and of chemical nitrogen differ on account of the presence of argon in the former, and that during the treatment with oxygen nothing is oxidised except nitrogen."[2] This assumption was known to be justified because experiment had proved that 'chemical' nitrogen obtained from magnesium nitride, which had been prepared by passing 'atmospheric' nitrogen over ignited magnesium, was of the density usual to chemical nitrogen. "It is

[1] P. 350 *b*. [2] P. 351 *b*.

therefore seen that 'chemical' nitrogen, derived from 'atmospheric' nitrogen, without any exposure to red-hot copper, possesses the usual density."[1] The density thus obtained was 20·6. Here again we have the suggestion of a hypothesis by the quantitative computation of a residual phenomenon. This hypothesis was then to be tested experimentally. This could not be done by direct weighing, as it was not found possible to collect by the oxygen method enough of the gas to fill the large globe employed for other gases. So a mixture of about 400 c.c. of argon with pure oxygen was weighed, and the density of argon calculated from the result, and found to be 19·7. The argon derived from the second series of positive experiments was found to have a density of 19·9. As the density of argon is thus proved to be different from that of nitrogen, this determination supports the conclusion already reached that argon is a distinct constituent of atmospheric nitrogen.

A careful examination of the spectrum of argon by Mr. W. Crookes disclosed the fact that argon "gives two distinct spectra according to the strength of the induction current employed." Nitrogen does the same, "but while the two spectra of nitrogen are different in character, one showing fluted bands and the other sharp lines, the argon spectra both consist of sharp lines."[2] Additional confirmation of the existence of argon was thus furnished. But further, Mr. Crookes says: "I have found no other spectrum-giving gas or vapour yield spectra at all like those of argon,"[3] and this is a proof that argon is a gas previously unknown. But the fact that it yields two spectra suggested the hypothesis that argon is really a mixture of two gases. This is an instance of how the solution of one problem opens up others by bringing to light new phenomena requiring explanation. Other considerations bearing on this point will be noticed later on.

The solubility of argon in water was next determined, and was found to be approximately that of oxygen and about two and a half times that of nitrogen. This con-

[1] P. 348. [2] P. 354 *a*. [3] *Ibid.*

clusion was proved by testing the deductive inference that, if so, argon would be found in increased proportion in the dissolved gases of rain water; the test experiment showed that the argon in 'water' nitrogen is in the ratio of 24:11 to that in 'atmospheric' nitrogen.

Experiments were conducted by Professor Olszewski on the liquefaction and solidification of argon. "Four series of experiments in all were carried out, two with the object of determining the critical temperature and pressure of argon, as well as measuring its vapour pressure at several other low temperatures, while two other series served to determine its boiling and freezing points under atmospheric pressure, as well as its density at its boiling point."[1] It was found that the critical temperature is $-121°\cdot0$ (that of nitrogen being $-146°\cdot0$), its critical pressure 50·6 atmospheres (that of nitrogen being 35·0), its boiling point $-187°\cdot0$ (that of nitrogen being $-194°\cdot4$), and its freezing point $-189°\cdot6$, at which temperature it solidifies into white crystals (that of nitrogen being $-214°\cdot0$).

It further appears from experiments made on the velocity of sound in argon "that argon gives practically the ratio of specific heats, viz. 1·66, proper to a gas in which all the energy is translational. The only other gas which has been found to behave similarly is mercury gas, at a high temperature."[2] "In the case of mercury the absence of interatomic energy is regarded as a proof of the monatomic character of the vapour, and the conclusion holds equally good for argon. . . . Now a monatomic gas can only be an element, or a mixture of elements; and hence it follows that argon is not of a compound nature."[3]

But whether it is an element or a mixture of elements was still doubtful. "The behaviour on liquefaction, however, seemed to prove that in the latter case either the proportion of the subordinate constituents was small, or else that the various constituents were but little contrasted. An attempt, somewhat later, by Ramsay and J. Norman Collie to separate argon by diffusion into two parts, which should have different densities or refractivities, led to no

[1] P. 355 *a*. [2] P. 352 *b*. [3] P. 353.

distinct effect. More recently Ramsay and M. W. Travers have obtained evidence of the existence in the atmosphere of three new gases, besides helium, to which have been assigned the names of neon, krypton, and xenon. These gases agree with argon in respect of the ratio of the specific heats and in being non-oxidizable under the electric spark. As originally defined, argon included small proportions of these gases, but it is now preferable to limit the name to the principal constituent and to regard the newer gases as 'companions of argon.' The physical constants associated with the name will scarcely be changed, since the proportion of the 'companions' is so small. Sir William Ramsay considers that probably the volume of all of them taken together does not exceed 1/400th part of that of the argon."[1]

If argon is an element its atomic weight is 40, for its molecule is identical with its atom, and the molecular weight of a gas is double its density, which in the case of argon is approximately 20.

"From the manner of its preparation it was clear at an early stage that argon would not combine with magnesium or calcium at a red heat, nor under the influence of the electric discharge with oxygen, hydrogen or nitrogen. Numerous other attempts to induce combination also failed. Nor does it appear that any well-defined compound of argon has yet been prepared. It was found, however, by M. P. E. Berthelot that under the influence of the silent electric discharge, a mixture of benzine vapour and argon underwent contraction, with formation of a gummy product from which the argon could be recovered."[2]

The determination of the inert nature of argon—to which it owes its name—answered all objections to the hypothesis of its existence based on the fact of previous non-observation. "For more than a hundred years before 1894 it had been supposed that the composition of the atmosphere was thoroughly known. Beyond variable quantities of moisture and traces of carbonic acid, hydrogen, ammonia, etc., the only constituents recognised were

[1] Lord Rayleigh: Article on Argon, *Ency. Brit.*, 11th ed., p. 478 *a*.
[2] *Ibid.*, pp. 477 *b*-478 *a*.

nitrogen and oxygen. The analysis of air was conducted by determining the amount of oxygen present and assuming the remainder to be nitrogen. Since the time of Henry Cavendish no one seemed even to have asked the question whether the residue was, in truth, all capable of conversion into nitric acid."[1]

Even such an imperfect outline as the above makes abundantly manifest that induction is by no means an easy process, or one that can be reduced to mechanical rules; that the procedure starts from, and is guided throughout by hypotheses; that number of experiments is appealed to only as a guarantee that only known conditions are operative; that the procedure of perceptual analysis is to establish a positive connexion, to purge this of exceptions and to limit and corroborate it by negative instances; and that one inductive enquiry gives rise to others.

5. **Henry VIII and the Parliament of 1529.**—It will be fruitful to compare the exactness of experiment and the certainty of conclusion possible in such a research as that just considered, in which experiment can be used throughout to verify or to disprove hypotheses, with the uncertainty of speculation and the balancing of probabilities which are the sole resources of the labourer in social or historical science. As an example of historical reasoning we will take Mr. A. F. Pollard's examination of the assertion that at the election of 1529 the House of Commons was packed with royal nominees.[2] Only the salient features of the argument are indicated.

Mr. Pollard contends that the House was not packed, but that the elections were free. This is the hypothesis he sets out to establish. He points out that "the election of county members was marked by unmistakeable signs of genuine popular liberty. There was often a riot, and sometimes a secret canvass among freeholders to promote or defeat a particular candidate."[3] Two illustrations are given, one of "secret labour among the freeholders of Warwickshire," and the other of a "hotly contested election between the candidate of the gentry of Shropshire

[1] *Ibid.*, p. 475 *a*. [2] Pollard, *Henry VIII*, p. 252. [3] *Ibid.*, p. 252.

and the candidate of the townsfolk of Shrewsbury." The first is of date 1534, and only on the assumption that it is typical of the times can it be used as evidence for 1529; the second is not dated. There is also the assumption, based on analogy, that a "riot" and a "secret canvass" are "signs of genuine popular liberty" of election.

The next instance taken has also only the force of an analogy. "In 1547 the council ventured to recommend a minister to the freeholders of Kent."[1] The electors objected, and in spite of the council's protest that they had no wish to deprive the shire of its "liberty of election," the government candidate had to find another seat. Since this incident took place presumably after the death of Henry VIII it has no probative value and again assumes that the conditions in 1529 were sufficiently similar to make such instances possible, though there is an absence of direct testimony to them.

But phrases like 'popular liberty,' 'freedom of election' may mislead owing to analogies from modern times. It is, therefore, pointed out that the boroughs were often close corporations of few electors. They were on account of their numbers especially susceptible to external influence, and therefore to that of the king. To this it is replied that there was every variety of franchise. In some it was almost democratic. This is supported by the example of London. The election to the Parliament of 1529 took place in the presence of a large company in the Guildhall with "no hint of royal interference, the election being conducted in the customary way, namely, two candidates were nominated by the mayor and aldermen, and two by the citizens."[2] The argument is here from silence: the absence of any hint of royal interference is not in itself conclusive since we are not certain that if there had been interference, the record of it would have remained. The fact, however, that the election took place in the presence of a throng of citizen electors makes the theory of interference improbable.

However, "in many boroughs, elections were largely

[1] *Ibid.*, p. 252. [2] *Ibid.*, p. 253.

determined by recommendations from neighbouring magnates, territorial or official." It is suggested that the latter would be subservient to the king. But it is clear that the nominations were not royal, and there is no evidence to show that such classes were necessarily supporters of the king. Royal ministers, of course, would be. Apart from them members would represent the views of their patrons, and these might be opposed to the Court. That there was opposition is supported by the fact that "in 1539 Cromwell's agents were considering the advisability of setting up Crown candidates against those of Gardiner, Bishop of Winchester."[1]

A letter to Cromwell, written in 1529, is quoted in support of the theory that the House of Commons consisted of Royal Nominees. On examination this is shown to be inconclusive. Cromwell sought Henry's permission to serve in parliament in order to be useful to the king. If Henry accepted his offer he was to be nominated for Oxford. But his election there was not regarded as certain—an obvious limitation of the royal power. If not elected for Oxford he was to put up for a constituency in the diocese of Winchester. As a matter of fact he sat for Taunton. This is not conclusive against royal nomination. It does show, however, that there were difficulties in the way of its exercise. It is admitted that there was a small number of royal boroughs in which the king had only the same influence as an ordinary patron.

The next hypothesis to be refuted is that constituencies were created in order to further royal influence. Returns show that as a matter of fact there were six boroughs in 1529 which had never returned members before. Two explanations are suggested: (1) Almost all the six had increased in population or importance, and it is suggested that their representation was due to that. On the general principle relating to the creation of constituencies the reasons given are adequate. (2) But there is an alternative suggestion—that they were created to pack the House of Commons. This is negatived by considering the futility

[1] *Ibid.*, p. 254.

of creating half-a-dozen boroughs. For "so small a number of votes was useless, except in the case of a close division of well-drilled parties, of which there is no trace in the Parliaments of Henry VIII. The House of Commons acted as a whole, and not in two sections. 'The sense of the House' was more apparent in its decisions then than it is to-day. Actual divisions were rare: either a proposal commended itself to the House, or it did not; and in both cases the question was usually determined without a vote."[1] The analogies drawn from present-day procedure are rejected because there were fundamental differences of conditions. The rejection of the second explanation does not prove the first, but it strengthens it. In the absence of further alternative suggestions, the first has considerable probability from 'the nature of the case,' that is, as a deduction from general principles.

Further, the creation of boroughs to serve the king's interest must be shown to be motived by the king's political needs. The only reasonable ground would be an opposition of interests between king and parliament. But there was no need to pack the parliament on this score, since both were averse from war and both had grievances against the Church. Their secular and ecclesiastical interests were identical. The creation of boroughs is also shown to be unnecessary on the ground that parliament could not be a centre of discontent. The members were chiefly from the landed gentry and the prosperous commercial classes. Now the grievances of the time were social and economic, and not as a rule political. For example, complaints were rife as to the enclosing of lands and trading monopolies. Assuming that Henry was averse to measures for redress, he had no need to fear that parliament would propose them, for members were themselves the chief offenders in these matters. Thus Henry had no adequate motive for packing his parliament.

Lastly there *was* systematic interference in elections during Henry VIII's reign, why not then in 1529? This is met by the hypothesis that systematic interference was

[1] *Ibid.*, p. 255.

a later expedient due to Cromwell. As for the facts "it was apparently tried during the bye-elections of 1534, and at the general elections of 1536 and 1539."[1] The motive is found in the necessity which Cromwell was under of finding support in his struggle with "the reactionary party in the council." (Notice the question-begging epithet 'reactionary.') The fact that shortly after the policy of interference was adopted Cromwell was executed is noted as significant, and is alleged to support the author's contention on the ground of a tentative generalisation which is illustrated from later times. "The packing of Parliaments has in fact been generally the death-bed expedient of a moribund Government. The Stuarts had their 'Undertakers,' and the only Parliament of Tudor times which consisted mainly of government nominees was that gathered by Northumberland on the eve of his fall in March, 1553; and that that body was exceptionally constituted is obvious from Renard's enquiry in August, 1553, as to whether Charles V would advise his cousin, Queen Mary, to summon a general parliament or merely an assembly of 'notables' after the manner introduced by Northumberland."[2]

The above example illustrates the difficulty of coming to definite and convincing conclusions in the absence of full and adequate testimony. The facts on which the argument is based represent a selection made from contemporary documents. In his condensed narrative the historian does not analyse the general value of his sources, nor test the sincerity and accuracy of separate statements, probably because they are of such a kind that there is no motive for calling their genuineness in question. Taking the facts as they are reported, he shows how they support his hypothesis, and the conclusions drawn from it. The whole narrative is avowedly a construction which the author believes to represent the truth. His attitude to history can be best expressed in his own words. Referring to numerous sources for the facts of Henry VIII's reign made accessible in recent years, he says,

[1] *Ibid.*, pp. 260-261. [2] *Ibid.*, p. 261.

"These sources probably contain at least a million definite facts relating to the reign of Henry VIII.; and it is obvious that the task of selection has become heavy as well as invidious. . . . Yet selection is inevitable, and arrangement essential. The historian has no option if he wishes to be intelligible. He will naturally arrange his facts so that they spell what he believes to be the truth; and he must of necessity suppress those facts which he judges to be immaterial or inconsistent with the scale on which he is writing. And if the superabundance of facts compels both selection and suppression, it counsels no less a restraint of judgment. . . . Dogmatism is merely the result of ignorance; and no honest historian will pretend to have mastered all the facts, accurately weighed all the evidence, or pronounced a final judgment."[1]

6. **Wave Theory of Light.**—As a last example we will take the establishment of a wider generalisation embracing many empirical laws, and give in outline the steps by which the wave theory of light gradually won general acceptance. The steps cannot be given in detail because of the somewhat difficult mathematical reasoning involved.

The wave theory assumes the existence of a medium called the luminiferous ether through which light is propagated in waves. It was suggested by analogy with the theory of sound. Both light and sound were known to be reflected according to the same law, and it was further known that sound is propagated in waves through a medium such as air or wood. Hence the suggestion that light is propagated in a similar way. But wave-motion from its very nature cannot be conceived except in some continuous medium which transmits the motion. Now light reaches us from the stars. Consequently in order to account for the arrival at the earth of light from the stars we must assume, if we adopt the wave-theory, the presence in the intervening space of a continuous medium capable of transmitting waves. Since the properties of the medium

[1] *Ibid.*, Preface, pp. viii-ix.

cannot be directly apprehended by the senses we can only arrive at them by the indirect method of enquiry.[1]

Huygens, in his *Traité de la Lumière* published in 1678, gave the earliest important exposition of the Wave Theory of light. That theory in its present form assumes that a candle or gas flame, the incandescent filament of an electric glow lamp, a glowworm, or any other material body generating light, does so by the propagation of vibrations through the luminiferous ether. The vibrations are propagated in the form of waves in every direction in such fashion that the wave-front has the form of a sphere. Moreover, around each particle of a vibrating medium as a centre other spherical waves are formed. The ether is assumed to fill all space and to permeate all matter, to have a definite density, and to possess elasticity.

When Huygens wrote, two important empirical laws in relation to reflection and refraction had been established. If a ray of light is reflected from a perfectly smooth surface, the angle between the ray of light and the perpendicular drawn to the surface at the point where the ray reaches it, is equal to the angle between the reflected ray and the same perpendicular. Both reflected ray and incident ray lie in the same plane. On the other hand, if the smooth surface is that of a transparent medium, a second ray starts from the point of incidence and traverses the transparent medium. This is termed the refracted ray, and makes an angle with the perpendicular as before defined which bears a constant ratio to the angle of incidence. The value of the ratio in any particular case depends on the nature of the two media through which the light passes. Huygens showed that these laws of reflection and refraction follow at once if light is assumed to be a wave motion with a speed of propagation that is the same for each medium, but different for different media.

So far the theory was capable of explaining important phenomena of light, but as Huygens formulated it, there were other phenomena which it did not explain. It did not account for the fact that light travels in straight lines.

[1] See Edser, *Light for Students*, p. 286.

On analogy with sound it seemed as though light should be seen, as sound is heard, when a material obstacle intervenes. On this ground Newton rejected the theory. "If light consisted in vibrations it would, like sound 'bend into the shadow.'"[1] He preferred, therefore, the rival corpuscular theory that light consists of a great number of material particles travelling at a great speed and emitted by luminous bodies. This theory easily explained the rectilinear propagation of light since the particles themselves were conceived to move in straight lines. It also accounted for the laws of reflection and refraction, though it did not explain as well as did the wave theory how there could be both reflection and refraction in the case of a transparent medium. Two assumptions were made: first, that the luminiferous particles are perfectly elastic; second, that they travel faster in a denser medium. Granted the perfect elasticity, the laws of reflection could be easily deduced: granted the more rapid propagation in a denser medium, the laws of refraction at once followed. Thus the corpuscular hypothesis was moulded to fit the facts. But neither hypothesis gave a satisfactory explanation of all the facts. After considerable hesitation Newton pronounced in favour of the corpuscular theory, and his great authority caused the wave theory to fall into disrepute until it was revived at the beginning of the nineteenth century by Young, in England, and Fresnel in France.

To these men was due the first statement and development of the *principle of interference*. When any medium is set independently vibrating at two points very near together, the wave-motion with its centre at one point will overlap and interfere with the wave-motion which has its centre at the other. When the crests or the hollows of the waves from each source coincide they accentuate the separate effect of each, but where the crests from one meet the hollows from the other the separate effects are neutralised. In the first case the vibrations differ in phase by a wave length or some multiple of a wave length: in the second, they differ by a half wave length or some odd

[1] Cajori, *A History of Physics*, p. 87.

multiple of it. Now if light be due to wave-motion there should be cases in which waves from two sources of light interfere with one another so as to give increased brightness where the waves help one another, and to produce darkness where they act in opposite directions. Fresnel produced the phenomena by experiment. A beam of sunlight was introduced into a darkened room and focussed at a particular point. This point was reflected from two mirrors arranged at such an angle that the two streams of light followed a slightly different path, but near enough together to produce the phenomena of interference if the wave theory were true in fact. It was found that alternate bright and dark bands of colour were produced as the theory had led him to expect.

The experiment was varied by passing the white light through glass of different colours, but the results only served to confirm the principle of interference. With red light, for example, the bands were alternately red and black. Newton had been familiar with the phenomena in another form and had explained it by means of the corpuscular theory with the aid of one or two further assumptions as to the mode of action of his light particles. His explanation was "reasonable and ingenious," but one of his assumptions has been shown experimentally to be unwarranted. "Fresnel's experiment gives decisive evidence in favour of the wave theory of light. That light when added to light should produce darkness is incomprehensible on any theory of the material nature of light." [1]

But the difficulties of the wave theory were so great in themselves that it did not appear decisive at the time. Fresnel had, however, lessened them by showing that the approximate rectilinear propagation of light could be explained by wave-motion. Mathematical considerations prove that on the assumption that the length of a light wave is very small in comparison with ordinary magnitudes the path of a ray of light would not differ appreciably from a straight line. It can be shown by independent methods "that, as a matter of fact, the wave length of

[1] Edser, *Light for Students*, p. 323.

light varies between $0{\cdot}4 \times 10^{-3}$ mm. and $0{\cdot}8 \times 10^{-3}$ mm., according to its colour,"[1] and this length is of the requisite degree of smallness.

But in strict theory "waves of such lengths should exhibit a slight tendency to bend round corners, and the effects of this should be observable under appropriate conditions."[2] If light travelled in straight lines the shadows cast by objects not illuminated by other sources of light should have clear cut outlines, and be of definite geometrical shape. Grimaldi in the seventeenth century had observed phenomena at variance with this deduction. He introduced a pencil of light into a dark room through a small hole. "The shadow cast by a rod held in the cone of light was allowed to fall upon a white surface. To his surprise he found the shadow wider than the computed geometrical shadow; moreover, it was bordered by one, two, and sometimes three coloured bands. When the light was very strong, he saw, in addition, coloured bands inside the shadow itself. On replacing the rod by an opaque plate with a small hole in it, the illuminated circle was found larger than it should have been on the supposition that the rays travelled past the edges of the hole in exactly straight lines. This and other experiments established the fact that light bends very slightly round a corner. He called the new phenomenon 'diffraction.'[3] Diffraction was explained on the emission theory "by means of hypothetical laws of attraction and repulsion between the light corpuscles and the edges of the object causing diffraction."[4] At a later date the phenomenon was studied experimentally, and the facts were found to be in accordance with deductions from the assumed propapagation of light in waves through ether as a medium. Again, both hypotheses explained the phenomenon.

A further difficulty in the way of accepting the wave theory arose from the polarisation of light. If a ray of light be passed through a rhomb of Iceland spar it will

[1] Edser, *ibid.*, p. 427. [2] *Ibid.*, p. 427.
[3] Cajori, *History of Physics*, p. 88. [4] *Ibid.*, p. 143.

take two different paths. One of the two rays is called the *ordinary* ray, the other the *extraordinary*. It is the latter which is said to be polarised. Malus discovered that similar phenomena are produced by reflection. "At this time no explanation of polarisation had been given by the wave theory, which was in great danger of being overthrown by the new mass of evidence furnished by Malus. Thomas Young wrote in 1811 to Malus, who was a pronounced partisan of the emission theory: 'Your experiments demonstrate the *insufficiency* of a theory (that of interferences), which I had adopted, but they do not prove its falsity.'"[1] In other words, he did not feel called upon to abandon his hypothesis, but felt that he must seek some modification of it which would explain the new facts brought to light. As the wave theory then stood, the wave-motion of light was conceived as similar to that of sound, and therefore the vibrations were thought to be only longitudinal, that is, in the direction of the ray. This assumption did not account for the direction of the polarised ray. But Fresnel assumed that in this case the vibration was in a direction transverse to the ray, and then showed that the consequences of this assumption were borne out by the facts as experimentally determined. Thus with a slight modification the wave theory was proved to explain adequately all the phenomena of polarisation.

But the corpuscular hypothesis was not yet dead. For example, when the rich colours produced by polarised light were discovered by Arago in 1811, "partisans of the two rival optical theories hastened to find explanations of this phenomenon. . . . On the undulatory theory explanations were given first by Young, then more fully by Arago and Fresnel. On the corpuscular theory, the facts were accounted for by Biot in a complicated research of great mathematical elegance."[2] Moreover Brewster, who did much to further the study of polarisation in crystals, asserted that "his chief objection to the undulatory theory of light was that he could not think the Creator

[1] Cajori, *op. cit.*, p. 146. [2] *Ibid.*, p. 147.

guilty of so clumsy a contrivance as the filling of space with ether in order to produce light."[1]

The claims of the rival hypotheses as alternative explanations were not decided until 1850. "Newton had deduced the laws of refraction from his corpuscular theory of light by assuming that the luminiferous particles travelled faster in a denser medium. . . . The wave theory reverses this. It also explains the phenomenon of refraction as depending on the different velocities in the two media; but it requires that the velocity should be less in the denser medium."[2] Foucault devised an apparatus by means of which the velocity of light in air and in water was directly measured. He found that the velocity was greater in air than in water. But water is the denser medium, and on the corpuscular hypothesis light should travel faster through it. The experiment, therefore, definitely disproved that hypothesis.[3]

Since then the wave theory has been regarded as established. But it must be noted that Foucault's experiment does not prove it: it only disproves the rival hypothesis, and shows that the wave theory, if true, accounts for the empirical law of refraction. To establish the wave theory beyond cavil it would be necessary to show that no other hypothesis explains the facts; though none other has been suggested, it would not be safe to say that no other is possible. However, the probability of the truth of the wave theory is very great owing to the wide range of empirical laws which can be deduced as consequences from it. The greatest problem which the theory raises is the nature of the ether which it postulates. To form an adequate conception of a medium filling all space, with some of the properties of a solid and yet not perceptible to the senses, is a task which waits for its fulfilment in the future.

[1] Tyndall, *Six Lectures on Light*, p. 49, *cit.* Cajori, *op. cit.*, p. 148.

[2] Eggar, *Wave Motion, Sound, Light*, p. 530.

[3] Cf. also pp. 351-352.

CHAPTER XXXIII.

QUANTITATIVE DETERMINATION.

1. **Measurement.**—We have considered in the preceding chapters how the existence of a necessary relation between phenomena can be established. But all perceptible facts are measurable, and, consequently, a relation between them is stated with perfect definiteness and exactness only when it is expressed in quantitative form. We must not only be able to say generally that *If S is A it is X*, but that for every definite value a_1 of A, there is a corresponding definite value x_1 of X. Thus, as Mach says, "The laws of nature are equations between the measurable elements of phenomena."[1]

No doubt the attainment of precise mathematical statement of law is only possible in the most advanced branches of science. Every branch of knowledge is at first merely qualitative, or at best very roughly quantitative. The ancient Chaldean astronomers, for example, were satisfied to record an eclipse to the nearest hour, but now the time is fixed to a very small fraction of a second. As the power of exact measurement is increased, science advances; and thus the invention of a new and more exact instrument of measurement has, as Jevons remarks, "usually marked, if it has not made, an epoch" in the branch of science to which it is applicable. This dependence of the advance of science on progress in the power of measurement is forcibly illustrated by De Morgan. "Had it not been," he remarks, "for the simple contrivance of the balance, which we are well assured (how, it matters not here), enables us to poise equal weights against one another, that is, to detect

[1] *Science of Mechanics*, p. 502.

equality and inequality, and thence to ascertain how many times the greater contains the less, we might not to this day have had much clearer ideas on the subject of weight, as a magnitude, than we have on those of talent, prudence, or self-denial, looked at in the same light."[1] Indeed, as scientific thought is exact thought, physical science cannot strictly be said to exist except in a quantitative form. "Numerical precision . . . is the very soul of science; and its attainment affords the only criterion, or at least the best, of the truth of theories, and the correctness of experiments. . . . Indeed, it is a character of all the higher laws of nature to assume the form of precise *quantitative* statement."[2]

The laws of nature are, however, expressions of the relations between phenomena. Now, every concrete fact is complex; that is to say, it is due to the co-operation of several conditions, it is the embodiment of a plurality of relations. Its whole nature, therefore, can never be expressed by one law, for each law is the statement of only one of these relations. Hence, each law is highly abstract, and is hypothetical in essence, even when expressed in a categorical form, as it states not what does happen, but what would and must happen under certain defined conditions. But it is frequently impossible to secure these conditions unmixed with other and interfering conditions. For the relations which meet in a concrete fact are not independent, and they frequently modify or counteract each other. Thus, it happens, that many quantitative laws—as, *e.g.* the law of inertia—are never exactly realised in fact. Such a law obviously cannot be simply read off from the facts which more or less fail to fulfil it. It is a hypothesis of the quantitative relation which would hold did it exist alone, and its truth is established indirectly by showing that the actual deviations from it can themselves be calculated as the effect of the interference of other conditions.

This becomes more evident, however, when we remember

[1] *Formal Logic*, p. 175.
[2] Herschel, *Discourse on Natural Philosophy*, §§ 115-6.

that facts are known to us only as they are observed by us, and that all observation is determined not only by the fact but by the observer. All observation, whether made with or without instruments, rests at bottom upon the accurate interpretation of sense impressions; for instruments and other artificial means of improving observation can do nothing but bring the observed object, by breaking it up, magnifying it, and so on, within that range of moderate quantities which can be gauged by our senses. But how erroneous our interpretation of sense impressions may be is evident to anyone who considers, for instance, how much larger the moon appears to us when it is just above the horizon than when it is in its zenith, or again how the shape or size of the same object seems to vary with its distance from, and relative position to, the observer.

It is true we never attempt to estimate absolute, but only relative, magnitude; in other words, all measurement is comparison with a standard unit. What character of unit should be employed for the measurement of any kind of quantity is a matter of the greatest practical importance, but it is one which concerns the particular branches of knowledge, not general logical theory. The recognition of this comparative character of measurement, however, makes clear that its accuracy is relative to our power to distinguish differences by our senses. And this power is limited, even with the most delicate instruments. "Few measurements of any kind," says Jevons, "are exact to more than six significant figures, but it is seldom that such accuracy can be hoped for."[1] When, then, it is said that two magnitudes are equal, all that is meant is that our most exact instruments fail to make apparent any difference between them; but it does not follow that with more delicate instruments some difference would not appear. Thus a rough balance would indicate equality between two weights which a more delicate instrument would show to differ. A medical prescription could not be satisfactorily made up with a pair of grocer's scales.

Further, the actual correctness of measurements even

[1] *Principles of Science*, p. 303.

within this limit of possible accuracy cannot be assumed. This is evident from the fact that two careful measurements of the same magnitude never exactly agree, and, therefore, one or both are erroneous. Indeed, "we may," as Jevons says, "look upon the existence of error in all measurements as the normal state of things."[1] Now 'error' means a discrepancy between the magnitude given by actual measurement, and that estimated as the application of the law to this particular case. It might seem, at first sight, that the law was, therefore, shown to need modification and restatement. But this is not necessarily the case. If, as the experiment is repeated more and more carefully, the results approximate more and more to the law, then the law is confirmed: exact coincidence must never be expected. For instance, a student "learns that two pints of steam at a temperature of 150° Centigrade will always make two pints of hydrogen and one pint of oxygen at the same temperature, all of them being pressed as much as the atmosphere is pressed. If he makes the experiment and gets rather more or less than a pint of oxygen, is the law disproved? No; the steam was impure, or there was some mistake. Myriads of analyses attest the law of combining volumes; the more carefully they are made, the more nearly they coincide with it." Or again, "The place of a planet at a given time is calculated by the law of gravitation; if it is half a second wrong, the fault is in the instrument, the observer, the clock, or the law; now, the more observations are made, the more of this fault is brought home to the instrument, the observer, and the clock."[2]

It thus follows that our laws are more exact than our observations and experiments can ever be, and that, consequently, they cannot be derived from mere comparison of the results of these observations, but are hypotheses which are more or less exactly verified by those experiments. Indeed, even were our measurements exact, yet they are only of isolated quantities, while the law gives the general formula of relation which claims to be applic-

[1] *Op. cit.*, p. 357. [2] Clifford, *Lectures and Essays*, pp. 91-92.

able to all other degrees of magnitude of the same phenomenon. The law is thus seen to be a result of the synthesising activity of mind, an activity which leads us to think of all nature as constituted by universal and definite laws.

As, then, every measurement is more or less inaccurate, the question next arises as to whether, and how far, such *error can be explained. In some cases it can; allowance* may be made for the influence of modifying conditions when they are known to exist; *e.g.* for friction in any experiment dealing with the law of inertia; for temperature in experiments with the pendulum, or in measuring with metal rods. In other cases, there is a known and constant source of error in the instrument by means of which the observation is made—*e.g.* a clock may be known to be a second too fast or too slow. Again, there is with all observers a 'personal equation' which is practically constant, as it is always in the same direction and of the same average amount; that is, one observer will always note the time of an observation a little earlier than another. Thus, "the difference between the judgment of observers at the Greenwich Observatory usually varies from $\frac{1}{100}$ to $\frac{1}{3}$ of a second, and remains pretty constant for the same observers."[1] The average amount of this personal error may then be calculated and allowed for.

But there are other errors which cannot be explained. They are due to causes which are unknown, or whose influence is so inconsistent that it cannot be calculated. They may result from the operation of some law not known to the observer, or to the imperfection of man's *powers of observation and of his instruments. On this* we will quote an excellent passage from De Morgan's *Essay on Probability*: "To note a measurable phenomenon without any error at all, would require sight and touch by which every magnitude, however small, could be perceived and correctly estimated. Such senses belong to no one, and the degree of approach towards perfection not only varies with the observer, but is different at different times with the same observer. Many errors to which

[1] Jevons, *op. cit.*, pp. 347-348.

instruments are subject ought in strictness to be classed under the first head; if, for instance, an astronomical circle gradually change its form, or undergo daily expansion and contraction by variations of temperature, the diversity of results which such a piece of brass will show are certainly subject to laws, and might be predicted, if we possessed sufficient knowledge of the constitution of the metal, and the laws which regulate the effect of pressure, temperature, moisture, etc., upon it. But so long as such laws are unknown, and the variations do not follow any distinguishable rule, their effect upon general results differs in nothing perceptible from that of the observer's own errors, with which they are mixed up in the particular results of observation."[1] When, therefore, allowance has been made for all known sources of error, there yet remain discrepancies whose causes, character, and magnitude are unknown. We must, then, consider briefly how the scientific enquirer seeks to eliminate such chance errors.

There are two broad cases: first, when the effective conditions of a phenomenon are unknown; secondly, when measurements actually made differ slightly from each other. In the former, appeal is made to the Theory of Probability; in the latter, to methods of eliminating the influence of sources of error in measurements.

2. Probability.

(i) **Basis of the Theory.**—When, in any particular case, we do not know what conditions are operative, we cannot tell, on the one hand, what result will appear, nor, on the other, can we say positively what conditions have produced a certain given event. In such cases we are accustomed to speak of the occurrence as due to chance. But our whole conception of the unity of nature forbids the idea that any element of reality can be really casual. Every detail is, in the strictest sense, necessary, and determined absolutely by conditions—all is causal, nothing is casual. Were our knowledge complete, then, the idea of chance would disappear; it is due solely to the imperfection

[1] Pp. 130-131.

of that knowledge. This imperfection is, of course, greater in some cases than in others; it may affect the event as a whole, or it may affect only some particular aspect of it. But, even with imperfect knowledge, we are often called upon to come to a decision or to act. The question then arises as to what we ought rationally to expect.

The estimation of such rational expectation is the province of the Theory of Probability. Probability is, thus, seen to be subjective in the sense that, when we say that the probability that an event will happen in a certain way is $1/n$, what we mean is that the relative amount of knowledge and ignorance we possess as to the conditions of the event, justify that amount of expectation. The event itself will happen in some one definite way, exactly determined by causation; the probability does not determine that, but only our subjective expectation of it. Indeed, without the assumption of the necessity of the actual event, there would be no basis for the calculation of probability. The very foundation of the theory is that possibilities can be limited to a definite finite number. We know, in other words, that the general relation *If S is A it is X* holds; but we do not know what particular form —x_1 *or* x_2 *or* . . . x_n—X will take, because we do not know in what exact form—a_1 *or* a_2 *or* . . . a_n—the general condition A will be realised, though we are bound to assume that every definite form a_n necessitates one perfectly definite form x_n.

It is from this combination of knowledge and ignorance that the calculation of probability starts. Its logical basis is thus seen to be the disjunctive judgment. But certain conditions must be fulfilled by the alternative specifications which form its predicate. They must first, as has been said already, be exhaustive of the whole range of possibility. Secondly, they must be definite, and therefore, in agreement with the principle of identity. Thirdly, they must be mutually exclusive. As we saw in our discussion of the disjunctive proposition, the form of the proposition does not guarantee this exclusiveness though the nature of the alternatives often does. But where there is any doubt on the matter, the disjunction may always be so written

as to make all the alternatives *formally* exclusive—thus *S is either* $m\bar{n}$ *or* $\bar{m}n$ *or* mn expresses, with formally exclusive predicates, the disjunctive proposition *S is either m or n*. Lastly, the alternatives must be of equal value, *i.e.*, if they are given—as they always may be—as alternative consequents in a hypothetical proposition, those consequents must be equally likely.

If the disjunctive proposition is given in categorical form, this condition postulates that the alternative predicates either shall be equal and co-ordinate specifications of the common genus, or shall refer to equal extents of denotation. Thus, if an urn contains one black and six white balls, the disjunction that a ball drawn will be either white or black is doubtless correct; but it is not a basis for the calculation of probability, for black and white are not specifications of equal value of the genus ball. Or, stating the proposition in hypothetical form—If a ball is drawn, it is either white or black—the consequents are not equally likely, owing to the preponderance in number of the white balls. And generally, this condition of equivalence can never be fulfilled when one of the alternatives simply negatives the other, owing to the indefinite range of reference of the negative term. Indeed, such a disjunction would not fulfil the second condition given above, which requires that each alternative shall be definite.

But it is necessary to enquire into the justification for regarding the alternatives as equally likely. This justification must be found in our ignorance as to what precise conditions will be operative. For example, if a penny is tossed, it will fall with either head or tail uppermost. Now, which will be uppermost in any particular throw will be exactly determined by such conditions as the position of the coin at starting, how it is grasped in the fingers, the force and direction of the twist, etc. But what special form these conditions will take we are totally ignorant. We know that if S (the coin) is tossed (A), it will fall with one side uppermost (X); but as we do not know the form a_n which A will take, we do not know whether X will appear as x_1 (heads up) or x_2 (tails up).

There is, thus, in every such calculation a basis of

knowledge. We know the coin will lie on one of its sides and not on its edge; but we have no reason to expect one side rather than the other to be uppermost, that is, we have no reason to believe the chances to be unequal. But further, we also know that in a long series of throws the sides which come upwards will succeed each other very irregularly, and yet with an approximation to equality in frequency. We have thus, in addition, objective and positive reason to believe that the chances for head and tail are at any rate approximately equal.

Experience thus tends to verify the hypothesis of equal probability to which we are led on the subjective ground that we know no reason to expect one result rather than the other. For when this is the case, we expect that these unknown conditions will in the long run make as often for one result as for the other, and, therefore, that the possible results will be realised with equal frequency. Of course this can never be exactly verified, as that would involve the examination of an infinite series of occurrences, which is impossible. To stop in a series of observations—*e.g.* of the throws of a penny—when the possible events have all happened an equal number of times, would obviously be to beg the question; for a continuance of the series would, in all likelihood, show a divergence from equality. But as, in a long series, the approximation to equality is always fairly close, experience may be said to give as full confirmation of the theory as the case permits.

The calculation of probability is, nevertheless, not dependent upon the actual experience of any series. It is possible, not only when we have grounds for believing the chances to be equal, but when we have no reason for assuming them to be unequal. In fact, our whole data is knowledge of the number of equivalent possible cases, and the absolute absence of any ground for preferring one rather than the others. We may calculate the probability just as well when only one of the possible cases can be actually realised—*e.g.* *one* throw of a penny—as when we are dealing with a series in which all the alternatives will be actually realised an approximately equal number of times.

It is said sometimes that the idea of equally likely alternatives is based upon equality of belief in their occurrence. This is to make the question too entirely and individually subjective. The belief of any individual is influenced by many conditions, some of which are generally more or less irrational and independent of any valid justification. Thus, belief in the possible occurrence of any event would vary with different individuals. But the probability does not vary with it. It is not, therefore, actual belief, but what ought to be believed, that is defined by probability. And what ought to be believed is but another name for rational expectation, that is, expectation based upon the proportion of available knowledge to ignorance. Probability is, therefore, a measure of knowledge, and so defines what ought to be believed, instead of being based upon what actually is believed.

From what has been said, it is evident that the amount of probability is not affected by considerations of time. Whether the calculation refers to an event in the past or in the future, or to the truth of a universal proposition, is immaterial; it is in every case based on the same principles. No doubt most illustrations of the doctrine are drawn from games of chance, as such games exemplify the conditions in the simplest way, and these have generally a reference to future expectation. But our knowledge and ignorance may be combined with reference to a past event in the same way as to one in the future, and we must calculate the probability of the reality of that event on the same principles. Thus, the theory of probability is applicable to the credibility of testimony, as well as to the prediction of a future occurrence. No doubt, when an event has occurred, we can frequently get more exact knowledge of it, and then, of course, the need for the calculation of its probability is, to that extent, removed; but so long as we do not know exactly what has happened, but only that one out of a certain number of alternatives has occurred, the appeal to probability remains.

The distinction so frequently drawn between probability before and after the event is due partly to the neglect of this consideration and partly to failure to make the alter-

natives equal in value. Thus, for example, it is extremely improbable that a hand at whist should consist entirely of trumps. Yet the probability of this is no less than that of any other one definite hand. It is its interesting character which draws special attention to it, and causes us to recognise how enormous are the odds against it. This we do not recognise in the case of other hands. If, then, a person told us he had held a hand of thirteen trumps the previous evening, we should probably feel more hesitation in believing him than if he told us he had held a hand consisting of certain definitely named cards. Yet the antecedent improbability would be no greater in the one case than in the other. Were the person, however, to claim that he had, previously to playing, written down the contents of a certain hand, and that he had actually been dealt that hand, we should probably hesitate to receive his statement just as much as if he told us he had been dealt thirteen trumps; for the previous defining of the hand would have made the odds against it as apparent as in the other case.

When we hesitate to believe the statement of such coincidences it is because we feel that the odds against the occurrence were antecedently very great, and we balance that with the odds in favour of the credibility of the witness. If we do not doubt his credibility, we receive the statement in spite of its antecedent improbability, for to assume that the extremely improbable is impossible is to fall into a dangerous fallacy. The way in which we use our terms in discussing such a case shows that probability and improbability are not opposed terms in the theory of probability; improbability only means a very low degree of probability. It is, however, very vague in its reference; so, in all calculations, the probability only is spoken of, and this may, of course, be very high or extremely low.

(ii) **Estimation of Probability.**—The full exposition of the applications of the theory of probability would take us far beyond our province into the region of mathematics. We are here concerned only with the logical character of the theory, and we will pursue its developments only so far as is necessary to show how it is based

upon various modes of combining disjunctive judgments, and deductive reasoning from those combinations. The primary difficulty always is to secure in each case a full statement of exhaustive, definite, exclusive, and equally likely, alternatives; and it is in the determination of this that the mathematical doctrine of combinations and permutations is mainly appealed to.

(*a*) *Probability of Simple Events.* If we have to consider simply one set of alternative and co-ordinate possibilities, our data may be expressed in the single disjunctive proposition A *is* a_1 *or* a_2 *or* . . . . a_n, where we have n alternatives fulfilling the four conditions enumerated above. As there is no reason for preferring any one of these alternatives, the probability of each is equal. But as the alternatives are assumed to exhaust all possibilities, it is obvious that their sum must be equivalent to certainty, for we know A will be realised and are only ignorant as to which of the forms $a_1\ a_2$ . . . a_n that realisation will take. This certainty is most appropriately represented by unity. The probability of each of the n co-ordinate alternatives $a_1\ a_2$ . . . a_n is, therefore, represented by the fraction $1/n$. On the same lines, impossibility, *i.e.* the absence of any chance of realising A in any form, would be represented by o/n, *i.e.* by zero.

Now, if there are n possibilities, the chances against the realisation of any particular one are obviously $n - 1$; for the sum of the chances for and against must be the total number of chances as $n/n = 1$. This is expressed by saying the *odds* against any one particular case are $n - 1$ to 1. Now, if a number of similar but distinct possibilities are massed together as one alternative, that alternative must be given its true value. For example, if there are n balls in an urn, of which one is white and the rest black, then the chance of drawing black is $(n - 1)$ times that of drawing white, and the probabilities are not equal, but are represented by $1/n$ for white, and $(n - 1)/n$ for black.

(*b*) *Probability of Compound Events.* By a compound event is meant one in which two or more simple events occur in connexion with each other. The logical basis of

the probability of such an event will, therefore, be found in the combination of two or more disjunctive judgments, fulfilling the conditions enumerated above. The greatest care must be taken in the statement of the alternatives to make them both exclusive and exhaustive.

(1) *Independent Events.* If we have two independent disjunctive judgments, *i.e.* judgments whose predicates are neither necessarily connected, nor necessarily incompatible, with each other—

A is either v or w
A is either x or y or z,

then by combining them we get the judgment

A is either vx or vy or vz or wx or wy or wz.

Similarly, if the disjunctive propositions are also hypothetical—

If S is a, it is either v or w
If S is b, it is either x or y or z,

then the combination will give

If S is both a and b, it is either vx or vy or vz or wx or wy or wz.

In each case the number of alternatives in the joint proposition is the product of the number of those in the single propositions, and as a result, the probability of the joint event is equal to the product of the probabilities of the simple events which it embraces. Thus, in the above symbolic examples, the probability of v is $1/2$, and that of x is $1/3$, whilst the probability of vx is $1/6$. Generally, if the probability of an event, A, is $1/m$, and the probability of another event, B, is $1/n$, then the probability that A and B will both occur is $1/mn$. But, as the direct basis of the joint probability is a single disjunctive proposition, the distinction between single and compound events is seen to be somewhat arbitrary, and to be nothing more than a distinction between two ways of regarding an occurrence.

As an example we will take the following simple case. Let there be two urns each containing three balls, of which two are white and one is black. What is the probability of drawing a black ball from each? It might seem at first

sight as if the two drawings give only four alternatives, viz. *ww, wb, bw, bb,* where *w* and *b* denote white and black respectively. But this is to neglect the fact that there are twice as many white as there are black balls; in other words, the alternatives are not of equal value. If we symbolise the contents of the first box as w_1, w_2, b_1, and that of the second as w_3, w_4, b_2 we then have the separate disjunctives, in which this objection is removed—

$$A \text{ is either } w_1 \text{ or } w_2 \text{ or } b_1$$
$$B \text{ is either } w_3 \text{ or } w_4 \text{ or } b_2$$

Then the combined disjunctive is

$$AB \text{ is either } w_1w_3 \text{ or } w_1w_4 \text{ or } w_1b_2 \text{ or } w_2w_3 \text{ or } w_2w_4 \text{ or } w_2b_2 \text{ or } b_1w_3 \text{ or } b_1w_4 \text{ or } b_1b_2$$

where the number of alternatives is seen to be nine, only one of which is *bb*, and, consequently, the probability of drawing two black balls is 1/9, which is in accordance with the general rule given above. If we examine the other alternatives in the final proposition we see that there are four cases of two white balls; therefore, the probability of drawing a white ball both times is 4/9. This is also in accordance with the general formula; for the probability of white in each case separately is 2/3, and, therefore, the probability of white in both cases is $2/3 \times 2/3 = 4/9$. Lastly, as the probability of white is in each case 2/3, and that of black is 1/3, so 2/9 is the probability both that the drawings will give first white and then black, and that they will yield first black and then white; this is shown to be true by the fact that the final proposition yields two cases in which white is followed by black and two cases in which black is followed by white. The sum of these probabilities is, of course, unity, as one of the alternatives must occur. The general principle is thus illustrated at every point.

A similar example is found in the throwing of dice. If one die is thrown the probability that it will fall with the side bearing six pips uppermost is 1/6, as the die has six sides, and the probability of being uppermost is equal for them all. If a second die is thrown, the probability that in that throw six will be uppermost is also 1/6; con-

sequently, the probability that six will be thrown in each of two throws is $1/6 \times 1/6 = 1/36$. In this case, it is obviously immaterial whether two dice are thrown simultaneously or whether the same die is thrown twice successively. So in the former example, whether the two drawings are made from two urns each of which contains two white balls and one black one, or successively from one of those urns, makes no difference, so long as the constitution of the one urn is kept constant by returning after the first drawing the ball which had been drawn. This again illustrates the fact that the distinction between simple and compound events is an artificial one.

(2) *Dependent Events.* This leads us to the case in which the events are not independent. This, however, in no way differs in principle from the one we have just considered, though still greater care is required in stating the alternatives. The case, stated generally, is when the second event depends in some way on the first, so that the probability of the realisation of the second is modified by the occurrence or non-occurrence of the first.

Perhaps, this will be best understood if we begin with the examination of an example. Starting, then, with the same supposition as before, of an urn containing three balls, two of which are white and one is black, we will ask what is the probability of drawing white twice in succession, if the ball extracted at the first drawing is not returned? In this case the constitution of the box is obviously different at the second drawing from what it was at the first, and the probability of the second drawing yielding white depends upon the result of the first drawing. If black is drawn the first time, for example, nothing but white balls remain in the urn, and the second drawing must, therefore, certainly yield white. Whilst, if white is drawn the first time, there remain only one white and one black ball in the urn, and the chance of drawing white at the second draw is, in that case, 1/2. This last case is the only one which concerns us; as in the former, the first drawing yielding black obviously makes it impossible to get white at both the first two drawings. The two consecutive drawings from the same urn are, therefore,

identical with a simultaneous drawing from two urns, one of which contains two white balls and one black ball, and the other contains one white and one black ball. The disjunctions may, therefore, be written—

$$A \textit{ is either } w_1 \textit{ or } w_2 \textit{ or } b_1$$
$$B \textit{ is either } w_3 \textit{ or } b_2$$

where w_3 represents in the case of the single urn whichever of the white balls, w_1 *or* w_2, is not extracted at the first drawing. Combining these we have

$$A\,B \textit{ is either } w_1w_3 \textit{ or } w_1b_2 \textit{ or } w_2w_3 \textit{ or } w_2b_2 \textit{ or } b_1w_3 \textit{ or } b_1b_2$$

and there are seen to be six alternatives, including two cases in which two white balls are drawn. The probability of this drawing is, therefore, $2/6 = 1/3$. Similarly the probability of drawing two black balls is seen to be $1/6$. The same result is reached if we reason in this way—The chance of drawing white the first time is $2/3$. But only if white is drawn the first time is the second drawing proceeded with. Hence the probability of the second drawing taking place at all is $2/3$, and the probability of that drawing yielding white is $1/2$, as it is made when the urn contains only one white and one black ball. Hence, the probability of a second white ball being drawn is $2/3 \times 1/2 = 1/3$.

A similar example is the enquiry into the probability that if a penny is tossed up three times, head will fall uppermost each time. The probability that head will appear at the first throw is $1/2$, as head and tail are the only two alternatives. The probability of the second toss taking place is, therefore, $1/2$, and the probability of its yielding head is $1/2$. The probability of two consecutive heads is, therefore, $1/4$. But only when this is secured does the third throw take place; the probability of there being a third throw is, therefore, $1/4$, and the probability that it will give heads is $1/2$. Therefore, the probability of three consecutive heads is $1/8$. The original disjunctions are—

$$A \textit{ is either } h_1 \textit{ or } t_1$$
$$B \textit{ is either } h_2 \textit{ or } t_2$$
$$C \textit{ is either } h_3 \textit{ or } t_3$$

and the combination is

ABC is either $h_1h_2h_3$ or $h_1h_2t_3$ or $h_1t_2h_3$ or $h_1t_2t_3$ or $t_1h_2h_3$ or $t_1h_2t_3$ or $t_1t_2h_3$ or $t_1t_2t_3$

where it is seen that only one case out of eight secures three heads.

Thus, the probability of the conjoined event is seen to be in this, as in the former case, the product of the probabilities of the separate events, and this is found to hold both when one event is contingent upon the occurrence of another, and when the separate events are independent. The results are different in the two cases, because in the former the number of possibilities of the second event is changed by the occurrence of the first event. But the general formula given in the last section (page 459)—If the probability of A is $1/m$ and the probability of B is $1/n$, then the probability $A B$ is $1/mn$—holds in every case.

(3) *Events which can happen in a Plurality of Ways.* When we have obtained a correct statement of alternatives fulfilling the conditions of exhaustiveness, definiteness, exclusiveness, and equal likelihood, we sometimes find that several of them are followed by the same consequent. For example

S is either a or b or c,
If S is a it is x,
If S is b it is x,
If S is c it is y.

Here it is obvious that the result is that *S is either x or y*, but that the probability of x is twice as great as that of y, as x will occur both when a is realised and when b is realised, and the probability of both a and b is seen, from the original proposition, to be 1/3. Hence, the probability of x is 2/3. If in the original disjunction the probability of a had been 1/2—*i.e.* if the proposition when accurately stated ran *S is either a or a or b or c*—then the probability of x would, on the conditions given in the hypothetical propositions above, be 3/4. Thus to put the matter generally: When several distinct and exclusive conditions have the same consequence, the probability of that consequence is the sum of the probabilities of those conditions. This,

it will be seen, is not a new principle, but a simple deduction from the conditions which a set of alternatives must satisfy in order to form a basis for the calculation of probability.

As a simple example, we will ask what is the probability that head will come uppermost in one or other of two throws of a penny. Here there are four alternatives in the original disjunctive proposition. The throws are either *hh or ht or th or tt,* and any one of three of these secures a head. Of course in the first case, when head is once secured the second throw would not be made; but the case must not be omitted, as there are two possible sequences in which head comes first. This is seen quite clearly when it is remembered that two consecutive throws of the same coin and the simultaneous tossing of two coins are identical as a basis for computing probability. We reach the same result by a line of reasoning similar to that we appealed to in the last case [*see* (2)]. The chance of getting head at each toss is 1/2; but the second toss is contingent on the failure of the first to give heads; the probability of the toss itself is, therefore, 1/2, and consequently the probability of its giving heads is $1/2 \times 1/2 = 1/4$. Hence, the total probability of getting heads in one or other of two tosses is $1/2 + 1/4 = 3/4$.

For another illustration we will take the probability that a throw of two dice will yield a certain definite number of pips, say seven, irrespective of how that number is divided between the faces of the two dice. Our original disjunction here consists of thirty-six alternatives; each of which is a condition fulfilled in one throw, and yielding a particular consequence. Upon examining them, we find that the number seven results from six of these alternatives, as it may be composed of $4 + 3$, $5 + 2$, or $6 + 1$, and each of these may occur in two ways, according as the larger number appears on the first or the second die. The probability of seven being thrown is, therefore, $6/36 = 1/6$. Similarly, the probability that each die will throw the same number of pips is 1/6, as there are obviously six possible cases in which a pair of numbers can be shown.

(*c*) *Probability of Alternative Conditions.* We will now consider what is often called "inverse" probability, which

stands in the like relation to the direct probability we have been hitherto dealing with as induction does to deduction. As the same principles of reasoning govern both those methods of inference, so here we shall find no new principles. The problem is—given that a certain event has happened, to which of a certain number of alternative causes or essential conditions is it probably due, each being conceived possible? This is the problem that has to be solved in endeavouring to reach the true magnitude of a phenomenon from more or less inaccurate measurements.

The problem, obviously, presents more practical difficulties in its solution than do those we have already dealt with, as we have often no certainty that we have correctly stated the alternatives. But the principle on which we proceed is the simple one that the condition from which the actual event would most probably follow is most probably the condition from which it actually did follow. We have, then, to take account of two probabilities—(1) that the assumed condition was actually existent, and (2) that the event would follow from it if it were present; the probability of the condition is then the product of these two probabilities. This is seen to be only the inverse mode of stating the direct probability of a compound event.

To take a simple example. An urn is known to contain three balls but we are ignorant of their colour, and after each drawing the ball drawn is returned to the urn. If one white ball is drawn we have no reason to conclude anything about the possibility of drawing or not drawing a ball of another colour. If the next drawing gives a black ball we have two alternatives, but we are not certain that they are exhaustive of all the possibilities. If, however, we continue drawing and get nothing but white and black balls, the probability that there are no balls of any other colour present becomes rapidly stronger. If, for instance, the urn contains one, and only one, red ball, the probability that that ball would not be drawn at any one drawing is 2/3. The probability, therefore, that it would not emerge in four drawings is only $2/3 \times 2/3 \times 2/3 \times 2/3 = 16/81$, whilst that it should not appear in eight draw-

ings is only $2^8/3^8$ which is less than 1/25, and with every drawing this probability decreases. On the other hand, if the urn contains only white and black balls, the result obtained would be a necessity. This assumption is, consequently, by far the more likely.

As a slightly more complex example, we will assume it known that the urn contains three balls which are known to be either white or black, and we will estimate the probability of the various possible proportions of white and black balls in the urn. Before the drawings commence there are four possible alternatives—*www, wwb, wbb, bbb.* The probability of each alternative is, therefore, 1/4, and that of any one particular ball is $1/3 \times 1/4 = 1/12$. The case is, indeed, the same as having twelve balls—six white and six black—to draw from. The first drawing, however, giving a white ball, the alternatives are reduced to three, *bbb* being put out of court. We then consider the relative probability of the remaining alternatives.

As there are six ways in which *w* can be drawn, the probability for each individual white ball is 1/6. The probability, therefore, that the white ball actually drawn came from *www* is $3/6 = 1/2$; that it came from *wwb* is $2/6 = 1/3$, and that it came from *wwb* is 1/6. The most probable alternative, therefore, at the end of the first drawing is *www.* If, however, the second drawing gives a black ball, *www* is proved to be impossible, and the alternatives are reduced to *wwb,* and *wbb.* And so far, there is no reason to prefer one to the other. If, however, the next drawing is *w,* so that in three drawings *w* appears twice and *b* only once, the alternative *wwb* is the more probable. For assuming *wwb* to be real, the probability of *w* is at each drawing 2/3 and that of *b* is 1/3. Hence, the probability of getting a single combination giving *w* twice and *b* once is $2/3 \times 2/3 \times 1/3 = 4/27$. But this combination may be drawn in three different orders, viz. *wwb, wbw, bww.* Therefore the total probability of getting one of these three orders is the sum of them all, and as each is 4/27 the sum is $3(4/27) = 4/9$. On the other hand, on the supposition of *wbb*, the probability of getting *w* twice and *b* once is $2/3 \times 1/3 \times 1/3 = 2/27$. This may also happen in

three ways, and, therefore, its total probability is $3(2/27) = 2/9$. Thus the odds in favour of the first hypothesis are 2 to 1, and its probability is, therefore, 2/3.

As another example we will take a case of the probability of testimony. Suppose that two witnesses, the probability of whose accuracy is 3/4 and 2/3 respectively, agree in affirming the occurrence of an event which in itself is as likely to have happened as not to have happened; *i.e.* whose antecedent probability is 1/2. What is the probability that the event really did happen? There are two possible and equally likely antecedents—that the event did happen, and that it did not happen. On the hypothesis that it did happen the probability of both witnesses stating that it did is the probability of their both telling the truth, *i.e.* $3/4 \times 2/3 = 6/12$. On the hypothesis that it did not happen, the probability that both witnesses should assert that it did is that of their both speaking falsely, *i.e.* $1/4 \times 1/3 = 1/12$. The odds are, therefore, 6 to 1 that the event did happen; *i.e.* the probability that it really occurred is 6/7.

(*d*) *Probability of the Recurrence of an Event.*[1] If we know nothing whatever of the conditions of an event, but only that it has occurred, we may estimate the probability that it will occur again. At first sight it might seem that the chance of its occurring again under the same conditions is exactly equal to that of its not recurring; *i.e.* that for every occurrence the probability would be 1/2. But this would lead to the paradoxical result that continued repetition of an event gives us no grounds for expecting it to occur again, and so that even infinite uncontradicted experience of the occurrence would not increase our rational expectation that it should occur again. It is, however, a miscalculation, as it overlooks the fact that continued occurrence testifies to the persistence of the conditions which produce the event, and even though we do not know what those conditions are, yet as evidence of their existence increases, we are justified in

[1] The solution of this problem is that given by Lotze, *Logic*, § 282 (5).

expecting more and more strongly their continued existence.

We must, therefore, take account of all the times the event has occurred, and in stating our original alternatives each of these must count as one. For instance, if an event has happened once, that is one reason for expecting its recurrence. But if we put that on one side for the moment, the chances for its occurrence again would be equal. There are, therefore, two reasons to expect its recurrence and only one for expecting it not to happen again. The odds for its recurrence are, consequently, 2 to 1 and the probability of the event happening again is 2/3. So, generally, if an event has happened m times, that gives m alternatives in the original disjunction; the possibilities that it may occur again and that it may not do so, add 2 more. Consequently, the total number of alternatives is $m + 2$ of which $m + 1$ are favourable. The probability that the event will occur once more is, therefore, $(m+1)/(m+2)$. For instance, if the sun has risen daily for five thousand years, the probability that it will do so once more is 1,826,214/1,826,215.

It is thus evident that with continued uncontradicted experience, the probability of a single repetition of the event rises very high indeed, and that it increases with the growth of such experience; the larger the value of m, the nearer the probability approaches to certainty. But this probability rests on the assumption that continued experience testifies to the persistence of the conditions which produce the event, and this assumption is itself only probable. The formula, therefore, measures the probability of the recurrence of the event only indirectly: it directly measures the probability of this probability.

It will be seen that this calculation of probability is the true basis of induction by simple enumeration. The formula shows that with wide and uncontradicted experience the probability that an empirical law which summarises that experience will hold good in one more case is very high. But it also shows that extension of it beyond the realm of actual experience becomes increasingly uncertain with increase in the width of that extension. For, if the formula is written to show the probability that

an event which has occurred m times will happen n times more, it becomes $(m + 1)/(m + n + 1)$; for $m + 2$ in the original formula $= m + 1 + 1$, where 1 is the number of new cases, *i.e.* n—which obviously decreases in value as n is increased.

Again, another modification of it shows how the actual experience of the failure of the event weakens the probability of its recurrence. For if an event has occurred m times and failed to occur n times under circumstances where it might have been expected to happen, then there are already $m + n$ cases; the possibilities that it may or may not occur again add 2 more, and thus, the probability for its recurrence is $(m + 1)/(m + n + 2)$, which decreases as n increases. In this case, the extension to p more cases becomes still more hazardous, as its formula is $(m + 1)/(m + n + p + 1)$, where $(m + n)$ in the denominator corresponds to m in the original formula, as it expresses the total number of observed cases, and p corresponds to n as expressing the number of unobserved cases, the probability of whose occurrence it is desired to estimate.

3. **Methods of determining Magnitude.**—As different measurements of the same phenomenon vary, the ascertainment of the true magnitude is an inductive problem which can only be solved with a high degree of probable approximation. We have to decide what assumption as to the true magnitude will lead with the greatest probability to the values actually obtained, on certain hypotheses as to the character and mode of action of the unknown conditions to which the errors are due. We have the permanent cause which yields the true magnitude, and in addition an indefinite number of interfering conditions, more or less insignificant in amount, of whose number and influence we are ignorant. The question is—What assumptions are we justified in making with regard to these conditions; and, as a consequence of those assumptions, what practical methods are open to us of correcting our observations of magnitude?

(i) **The Method of Means.**—When we have different results from several measurements of the same pheno-

menon, we know that all but one *must* be, and all *may* be, erroneous. As the number of measurements is increased, the probability of any particular one being exactly right gets less and less, and may, therefore, in our discussion, be left out of account. If then we have two different measurements, four alternatives are open to us—either both are too large, or both are too small, or either one is too large and the other one too small. If we are really entirely ignorant of any conditions likely to influence the result one way or the other, these alternatives are equally probable. If, however, the measurements are made with different instruments which have not the same degree of precision, or if one is made by a method more likely to yield accurate results than that used in the other case, we should prefer the measurement made under the conditions most likely to yield an accurate result. "Any method of measurement which we know to avoid a source of error is far to be preferred to others which trust to probabilities for the elimination of the error. As Flamsteed says: 'One good instrument is of as much worth as a hundred indifferent ones.'"[1] But no methods or instruments give invariable and absolutely truthful results, though in all scientific work the methods are so well considered and so carefully employed, and the instruments are so delicate and precise, that we may regard the occurrence of any considerable error as a practical impossibility, and may assume that the larger a possible error is, the less is its chance of occurrence.

If, then, we assume that the two measurements, A and B, were made by methods and with instruments equally likely to give an accurate result, and if we are absolutely ignorant of any ground for preferring one measurement to the other —that is, for expecting one kind of error rather than another—we are justified in assuming that in *each* measurement it is equally probable that the result obtained is too large or too small. From this it follows that the probability either that *both* measurements are too large or that both are too small is 1/4, while the probability that one

[1] Jevons, *Principles of Science*, p. 391.

is too large and one too small is 1/2. Thus the true magnitude most probably lies within the limits of A and B.

The same result is obtained from the hypothesis that small errors are more probable than large, for it would follow from this that the most probable magnitude is that which gives the least sum of errors. But if the true magnitude is between A and B the sum of the errors is $A - B$, while if it exceeds A, or is less than B, by n, the sum of the errors of the two measurements is $A - B + 2n$. As the number of measurements is increased, the probability that the true magnitude lies within the extremes rapidly increases. For example, if we have six measurements, the probability either that all of them are too large or that all of them are too small may be calculated by the mathematical theory of probability to be only 1/64, while the probability that the true magnitude lies between the extreme measurements is 62/64. The odds are, therefore, 31 to 1 in favour of that hypothesis.

The question now arises as to *what* intermediate value is the most probable. The mathematical assumption is that the number of causes of error is indefinitely large, while the influence of each is indefinitely small, and that each is equally likely to be operative in increasing or in decreasing the result. Under these conditions, the arithmetical mean —*i.e.* $(a + b + \ldots . n)/n$—of a number of measurements of the same phenomenon is most probably the true magnitude; for the errors so caused will balance each other, the sum of the positive and that of the negative deviations being equal.

When, however, as happens in a few cases, some condition is known to be operative which varies as the square of the distance, the geometrical mean—*i.e.* $\sqrt{ab}$—gives the more accurate result. For example, if the true weight of an object is sought by weighing it successively in the two scales of an imperfect balance, as gravity is an operative condition, the true result will be the geometrical mean of the result, though as, in small numbers, this differs but little from the arithmetical mean, the latter is generally taken as more easily calculated.

The arithmetical mean of the actual results of measurements is, then, the true magnitude—or the most probable approximation to that magnitude—in the great majority of cases when we are dealing with a single phenomenon. The errors in such measurements are related to each other as simply more or less in one fixed direction, and may be graphically represented by setting off on a straight line, on the two sides of a zero point which indicates the true magnitude—that is, no error—divisions which are proportional respectively to the positive and negative errors involved in the actual measurements.

(ii) **Method of Least Squares.**—But when we are dealing with compound magnitudes, in which two or more quantities are dependent on each other, the method of taking the arithmetical mean is no longer available. This case may be spatially represented by the deviations from the bull's eye of the shots on a target. The deviations are in all directions, but each can be resolved into a vertical deviation and a horizontal deviation. The graphic representation of each such resolution is a right-angled triangle, in which the actual deviation is the hypotenuse; and the square on this equals the sum of the squares on the lines which represent its resolution into vertical and horizontal deviations. When the total deviations are measured, the sum of their squares—on the supposition that the sum of the horizontal variations on the one side cancel those on the other, and that the vertical deviations neutralise each other in like manner—is easily shown by a geometrical construction to be the least possible.

The proof of the method of least squares involves an appeal to mathematics which would be out of place here. Its principle is: That magnitude is the most probable, whose assumption makes the sum of the squares of the errors of the actual measurements the least possible. It is an extension of the method of means, in that it indicates the most probable mean in cases which involve a plurality of arithmetical means. When we are dealing with only one magnitude the finding of the arithmetical mean gives the same result as the application of the method of least squares, of which it is but a particular case. And it

will be seen that this method shows the real agreement of the result obtained with the assumption that, as large errors are less probable than small, that result is most probable which gives the least possible sum of errors.

If we take three measurements of a single phenomenon and obtain three values, the intermediate one will always give the least sum of simple errors, whether it is the arithmetical mean or not. For if a, b, c be the three results the sum of errors involved in assuming b to be the true magnitude is—supposing them to be given in descending magnitude—$(a-b)+(b-c)=a-c$. But if the arithmetical average does not coincide with b but deviates from it by say n, then the sum of errors is $a-c+n$. For example, if our measurements are 15, 11, 10; then assuming 11 to be the true magnitude, the sum of errors is $4+1=5$; while 12, which is the arithmetical mean, gives $3+1+2=6$ as the sum of errors.

The same thing holds generally—the least sum of errors is obtained when the middle one of a series of measurements is taken as the true magnitude, if the series consists of an odd number of measurements; while if it consists of an even number, either of the two middle results, or a result intermediate between them, will yield the same, and that the least, sum of errors. But if we assume that the sum of the squares of the errors is to be the least possible, we find that this condition is fulfilled by the arithmetical mean alone. For example, on the supposition taken above, where the measurements were 15, 11, 10, if we assume 11 as the true magnitude, the sum of the squares of the errors is 17; but if we assume 12, the arithmetical mean, as the true magnitude the sum of the squares of the errors is only 14; and every other supposition will necessarily give a larger sum of squares of errors.

The method of least squares is, then, the most general mode of finding the true magnitude from a number of divergent measurements; but when these measurements involve one magnitude only, the simplest mode of applying the method is to take the arithmetical mean.

CHAPTER XXXIV.

SCIENTIFIC EXPLANATION.

1. **Nature of Explanation.**—Explanation is called for as frequently in common life as in science. In both it consists essentially in showing how the fact to be explained follows from some other fact or principle admitted to be true. It has, therefore, many degrees. The extent of the explanation offered in any given case will depend partly upon the knowledge of the individual to whom it is addressed, and partly upon the progress of knowledge as a whole at any particular time. For the kind of dependence which will suffice to account for a fact to one mind will be rejected as futile or inadequate by another. The plain man attributes a chill on the liver to the cold winds that have blown, and offers as the only explanation within his reach the observation that 'they always do affect me in that way.' This is not enough for the doctor, who seeks for reasons why the cold should affect the liver at all, and finds them in his general knowledge of the functions of that organ in relation to the rest of the body and in the effect of cold upon them. His explanation takes a wider range, but he too may come to a point where he is stopped by the demand for a reason which has not yet been found. Now almost any kind of explanation may be of use to this or that individual, but logic is concerned only with the general nature of explanation, and this consists in assigning any given fact a place in the system of knowledge by establishing its relations with other facts within the system.

We must, then, distinguish between logical explanation and explanation which is merely popular. The latter looks at the individual mind, and seeks to bring the new

phenomenon into relation with something already there. In many cases such explanation will be utterly inadequate from the scientific point of view. It may be merely a rough analogy or illustration, as is constantly the case with the only explanations which can be given to children. Logical explanation, on the other hand, does not regard the individual mind as such, though, of course, it is reached and expressed by some individual mind. It expresses the explanation in terms of the most advanced knowledge on the subject, and is itself, when first grasped, an addition to knowledge. Thus the moon's motion is logically explained by the theory of gravitation: it is popularly explained by saying that "she is a falling body, only she is going so fast and is so far off that she falls quite round to the other side of the earth, instead of hitting it; and so goes on for ever."[1]

Scientific explanation is only possible on the assumption that every fact is determined by certain conditions, so that if the conditions were otherwise the fact would be different. Explanation is the full and explicit statement of those conditions. Logically, the whole fact needs explanation, and obtains it to the extent to which its place can be assigned in a system of knowledge. Practically, when any particular point needs explanation it is one aspect of the whole complex phenomenon. The chemist, for example, seeks chemical explanations: the physicist aims at determining the physical conditions. The rest of the phenomena are passed over as having no bearing on the purpose in hand.

The problem is approached through classification[2]: but classification is not explanation though it may be nearly related to it. Suppose that the attempt to classify on the principle of modification through descent has been successful, we have still to ask why the course of descent has been of one kind rather than of another. There must be some ground for the development of diverse species from one genus. Classification reveals the fact by carefully noting important resemblances and differences, and then leaves it.

[1] Clifford, *Lectures and Essays*, p. 102. [2] Cf. pp. 81-83; 86-89.

But the very resemblances and differences revealed may suggest the directions in which the reasons for them should be sought. Resemblance "presumes identity of causation"[1] and difference demands explanation. Thus there is an important distinction between classification and explanation however closely they may be related. "In classifying we describe what is; in explaining, what must be, and why it must be."[2]

Indeed, necessary relation is the keynote of explanation. It is never thoroughly made unless the facts to be explained are proved to be absolutely dependent on certain definite conditions. The form in which such dependence is ideally best expressed is that of the reciprocal hypothetical propositions—*If S is M it is P; If S is P it is M.* These it is the business of induction to establish, so that the ideal of explanation is the same as that of induction.

Indeed all induction is progressive explanation. It may be incomplete, as when we resort to analogy or a hypothesis as yet unconfirmed, or rest content with a simple causal connexion. Nevertheless in each case we have a partial explanation, through which alone science can advance to more perfect knowledge. But not every explanation is inductive in form. When the relations to which we appeal as the necessary conditions of the phenomenon to be explained are axiomatic, or have already been established as true, we may often show deductively the dependence of the one on the other. This is so in mathematics and in sciences, such as astronomy, which are largely mathematical. Moreover, the statement of an explanation arrived at inductively may be expressed in deductive form. With these distinctions we are already familiar.

But whether the principles on which scientific explanation rests are already established or not is logically immaterial. They may become known in the very process of explanation. This is exemplified again and again in induction. The wave theory of light began as an unproved assumption. By means of it the particular phenomena of light were explained, and in the explanation was found

[1] Hobhouse, *Theory of Knowledge*, p. 473. [2] *Ibid.*

the proof of the truth of the theory. Here is a further difference from popular explanation. "Nothing is popularly accepted as an explanation in the way of analysis or interposition, unless the laws thus introduced are already known and accepted. The scientific man, however, does not much care whether these laws are already known or not: if they are, so much the better: if not, they must become known. For speculative purposes they equally in either case help him on towards that simplification of nature and reduction of all things to the fewest generalisations, which is his one great aim."[1]

2. **Generalisation.**—All explanation involves generalisation: it seeks to detect the universal in the particular. The particular may be a particular fact or an empirical law. A particular fact can be shown to be dependent on some other facts, and by the direct methods of induction it is often possible to establish the mode of dependence and to express it in an empirical law. So far there is partial explanation. But an empirical law only describes the phenomenon, and is itself in need of explanation. Now an empirical law is already a generalisation, and any further explanation must be sought in the establishment, by the indirect methods of induction, of its connexion with other laws of wider generality.

Thus the explanation of an empirical law frequently consists in the proof of one of those far-reaching statements of necessary relation, such as the law of gravitation, the principle of the conservation of energy, or the atomic theory of matter, by which the systematic unity of the world is constituted. To explain, in short, in the fullest sense of the word is to think the matter explained as part of a system.[2]

Whether we are explaining a particular fact or an empirical law there is no real difference in aim. In each case we endeavour to assign the essential and invariable conditions which account for the fact or law in question. When this has been accomplished we have passed beyond

[1] Venn, *Empirical Logic*, p. 509. [2] Cf. Ch. I., § 1.

the particular; for in stating the essential conditions we imply that wherever they are fulfilled the fact or law will be exemplified, though no particular instance is indicated. The fact or law has been generalised.

Generalisation, then, consists in the establishment of identity of conditions. The phenomena generalised will exhibit differences which may be great or small, but they must be exactly similar in other respects if they are to rank as instances of the same thing. But there may be similarity which is of no importance for scientific explanation, as when two things diverse in other respects resemble each other in colour.[1] Resemblances and conditions are terms of different import. The former may or may not serve as a basis for scientific generalisation: it depends on whether they do, or do not, stand in a necessary relation to the phenomena compared. If they do, then the resemblances are also conditions, and it is on a precise similarity between them that generalisation is based.

Such generalisation is not an inference from number of instances as such. "There is all the difference in the world between the judgment, 'An indefinite number of cases of $A - B$ exist' and 'The relation $A - B$ is universal.' In the first case, we may also have $A - C$, $A - D$, $A - N$. In the second, we may have none of those, but always $A - B$, and nothing but $A - B$."[2] The mere repetition of instances does not justify the passage from 'many' to 'all.' The transition can never be made with certainty: the conclusion always remains one of greater or less probability. But as soon as the instances are analysed and the essential conditions ascertained, the generalisation that these conditions universally determine the result is made, and is expressed in the hypothetical judgment, *If S is M it is P* Subsequent confirmatory experience may strengthen our belief that the analysis has been correctly made, but it does not add to the necessary conditions known. Indeed knowledge of them may, in very favourable cases, be attained by the careful examination of a single instance, though as a rule a plurality of instances

[1] Cf. below, pp. 479-481. [2] Hobhouse, *Theory of Knowledge*, p. 241.

facilitates the difficult task of analysis, not in virtue of the plurality, but by aiding the effectual isolation of the conditions sought. The validity of the generalisation rests, then, not on the number of instances examined, but upon the fundamental postulate of knowledge that every element of reality is definitely determined, and that always in the same way, so that the relation between a phenomenon and its conditions cannot vary. Without this assumption generalisation would be unjustified.

In all valid generalisation, then, we attempt to set forth a necessary relation between a phenomenon and its conditions; and by necessary we simply mean that given the conditions the phenomenon will exist whatever the circumstances may be. But this is a purely abstract statement. We desire to know, not only that under certain conditions a given result will follow, but whether in fact those conditions are exemplified in reality. It is not enough to know that *If S is M it is P*; for the advance of real knowledge we must ascertain *when* S is M.

In both these steps of generalisation there is room for error. In the former error may exist in that *S is M* may be an imperfect statement of the essential conditions of P. Either some elements may be omitted or other elements than the essential conditions may be introduced. The conditions are obscured by surrounding circumstances, and it is on their exact determination that the justification of the generalisation rests. Hence it is not sufficient that *S is M* should *contain* the ground of P: for extraneous elements may also be present. The circumstances must be analysed, and all that is unessential eliminated until we can assert that S being *M is* the ground of P. We have already examined the inductive methods by which such an elimination is effected.

In the latter step of the generalisation there is also room for error. The point here is whether the essential conditions in this new case *are* exactly those expressed by *S is M*. We are apt to be misled by outward resemblance. No doubt phenomena which are determined by the same essential conditions do frequently resemble one another in an external way. But however strong the outward resem-

blance may be, in scientific thought we can never trust to it. A reliance on resemblance, for example, would class the sea-anemones in the vegetable rather than in the animal kingdom, and indeed for many years biology contained several instances of such false generalisation.[1] On the other hand, the outward resemblance may be of the slightest, and yet the essential conditions be identical. "Between the brilliant explosive discharge of a thunder-cloud and the gentle continuous current produced by two pieces of metal and some dilute acid there is no apparent analogy whatever. It was therefore a work of great importance when Faraday demonstrated the identity of the forces in action."[2] Obviously the determination of identity of conditions depends on insight in the particular branch of science concerned; it is not a part of logical theory but of experimental practice.

The aim of science is to reach a full knowledge of essential conditions. But this aim is only being progressively realised. The general process by which truth is attained through induction reveals a more exact determination of conditions from simple enumeration of instances on the ground of a general resemblance, through analogy, to complete scientific induction, and at all stages the hypothesis is a tentative generalisation. Enumerative induction suggests an empirical generalisation that *Every S may be P.* But that the resemblance on which it is grounded is felt to be due to an underlying identity is shown by the attempt made in analogy to reach this identity, and to find a possible ground for the empirical generalisation. The further inductive process is but the testing, moulding, and limiting of this suggested ground until the ideal is reached of a statement of conditions so exact that for every variation in the phenomenon an explanation can be found in a modification of the conditions.

Now it is not always possible to complete the process by reaching a full statement of the essential conditions. If we are confined, for example, to the direct methods alone we have to stop short at empirical generalisations: and in

[1] Cf. p. 376. [2] Jevons, *Principles of Science*, p. 612.

many sciences this is as far as explanation has yet gone in regard to the bulk of the phenomena which they investigate. Even then an empirical generalisation is only regarded as a step towards some ultimate principle from which it may be derived. Astronomy has found a principle which appears at present to be ultimate in the law of gravitation, though there are indications that it may be derived in the future from a yet wider generalisation: medicine is still largely empirical. The characteristic of an empirical generalisation is that it describes exactly what has to be explained by formulating the law of its occurrence. This is itself a partial explanation of it, but for scientific explanation we must know *why* that law and no other gives an exact description of the phenomena. The answer, if found, will be a wider generalisation giving the conditions which strictly determine the empirical generalisation, and hence the particular facts on which it is based. We have, then, two degrees of generalisation, (i) empirical generalisations, (ii) established truths. The latter alone fulfils the logical ideal.

(i) **Empirical Generalisations.**—An empirical generalisation is one which is descriptive, and not in any real sense explanatory. It is itself in need of explanation. For example, take the generalisation that the pébrine disease in silkworms is due to the presence of certain corpuscles at various stages of the silkworms' growth. This answers the question to what is the disease due, and suffices for the devising of practical measures to prevent it. But it throws no light on the problem why corpuscles of that particular kind should be associated with the appearance of a special disease. It has been reached by observation and experimental investigation, and is a succinct description of the essential fact in relation to the mode in which the disease is caused. Nevertheless it stands in relative isolation from other knowledge. It is not seen to be derived from some more general principle of disease: in other words, it is unexplained.

Empirical generalisations result from observation and experiment. They arise, therefore, in the earlier stages of scientific enquiry and furnish the material for wider

theories. When they are seen to be derived from principles of greater generality they are no longer merely empirical. Hence it will often happen that a generalisation which was at first empirical will cease to be so classed. Thus Galileo determined the law of falling bodies empirically, and it remained an empirical generalisation until Newton showed the reason for it as a necessary consequence of the principle of gravitation. While a generalisation remains empirical its grounds are wholly or partially unknown.

Contemporary knowledge furnishes numerous examples of such unexplained uniformities, but explanation is always regarded as ultimately attainable. This belief is based upon a realisation of the necessary unity of nature. For where there is observed uniformity of occurrence there is always a presumption that the conditions determining the phenomenon are constant, and we may expect in time to discover what they are, so that the uniformity ceases to be isolated from the rest of knowledge. That there must be definite determining conditions is the basis of all expectation of the recurrence of an event when the determining conditions are unknown, and this expectation rationally increases with the extension of uncontradicted experience of the event. In other words, the oftener an empirical generalisation is exemplified, the greater is the probability that there is a necessary and universal relation underlying it.

But while the generalisation remains empirical, it must never be assumed to be universally and necessarily true. Its statement can never be more than categorical; we can at most say *Every S is P*, not *If S is M it is P*. Hence the logical rule that empirical generalisation can only be safely extended to adjacent cases. If we pass to cases which vary widely from those observed, or exceed the limits of time and space within which the observation has been made, we lessen considerably our assurance that the unknown conditions remain the same. It is an empirical generalisation that the expectation of life in certain dangerous trades is small: it would not do to apply the same generalisation to healthier occupations. Nor should we be justified in

extending generalisations, for example, as to the food and clothing of northern peoples to natives of the tropics. To provide flannel waistcoats for them shows more charity than knowledge. Indeed, the wider the scope over which we try to extend an empirical generalisation the less is our confidence that it will hold true, for the less ground have we for assuming that the essential determining conditions will remain unchanged.

An interesting class of empirical generalisations is composed of what are often called 'Statistical Laws,' but which are better termed *Statements of Statistical Uniformity*. Many problems are so complex that as a preliminary to closer analysis it is found advantageous to procure statistics of the phenomena under investigation. This is especially the case in sociological and political enquiries. We may in this way gather statistics of trade, education, population, and so forth. The mere counting of units would not be profitable. In the selection of the kind of unit and the nature of the statistics a definite question or series of questions is as a rule implied. "It might seem that in taking the average rate of mortality on the basis of the returns of local officials, etc., the figures of themselves disclosed the fact that the rate was higher for infants under two years of age than in later periods of life. But the total average of deaths would never have shown this. It is only because the average for infants has been separately calculated, in the expectation that there might be a difference, that the difference has been found."[1]

Now when statistics are gathered in this purposive way they often reveal uniformities which point to the fact that there are at the back of the phenomena constant conditions which it would be well to seek. It is well known, for example, that the number of persons who commit certain crimes, who are born, or who die, in the course of a year, bears a remarkably uniform proportion to the total number of the inhabitants of any given country; there is, as we say, a pretty constant *average* preserved in many of the phenomena of social life. Again, the number of people in

[1] Creighton, *An Introductory Logic*, pp. 224-225.

England and Wales who commit suicide is found to remain very uniform from year to year. Moreover, the averages are found to vary with great regularity according to the months of the year, being highest in June, falling regularly to December, and then gradually rising again. Further, the proportion who commit suicide at different ages remains fairly constant. All this indicates that social and material conditions have remained largely unchanged.

It would be an error, however, to assume that such a statistical uniformity must necessarily continue. If it does we may conclude that the same general conditions have been in operation. But it would be rash to think that the conditions will necessarily remain unchanged in the future: and, since that is so, there is no more necessity in a statistical uniformity than in any other empirical generalisation. Yet from the constancy of such averages an inference as to their necessity has sometimes been drawn. Thus Buckle says: "In a given state of society a certain number of persons (about 250 each year) must put an end to their own life. This is the general law, and the special question as to who shall commit the crime depends of course upon special laws: which, however, in their total action, must obey the large social law to which they are all subordinate. And the power of the larger law is so irresistible, that neither the love of life, nor the fear of another world can avail anything towards even checking its operation."[1] Such a conception can only be true if the changes in society leave untouched the forces and motives which determine men to self-destruction.

The monotonous record of a statistical uniformity after it had once been established would serve no useful purpose unless there was liability to change. It is the deviations from the law which bring nearer that analysis of the conditions which we desire. If a particular crime is found to be on the increase, if suicide becomes more common, if the birth rate shows a proportional decline, we at once search for some variation in existing social and economic conditions which may account for the change.

[1] *History of Civilisation*, vol. i., p. 25.

In this way we may be put on the track of a generalisation which will explain, and not merely register, the phenomenon. For we use statistics as an aid to explanation. The principle which would make the facts intelligible is hidden from us. As soon as it is discovered our interest in the collection of statistics, at any rate as a method of gaining knowledge, vanishes. The facts counted and averaged are each one determined exactly, and if a uniformity appears that too has its exact conditions. But what they are it does not tell us. The average, for instance, neglects all the particular characteristics of the individual instances, and there is no means of showing that it is necessary without examination of those characteristics which it discards. "Such uniformities of numbers and averages are primarily mere descriptions of facts, which need explanation as much as the uniformity of the alternation between day and night; and the explanation can be found only where the actual conditions, the efficient causes, are forthcoming. But these are the concrete conditions of the particular instances counted, they are not directly causes of the numbers; it is only the nature of the concrete causes which can show it to be necessary for the effects to appear in certain numbers and numerical relations."[1]

(ii) **Established Truths.**—When the conditions of an empirical generalisation have been exactly determined the generalisation is established as a necessary truth, which may be defined as an established general judgment. It is not a summary of particular judgments, for it is not arrived at by summarising the records of events which have been often observed to occur. Such a process as we have more than once urged can yield nothing more than a greater or less degree of expectation. General truths are the result of such an analysis of reality as enables us to state the relation which holds between a phenomenon and its conditions. And when this relation is ascertained the principle of the unity of the world—without which all knowledge is impossible—compels us to think it as universally and necessarily true.

[1] Sigwart, *Logic*, Eng. Trans., vol. ii., p. 490.

Of course, if our analysis is inaccurate, if what we assume to be the conditions of the phenomenon are not really its conditions, then our general propositions are not true. Or again, if our analysis is inadequate; if what we take for the essential conditions includes other elements, or if we omit some essential elements of the conditions, then our judgment has not universal validity; it is, we may say, nearly true.

There is then room for mistake in the application to reality of the hypothetical judgments in which we express the conditions of phenomena. Every such judgment expresses a necessary relation between its antecedent and its consequent. If you accept the judgment as true, you accept the relation as universal and necessary. But the judgment may fail to express accurately the nature of reality. It may err concerning the particular relation with which it deals, or respecting the connexion of that relation with the other parts of the system of knowledge. However, in so far as the general judgment is true it is necessarily true, for necessity consists only in the connexion of a consequent with its ground. Uncertainty, when it exists, pertains to the truth of the judgment in the form in which it is expressed; that is, it attaches to the accuracy and adequacy of the analysis of conditions upon which it is founded. When this accuracy and adequacy are undoubted there is no uncertainty.

It is for this reason that the truths of mathematics are sometimes held to be of greater certainty than those of other sciences. The contrast between them has been pointed by calling the former truths 'necessary,' and the latter 'contingent.' But the supposed difference does not lie in the nature of the truths themselves: it is to be found rather in the greater difficulty which is experienced in physical and social science in determining what the conditions really are. A brief consideration of the logical nature of mathematics will make this clear.

In mathematics we can always fulfil the requirements of necessary truth by an exact statement of the conditions upon which our constructions and inferences depend. There are two broad divisions of the subject—the science

of numbers, and the science of space. In the case of numbers the two fundamental operations are those of addition and subtraction. So far as the operation is mathematical it is independent of any objects to which the numbers may refer. The notion of unity is constituted by our own mental act, and is conceived exactly. The addition of units is then a matter of construction. If we have $4 + 5 = 9$, the whole is seen to be constituted by the parts 4 and 5, and the nature of the parts and of the whole—4 units, 5 units, 9 units—is so exactly defined that wherever the same combination of parts occurs the whole must necessarily follow. The result can be generalised at once because we are sure that there are no extraneous conditions which might conceivably modify it on another occasion. The relation falls entirely within the number series, and that has a constant structure. All the modifications and extensions which appear in the development of the concept of number are made under precisely defined conditions, and from these are derived truths which are necessary in that they depend upon them for their ground.

It is so, too, in geometry. The definitions, for example, of geometrical figures are less determined for us than by us. They tell us how to construct the figures in thought, although we may never construct them in reality. Thus a circle is a figure described by a point moving at a fixed distance from another point in the same plane. This fixes the exact conditions upon which a circle depends, and from them flow all the numerous properties discovered in the theorems relating to that figure. Any one of these, as soon as it has been demonstrated, can at once be generalised to the constitution of any circle whatever, for no other conditions are necessary. It is true that there is also involved the assumption that space is uniform in its construction: but this again is a conception which for the purposes of the geometer can be exactly defined. The essential point is that in all his reasonings, whether about figures in space of two, or three, or even more, dimensions, he can state all the necessary conditions, and be sure that he errs neither by excess nor by defect. Indeed in mathematical construction the ideal aimed at in experiment is

realised. No unknown conditions are present to introduce an element of uncertainty into the conclusion.

The superior certainty of the conclusions of mathematics has been denied on the ground that its concepts and axioms are merely generalisations from sensuous experience. It is said that we know that $2 + 2 = 4$, that a triangle is a plane figure bounded by three straight lines, that things equal to the same thing are equal to one another, because we have found these relations exemplified again and again in experience. For example, whatever the objects counted, whether plums, chairs, or people, the uniform result is obtained, $2 + 2 = 4$: and that is the sole warrant for the generalisation. If this were true the necessity of mathematical truths could not be maintained. "For you have no ground for supposing that if $2 + 2$ could sometimes make 5, cases of its occurrence would have occurred in your experience. Everything becomes problematical; the frequency of any particular sum of $2 + 2$ is quite indeterminate, if the sum is indeterminate; and your experience may assure you that you have never found them making anything else than 4, but cannot assure you that you are never likely to do so. And so it is with geometrical principles."[1] The generalisation is a precarious induction by simple enumeration, and cannot survive a contradictory instance. The criticism of Professor James, for instance, that two plus two does not always make four in sense experience, since two drops added to two drops make one drop, is a sufficient logical refutation of the generalisation.

Nor is the case improved if for simple enumeration we substitute the direct methods of induction. Suppose that we vary methodically all the circumstances as far as possible and still find our geometrical or numerical relation constant, we shall still be left with an empirical generalisation. The truth is that the relations dealt with in mathematics are not such generalisations from observation, but are constructions of thought to explain the mathematical aspects of experience. So mathematical concepts are ideal

[1] Joseph, *An Introduction to Logic*, p. 510.

constructions. The lines and figures of pure geometry have no exact counterparts in the concrete things of experience. No line can be drawn which is perfectly straight. But actual lines differ from straightness in an indefinitely large number of ways. 'Straightness' is a mental construction which attempts to explain the differences of these actual lines as deviations from a norm. It is not that we are born with such ideas as 'straightness' ready-made, any more than we are born with such an idea as is expressed in the law of inertia. Nor is it that we can abstract straightness from various forms and degrees of crookedness: evidently it is not there to be abstracted. No, 'straightness' is a hypothesis of an ideal line. Similarly with the other concepts.

It may be granted, then, that the truths of mathematics have been reached by way of sense-experience, and are fundamentally attempts to explain that experience. Geometrical and numerical relations are known roughly and empirically before they are demonstrated exactly and hypothetically. Mathematics has no mysterious origin. Like physics and chemistry and the other sciences it is an attempt to think relations in such an ideal form that the variations we actually experience can be explained as due to interaction. If a line is not drawn perfectly straight it is because of deviations due to such things as roughness of surface.

But the physical sciences have to deal throughout with complex reality. They aim at explaining phenomena. When mathematics is applied to a similar purpose its results are seen to be, like those of the other sciences, approximations more or less close to what is actually found. But, as we saw in the last chapter, the deviation is interpreted as on the side of the observed phenomenon, not on that of the theoretical result. Only so, indeed, could the constant variations be explained.

Yet even here the fundamental relations dealt with in mathematics are few and simple. It is this which makes us more certain of their truth. From these few initial propositions vast consequences can be drawn deductively —consequences which on the basis of those original pre-

mises are necessarily true. But like all deductive conclusions they are true only in the sense in which their foundations are—that is, as ideal representations of certain relations considered apart from any embodiment in actual things.

In the physical sciences our results are more open to question just because they do attempt to establish what relations actually exist. Every deviation must be explained; every phenomenon traced to the operation of a complex set of conditions. In this it is evident that it is much more difficult to reach certainty than when we are concerned with one kind of relations only. Moreover our explanations themselves must show mathematical precision as a condition of adequacy: for such relations are found in all observable phenomena. But there is no logical reason why any 'law of nature' should not be established as certainly as a mathematical truth. The difficulty is entirely practical.

Green has summed up the whole question admirably: "The distinction, then, of the 'necessity' of mathematical truths from the 'contingency' of truths about nature, if it is to hold at all, is not to be understood as if it were only in mathematics, and not in natural Science, that what is once true must be always true, or as if natural laws were liable to change, mathematical laws not. The true distinction is between what is fully true and what is partially true. What is fully true once is fully true always, of a natural phenomenon no less than of a geometrical figure; but any proposition about a natural phenomenon is true of it only under conditions of which we do not know all, while a proposition about a geometrical figure, if true at all, is true of it under conditions which we completely know."[1]

The true ground of the necessity of a generalisation must be sought in its relations to the whole system of knowledge available. If the facts can be explained in no other way consistent with the unity of that system, the general proposition must be regarded as objectively true

[1] *Philosophical Works*, vol. ii., pp. 249-50.

and logically necessary. No doubt the imperfection of knowledge leaves here a possibility of error. Modifications of the unity of the system follow every development of knowledge. But there is a growing body of truth which remains undisturbed. It is very improbable that any new discovery will completely overthrow the achievements of any well-established science such as that of heat or chemistry, however profoundly it may alter its outlook or widen its scope. The probability that any judgment received as true by experts in any branch of knowledge will ultimately be proved to be false decreases. Uncertainty attaches to the accuracy and adequacy with which our analysis in any case has ascertained the essential conditions, and it is diminished when we have assigned to the phenomenon an integral place in the system of knowledge so far as it has been already constituted. For systematisation is the last step in explanation.

3. **Systematisation.**—Systematisation is then the stage at which laws are explained by their relations to laws of yet wider generality. An object or event is isolated until we can assign its cause in an empirical law; an empirical law is isolated until we can derive it from a wider generalisation. This derivation can only be accomplished by discovering between the two generalisations some element of identity which is seen to condition the laws of less generality. By detecting this element in a number of empirical laws it is often possible to show that all of them are dependent on one principle. This is the most important manner in which systems of knowledge are developed. The empirical laws are said to be *subsumed* under the principle from which they are derived. Thus the laws of falling bodies and of the movements of the tides are special cases of the law of gravitation.

The wider the scope of such a principle, the more satisfactory is the explanation. When the wave-theory of light is found to account for reflection, refraction, interference, and other phenomena of light which differ widely in the effects they produce, it has strong claims to acceptance, even though it cannot be shown that no other theory

would explain them as well. Indeed, it has become almost an axiom of explanation that the fewer and simpler the fundamental laws which form as it were the pivots of the system of knowledge, the better. The few theories that occupy this position, such as the theory of gravitation, the principle of evolution, the atomic theory, and the wave-theory of light cover between them a wide range of natural phenomena, and men do not despair of showing that heat, light, electricity, gravitation, are modes of one and the same force. The postulate of unity compels us to seek a single system of knowledge in which the complex is resolved into the simple, and the main theories are few and comprehensive.

Other modes of systematisation that have been distinguished are subordinate to that of subsumption. We may, for example, explain a particular phenomenon of which we have found the empirical law as the resultant of two or more independent causes acting simultaneously, as when the rise of a balloon is explained by the laws of fluid pressure and of gravitation:[1] or we may trace the laws of causes acting successively and show how a series of events arise out of one another. The latter type of explanation will be frequent when we are determining the course of historical development. But in neither case need we be fulfilling the ideal of explanation, though we are making our knowledge more systematic. The laws by which an empirical law is explained may themselves be derivative, and to trace a historical sequence and to reveal it as causal is only a part of explanation. Aspects of the sequence are compared, and historical knowledge becomes more systematic as these aspects are embodied in wider generalisations.

To systematise knowledge is to make a distinct advance. Empirical laws which separately rest on evidence that is not conclusive are strengthened by forming an indispensable link in a system. Results obtained in one branch of science often corroborate, or throw strong light on, similar conclusions in another. The independent evidence on

[1] Cf. Venn, *Empirical Logic*, p. 505.

which they are based gives them mutual strength. All laws cannot at present be shown to be derivative from established principles, and while our knowledge of the system is imperfect it is a gain to discover that laws which have not yet been derived from a common principle, not only do not conflict with one another but afford positive support to their respective conclusions.[1] The distinction between the ideal and the actual system of knowledge is well expressed by Mr. Hobhouse. After summing up the essentials of inductive process, he writes: "The resulting system, worked out ideally for all experience, and with all its points of interconnexion clear and certain would be the ideal knowledge; worked out clearly and definitely in this or that body of truth (such, perhaps, as mathematics and physics), less coherently and definitely in others (such, perhaps, as physiology), and strongly but indistinctly felt rather than pointed out in the great bulk of our 'common-sense,' every-day beliefs, it constitutes the knowledge which we actually possess."[2]

The theoretical limit of explanation is a complete and exhaustive knowledge of the system of the universe itself. We are bound to assume that ideally everything within the system admits of explanation, since with full knowledge we should see it in its proper connexions in the system both with all other of its elements and with the whole, and that is all that explanation implies. Of the whole system there can be for us no explanation outside itself. It is true that some laws appear to be ultimate in our present state of knowledge: but in accordance with the postulates of knowledge we attribute their apparently ultimate character to our comparative ignorance. For example, wide-reaching as the theory of gravitation is, it throws no light on the nature of gravitation itself. Hence the mind cannot rest content with the theory as an ultimate explanation. It recognises it as a problem which, it is not unreasonable to suppose, may be one day solved.

[1] Cf. Hobhouse, *Theory of Knowledge*, p. 403.

[2] *Ibid.*, p. 404.

4. Fallacies incident to Explanation.

(i) **Origin.**—As explanation involves generalisation it is evident that errors of generalisation will render our explanations futile. It is further evident that all generalisation from insufficient or misapprehended data must be fallacious. This means that such invalid results are all too easily accepted, for our discussion has at least brought out the truth that the interpretation of experience is a very difficult task. Had we no innate tendency to generalise our experiences, science could hardly begin, yet this very tendency is at the root of the danger, for it leads us to draw general conclusions from very inadequate knowledge. Children generalise wildly, and untrained minds show little improvement. One of the most difficult lessons the scientific worker has to learn is to be cautious.

(ii) **Empirical Generalisations.**—Generalisations are, we have seen, at first empirical; and in that stage can never be more than probably true. Here are evidently possible two main forms of error. We may generalise from too few instances, and even without having considered whether any exceptions can be met with or have been recorded. The generalisation is usually based on some striking feature common to the cases noticed, and cases which do not present that feature for that very reason escape notice. Hence such generalisations as that dreams are prophetic arose from the marking of coincidences between dreams and subsequent events and disregarding all dreams which did not thus obtain fulfilment.

Such an example brings out another mode in which empirical generalisations are often vitiated, in that too general a resemblance is taken as its basis. The amount of coincidence between dream and fulfilment is often extremely small; its very existence really awakens surprise and wonder; it seems something needing explanation, and the explanation is found in an assumed causal connexion.

The almost inevitable result of such imperfect analysis is the undue extension of the generalisation. Thus, to extend an empirical generalisation of some phenomena of social life, founded upon a survey of the condition of any

one country at some particular time, to other peoples and other times is to make a fallacious inference.

As one class of fallacies of this character Dr. Fowler mentions undue respect for authority. The influence on the opinion of the multitude of expressed judgments of well-known people on certain questions is well known to advertisers of quack medicines, promoters of public companies, and those generally who wish to attract public confidence to the goods they have to sell. The form of the argument involved would seem to be, that A is a man whose opinion on many subjects is of weight, therefore, his judgment is of weight on all subjects. "We have to learn," said Dr. Fowler, "not only that men are to be trusted exclusively within the limits of their own experience, in their own profession or pursuit, but that even within those limits their authority is apt to become tyrannical and irrational unless it is constantly confronted with facts and subjected to the criticism of others."[1] Similar remarks apply to an excessive regard for the authority of great writers. A striking example of this fallacy is found in the intellectual idolatry with which the schoolmen regarded Aristotle.

The second main source of error is to regard an empirical generalisation as a necessary law. This has long been known as the fallacy *Post* (or *Cum*) *hoc, ergo propter hoc*—if events are connected in time they are causally related. This, too, is committed when dreams are regarded as prophetic. The fallacy arises because we cannot rest satisfied with mere statement of fact—even of general fact. We need explanation, and we are often in far too great a hurry to reach it, so we fall into this fallacy by accepting as true the first suggestion that offers itself.

No doubt constant connexion in time suggests a deeper bond—but that bond is not always found in the events so connected—often it must be sought in a deeper reality. Day and night, for example, are causally connected with the daily rotation of the earth—but day does not cause night nor night day, constant as is their temporal succession.

[1] *Inductive Logic*, p. 292.

(iii) **Established Truths.**—Fallacy, of course, is committed not only when an empirical law is regarded as causal and necessary, but when a truth really so established is wrongly applied to concrete reality. Of this latter fallacy the most common examples are probably to be found in the consideration of cases where the operation of the law is frustrated by counteracting conditions. Especially is this liable to occur in the consideration of social questions. In illustration we will quote a passage from Sir G. C. Lewis: "It is to be borne in mind that, in estimating negative instances, due allowance must be made for the occasional *frustration of causes*. . . . For example: it might be argued, from the occurrence of several cases in which the absence of high import duties and of commercial restrictions was accompanied with abundance and cheapness of commodities, that the former was the cause of the latter. Certain instances might then occur, in which the former existed without the latter; but each of these exceptional cases might be accounted for, by showing that there was a special circumstance, such as a deficient supply, or interruption of intercourse by war or blockade, which partially obstructed, and for a time suspended, the operation of the former cause. Again: it might be shown, by the evidence of facts, that the operation of a new law had been generally beneficial, with the exception of certain districts, where its enforcement had been prevented or retarded by certain peculiar and accidental circumstances. Exceptions of this kind, which admit of an adequate special explanation, serve rather to confirm the general inference than to weaken it; inasmuch as they raise the presumption that, but for the partial obstruction to the cause, it would have operated in these as in the other instances where no obstructions existed." [1] In such cases we have, of course, an inadequate analysis of the conditions of the case to which we propose to apply the general law.

Generally, then, all illicit statement of law, and all wrong application of law, resolve themselves into inade-

[1] *Methods of Observation and Reasoning in Politics*, vol. i., p. 386.

quate analysis of facts. The statement of the law is erroneous when the conditions given include either more or less than just those essential conditions which are the ground of the phenomenon. Thus, if we take one of a number of related conditions as the total cause, our general law will be wrong, as its statement will embrace instances which do not really come under it. For example, to state that the boiling-point of a liquid depends only on the temperature would be to omit the equally essential condition of atmospheric pressure. Thus, to say that water boils at 100° C. is wrong; it boils at that temperature under the pressure of one atmosphere; that is, the normal atmospheric pressure at the sea-level. Up a mountain the boiling-point is different.

Again, in the analysis of phenomena it is possible to confuse the whole or a part of the cause with the effect. That it is by no means easy to determine in all cases which are the determining, and which the determined, elements in a complex phenomenon is illustrated by the fact that meteorologists are not agreed whether the copious and sudden downfalls of rain which usually attend thunderstorms are the cause or the effect of the electric discharge. The common opinion is that they are the effect, but Sir John Herschel held that they were the cause.

Another source of error is the neglect of the mutual interaction of cause and effect. This interaction, of course, may also lead to that confusion of cause and effect which we have just noticed. This again is well illustrated from social phenomena by a passage from Sir G. C. Lewis. He says: "An additional source of error in determining political causation is likewise to be found in the *mutuality of cause and effect*. It happens sometimes that, when a relation of causation is established between two facts, it is hard to decide which, in the given case, is the cause and which the effect, because they act and re-act upon each other, each phenomenon being in turn cause and effect. Thus, habits of industry may produce wealth, whilst the acquisition of wealth may promote industry; again, habits of study may sharpen the understanding, and the increased acuteness of the understanding may afterwards increase

the appetite for study. So our excess of population may, by impoverishing the labouring classes, be the cause of their living in bad dwellings; and again, bad dwellings, by deteriorating the moral habits of the poor, may stimulate population. The general intelligence and good sense of the people may promote its good government, and the goodness of the government may, in its turn, increase the intelligence of the people, and contribute to the formation of sound opinions among them. Drunkenness is in general the consequence of a low degree of intelligence, as may be observed both among savages and in civilised countries. But, in return, a habit of drunkenness prevents the cultivation of the intellect, and strengthens the cause out of which it grows. As Plato remarks, education improves nature, and nature facilitates education. National character, again, is both effect and cause: it reacts on the circumstances from which it arises. The national peculiarities of a people, its race, physical structure, climate, territory, etc., form originally a certain character, which tends to create certain institutions, political and domestic, in harmony with that character. These institutions strengthen, perpetuate, and reproduce the character out of which they grew, and so on in succession, each new effect becoming, in its turn, a new cause. Thus a brave, energetic, restless nation, exposed to attack from neighbours, organises military institutions: these institutions promote and maintain a warlike spirit: this warlike spirit again assists the development of the military organisation, and it is further promoted by territorial conquests and success in war, which may be its result—each successive effect thus adding to the cause out of which it sprung."[1]

[1] *Op. cit.*, vol. i., p. 375.

QUESTIONS AND EXERCISES.

These questions are in most cases taken from papers set at public examinations.

CHAPTERS I.-II.

1. "Logic deals with the form and not the matter of Thought." Carefully consider this statement; and distinguish Logic (*a*) from Psychology, (*b*) from Grammar.

2. "A man who has reflected upon the principles of reasoning is much less likely to be deceived than one who is guided unconsciously by assumptions which he has never examined." Discuss this and give illustrations.

3. Examine the following statements as to the province or function of Logic—

(*a*) The science not of Truth but of Consistency.

(*b*) The science of the operations of the understanding which are subservient to the estimation of evidence.

(*c*) A machine for combating Fallacy.

4. What do you understand by the distinction between Logic as a Science and Logic as an Art? Illustrate your answer by reference to different departments of logical doctrine or by reference to different modes of treating logical problems.

5. In what sense, if any, is Logic dependent for its principles on advance in scientific knowledge?

6. Discuss the statement that much valid thought precedes the study of Logic, hence the study of Logic is valueless for the purposes of valid thought.

7. Discuss the view of Logic implied in the following:—"Logic had begun its work, and in men of a certain temperament political logic is apt to turn into a strange poison. They will not rest until they have drained first principles to their very dregs. They argue down from the necessities of abstract reasoning until they have

ruined all the favouring possibilities of concrete circumstance." —(John Morley, *Oliver Cromwell*, pp. 239-240.)

8. Explain the meaning and significance of the fundamental Laws of Thought. Do they furnish more than a negative test of truth?

Show that in every judgment there must be some identity between subject and predicate, but that in no judgment can subject and predicate be wholly identical.

9. State the three fundamental Laws of Thought, explain their meaning, and consider how far they are independent of each other.

10. State the law of Sufficient Reason, and discuss its logical place and value.

CHAPTER III.

1. How does the study of terms enter into Logic? Distinguish between (1) general and singular, (2) abstract and concrete terms, and illustrate the difficulties that arise in applying the distinction.

Upon what grounds would you decide whether the name "The Right Honourable H. H. Asquith" has extension or intension only or both?

2. What is a connotative name? Give examples showing different ways in which the connotation of a name is such that the name is understood to be applicable to a single object. Distinguish such names from Proper Names.

3. What different senses have logicians attached to the expression —the intension of a term? Are there any names which simply denote objects without giving any information about the characteristics of those objects?

4. "When we consider the use or function of terms we find that they are never used merely to name things or merely to connote attributes." Discuss this. Has it ever been maintained, so far as you know, that terms may "merely connote attributes"? How far does the dispute about the connotation of proper names arise from a difference in the meaning attached to the word connotation?

5. Explain the distinction between the intension and the extension of terms. What is the meaning of the statement that as the one increases the other decreases, and what are the limits to the accuracy of the statement? Is it true that the more comprehensive term has less significance than the less comprehensive?

6. (*a*) Distinguish between Abstract and Concrete Terms. Would you say that a generic term such as 'animal' is more abstract

than a specific term such as 'horse'? Give reasons for your answer.

(*b*) *Upon what grounds have abstract terms, if they are names of* a simple attribute, been thought to be destitute of connotation? Examine this contention.

7. Describe the nature of Collective Terms; can they be distinguished from general terms?

8. Give the *logical characteristics of—organism, force, nationality*, His Eminence, our American Cousin, monopoly, The Renaissance, the judicature.

9. Describe the logical characteristics of the following terms—lame, crowd, colour, son, equation, unholy, ant, Lord Shaftesbury, the tallest man alive, the Lord Chancellor, non-Christian.

10. Discuss the question whether the following terms are respectively connotative or non-connotative—London, St. Paul's Cathedral, The Emperor of India, man, parallelogram.

CHAPTER IV.

1. Discuss the logical importance of the Heads of Predicables.

2. Mathematics has been said to be distinguished from Physical Science by the preponderance in the former of affirmations of Proprium. Account for this difference. Can you give any non-mathematical scientific examples of such predication?

3. What is a predicable? Enumerate and describe the different predicables, constructing propositions to illustrate them.

Distinguish analytical and synthetical propositions in terms of the predicables they respectively employ.

4. Distinguish between a Verbal and a Real Proposition. Consider under which head the following ought to be classed—(1) Whatever is, is. (2) Those organisms survive which are best adapted to survive. (3) Right is right. (4) Man is man (and master of his fate). (5) It is inadvisable to eat too much. (6) The expense of travelling on the continent depends on the habits and means of the traveller, and his mode of journeying, and likewise on the rate of charges made in the various countries. (7) We are able to reason because we have the faculty of ratiocination.

5. Give the Genus, the Differentia, a Proprium, and an Accidens of—gold, Darwinian, rhombus, house; and say why you choose what you give in your answer.

CHAPTERS V.-VI.

1. What are the chief uses in defining a term? and what are the chief classes of terms that are indefinable?

2. State the meaning of Definition, and carefully explain any technical terms that you use in your statement.

Comment critically, with examples, on (*a*) the distinction between the nominal and real definition, (*b*) the distinction between the definable and the indefinable.

3. Comment on the following definitions—

(*a*) A multiple is any number divisible by another number.

(*b*) A legislator is a member either of the House of Commons or of the House of Lords.

(*c*) A straight line is that which lies evenly between its extreme points.

(*d*) An idle person is one who will not work when he can.

4. Show by examples the nature of the mistakes that are likely to be made in trying to define such words as Verb (in grammar), Factor (in Arithmetic), Combination (in Chemistry), Pleasure (in Psychology). (Other words may be substituted by the examinee for those suggested.)

5. In what respects, if any, do the following definitions fail to satisfy logical requirements?—

(1) Causality is uniform antecedence in time.

(2) Strength of will is that which enables a person to persist in his voluntary resolutions.

(3) Life is the totality of vital phenomena.

(4) Orthodoxy is my doxy; heterodoxy is another man's doxy.

(5) Sound sensations are those which are produced by stimulation of the auditory nerve.

6. Discuss the place and importance of Genetic Definition in knowledge.

7. "Definition must be both the starting-point and also the end and aim of all thinking." How do you understand this statement?

8. Compare the definability of the following terms—book, square, dullness, Victoria, silver, exogens, thermometer, coal, brilliancy, the present Chancellor of the University of London, Cambridge, cart, this ear.

9. Point out the fallacy, if any, in each of the following—

(*a*) All responsible beings are rational; responsibility increases with the increase of rationality; some dogs are more

rational than some men; therefore some dogs are more responsible than some men.

(*b*) If I am to pass this examination, I shall pass it, whether I answer correctly or not; if I am not to pass it, I shall fail, whether I answer correctly or not; therefore it is of no consequence how I answer the questions.

(*c*) The laws of Nature never can be broken. Social law is a part of the general system of Nature; therefore it cannot be broken.

(*d*) To assault another is wrong; consequently a soldier who assaults another does wrong.

(*e*) I shall not join your society: you can't expect me to do everything.

10. Discuss the main faults in definition and illustrate them by examples.

11. What fallacies may lurk in (1) the ambiguity of the word 'all,' (2) the use of metaphor?

12. Define the term Fallacy as employed in Logic and distinguish a fallacy, as thus employed, from a false belief.

CHAPTER VII.

1. Explain what is meant by Definition and Division, and show their relation to each other. Criticise the following as divisions:—(1) Human beings into men, women, and children. (2) Fungi into mushrooms and toadstools. (3) Substances into ponderable, imponderable, and spiritual. (4) Chair into legs, back, and seat. (5) Governments into democracy, oligarchy, monarchy, and anarchy. (6) Organisms into plants, animals, and social organisms.

2. Give concisely the formal rules of logical division. How is the differentia of a species connected with the *fundamentum divisionis* upon which the division of the genus proceeds? Examine the following examples of division; and amend them where necessary—

(1) Triangles contain either a right angle, an obtuse angle, or an acute angle.

(2) Numbers are either positive or negative or whole or fractional or prime or composite.

3. Criticise the statement that a purely formal division is a possible process.

4. Criticise the following divisions—

(1) North America into Canada, the United States, and Mexico.

(2) Terms into Singular, General, Abstract and Concrete.

(3) Books into bound, and unbound.

(4) A piece of lime into whiteness, solidity, weight, and extension.

(5) A person into bones, flesh, feeling, and will.

5. Explain what is meant by Classification, and show what excellencies a classification should have, and to what faults it is liable.

6. Distinguish Logical Division from Natural Classification. State and illustrate the rules of logical division, and consider how far they can be realised in classification.

7. Distinguish between a natural and an artificial classification, giving illustrations. What are the precepts and precautions to be observed in selecting the basis of classification?

8. In the Theory of Classification, what do you understand by essential characteristic, constant, important, and accidental features or properties? Give examples.

9. How far can we distinguish kinds of Classification by reference to the purposes which they serve?

10. Explain and illustrate the difference between the Nomenclature and Terminology of a Science, and show briefly the principles upon which they should be constructed.

Are there any technical terms of a Science other than those which belong to its Nomenclature or Terminology?

11. Discuss the principle on which *subsumptive* classifications are made and the limitations to which it is subject in actual scientific work.

CHAPTERS VIII.-X.

1. What do you understand by a Categorical Proposition? What other forms of judgment can be expanded into Categorical Propositions.

2. Discuss the function of the copula in logical propositions.

3. What is the essential distinction between the subject and predicate of a logical proposition? Illustrate by examples.

4. What is meant by the Quantity of a Proposition? Why are singular propositions classed under universal propositions? Bring out carefully, with the aid of illustrations, the various meanings

or ambiguities attaching to the designations, some, any, all, few, most.

5. Define 'exclusive' and 'exceptive' propositions. What is the logical relation, if any, between them? Express the following propositions in logical form, and give the contradictory of each—

(*a*) The only man in the house was the landlord.

(*b*) Any one but an idiot would believe it.

(*c*) No admittance except on business.

(*d*) Members alone can vote.

What is the logical relation between the propositions "Only graduates are eligible" and "Some graduates are eligible"? Under what conditions is this logical relation valid?

[An *Exclusive Proposition* contains a word such as *alone* limiting the subject—*S alone is P*: An *Exceptive Proposition* excludes a portion of the denotation of the subject-term from the predication by some such word as *except*, *unless*—*Every S is P unless it is M.*]

6. Reduce each of the following to its strict logical form and indicate the quantity and quality of each logical proposition—

(*a*) All men are not liars.

(*b*) Any man would have done the same.

(*c*) Few are successful in achieving their ambitions.

(*d*) A few were not disheartened by failure.

(*e*) Scarcely any marksmen score the full total possible.

7. State in logical form; indicate the subject and predicate; and give the quality and quantity of the following—

(*a*) All that glitters is not gold.

(*b*) Axioms are self-evident.

(*c*) Among Englishmen many great generals are found.

(*d*) Not one of the enemy escaped.

8. Give a full discussion of Indesignate propositions.

9. Express as clearly as possible what you take to be the nature of a hypothetical proposition.

(*a*) Compare the import of the hypothetical proposition, *If S, then P*, with that of the categorical proposition, *All S is P.*

(*b*) How far can the disjunctive proposition be resolved into hypotheticals, and into how many?

10. Express in hypothetical form the following disjunctive propositions, taking account of the meaning in each case—

(1) A line must be either straight or curved.

(2) To obtain this post a man must possess either merit or influence.

11. Discuss, with examples, the question whether Hypothetical Propositions can be reduced to Categorical Propositions.

12. In what way should the Disjunctive Proposition be interpreted? What is its relation to (*a*) the Categorical Proposition, (*b*) the Hypothetical Proposition?

13. How far do distinctions of Quality and Quantity apply to Hypothetical and Disjunctive Propositions?

14. Express the following Hypothetical Propositions in the fundamental three-term form—

(*a*) If a man steals, the law should punish him.
(*b*) If the report is true, what you say is untrue.
(*c*) If you do that, I will punish you.
(*d*) If study is well done, the student will gain.
(*e*) If two parts of hydrogen combine with one part of oxygen, water is formed.
(*f*) If this work requires three hours a day, I cannot do it.

15. What is meant by the 'Import of Propositions'? State various views which logicians have held on this subject. Which view seems to you to be preferable, and why?

16. "Judgment consists in comparing together two notions or ideas of objects derived from simple apprehension, so as to ascertain whether they agree or differ."

Critically examine this view, and indicate carefully the relation you take to hold between notions, judgments, and inferences.

17. How far is it (*a*) legitimate, (*b*) adequate, to construe an affirmative categorical proposition as meaning that the subject is included in the predicate class?

18. Examine how far the import of categorical propositions can be represented by diagrams. Comment upon any diagrammatic scheme that has been adopted by logicians, and indicate the extent to which it succeeds or fails in satisfying the requirements of such diagrammatic representation.

CHAPTERS XI.-XIV.

1. Arrange the following propositions so as to show whether or not the truth or the falsity of one can be inferred either from the truth or the falsity of another; No intelligent persons are prejudiced; All unprejudiced persons are intelligent; Some unintelligent persons are unprejudiced; Not every prejudiced person is

unintelligent. Explain any technical processes which you may use in working out your answer.

2. Distinguish (*a*) contradictory and contrary terms, (*b*) contradictory and contrary propositions.

What precise meaning do you attach to the law of contradiction?

How is the square of opposition affected if 'some' is taken to mean 'some but not all'?

3. Give the contradictories of the following hypotheticals—

(1) If the material world is finite, it must have been created.

(2) If a tiger once tastes human blood, it becomes dangerous.

4. What immediate inferences can be drawn from each of the following four propositions?—

(*a*) If x is true, then y is true; (*b*) x would be true, if y were true; (c) x and y cannot both be true; (*d*) either x is true or y is true.

What conclusion (if any) could be drawn in each case if it were further known (1) that x is true, (2) that x is false?

5. Distinguish the logical nature of Immediate Inference from that of Opposition. Explain why it is impossible to convert an O proposition.

State the following in logical form and give their converse and contrapositive—

(1) None but young soldiers run away, and they only in their first engagements.

(2) Starvation wages are better than none at all.

(3) Not all the riches of the earth can bring happiness.

(4) Humour is not given to all.

6. State the following propositions in logical form, and give, where possible, the Converse and Contradictory of each—

(*a*) All knowledge is only recollection.

(*b*) We cannot all command success.

(*c*) That is all that I asked for.

How would you prove the rules for Conversion, Obversion, and Contraposition?

7. (*a*) On what grounds has it been proposed to exclude the process called 'immediate inference' from the sphere of inference proper? Discuss.

(*b*) State, giving examples with *S* and *P* used for terms, what can be asserted as to the truth of a proposition from—

(i) The falsity of its contrary.

(ii) The truth of its simple converse.

(iii) The truth of its sub-contrary.
(iv) The falsity of its converse.
(v) The falsity of its sub-alternate.

8. Draw as many inferences as you can from the following propositions, naming them by their logical names—

(*a*) A merry heart goes all the way.

(*b*) You can only support yourself on something which offers resistance.

9. Give, where possible, the contrary, the contradictory, the converse, and the contrapositive of the following—All tyrants are unhappy. Some crows are white. When sinners repent, sometimes they are not forgiven. If a man seeks pleasure he can never attain happiness. If Aristotle is right, Herbert Spencer is wrong.

10. Give the most concise proofs that you can of the rules for Conversion, Obversion, and Contraposition of Propositions.

11. Give, where possible, the contradictory, converse, and contrapositive of the following—

(i) Not all despots are cruel.
(ii) Only philosophers fail to see the difference between a post and my idea of a post.
(iii) He who is down need fear no foe.

12. Reduce to logical form and give the contrapositives of the following propositions—

(1) More haste less speed.
(2) None but himself could be his parallel.
(3) Only fools learn by experience.
(4) It is an ill wind that blows nobody any good.
(5) A miss is as good as a mile.

13. Express in ordinary categorical form each of the following propositions, examine the distribution of its terms, and give its contradictory and contrapositive—

(*a*) We smile not only when we are pleased.

(*b*) The price of an article does not always rise immediately after it is taxed.

(*c*) Any one may judge of B.'s conduct who examines the evidence.

Wherein lies the fallacy of the following?—A poet is a man, therefore a good poet is a good man.

14. Show how diagrams may be used to prove the rules of conversion and contraposition.

Show also, by means of diagrams, that we cannot infer from

"All A's are B" that "Some B's are not A"; nor from "Some A's are not B" that "Some B's are not A."

CHAPTERS XV.-XVII.

1. Briefly discuss the claim of method to a place in logical doctrine.

2. Discuss the relation between the processes called Induction and Deduction on the one hand, and those called Analysis and Synthesis on the other.

3. Compare the respective functions of logical analysis and synthesis. Is there any fundamental opposition between them?

4. Criticise the statement that science progresses through analysis to synthesis.

5. On what grounds would you describe mathematics as mainly a synthetic science, and chemistry as mainly an analytic science?

6. What are the essentials of a good method? How far can they be usefully expressed in rules?

7. Compare mathematical and historical inference.

8. Indicate concisely what you take to be the essential nature of inference.

9. Discuss the place and function of the empirical element in geometry.

10. Discuss the conception of system in relation to (*a*) a particular science, (*b*) knowledge as a whole.

11. Explain exactly the nature of the fallacies *Petitio Principii* and *Ignoratio Elenchi*. Examine, with the aid of illustrations, the mental conditions which may give rise to these fallacies.

12. Examine logically the following arguments—

(*a*) "If it be said that there is an evil in change as change, I answer that there is also an evil in discontent as discontent."

(*b*) "I speak not from mere theory. There exist at this moment practical illustrations of my assertions."

(*c*) An adverse decision, my lords, will seriously prejudice the political prospects of my client. I beg you, therefore, to weigh well a decision which, if unfavourable, will spell disaster for an honourable man.

(*d*) Who can deny that this measure will ameliorate the lot of our fellow-citizens when we reflect that it will raise the standard of comfort in every home?

(*e*) "He has spoken of that noble person and of his intellect in terms which, were I disposed to retort, I might say show the hon. gentleman to be possessed of an intellect which would justify me in passing over in silence anything that comes from such a man."—(Charles James Fox.)

(*f*) No man can witness the daily misery and oppression of the poor in our land without being stirred to redress their wrongs. I need say no more. That will be sufficient for all enlightened men to support this reform which has no other aim than to relieve their distress.

CHAPTERS XVIII.-XXI.

1. Define a categorical syllogism in its narrowest sense; and give in shortest form the necessary and sufficient rules for the validity of a syllogism.

2. If, in a valid syllogism, the conclusion be false, must the premises be likewise false; if the premises be false, can the conclusion be true; if the conclusion and one of the premises be true, can the other premise be false? Give reasons for, and illustrate, your answer.

3. Give the meaning of the *Dictum de omni et nullo.* How is the proper statement of it affected by the method of interpreting the import of propositions in extension or intension, or both?

4. (i) State the *Dictum de omni et de nullo,* and prove from it the "Special rules for the First Figure."

(ii) What proof would you offer of the rules of the Hypothetical Syllogism? Give concrete examples in illustration of the rules. Comment on the term 'reduction' as used of the transformation of a hypothetical into a categorical syllogism, or *vice versa.*

5. What is a hypothetical syllogism? Explain the reason for the rule—affirm the antecedent or deny the consequent. To what fallacies of the categorical syllogism do the violations of this rule correspond?

Explain how the rule is connected with the doctrine of the plurality of causes, and why it is correct in spite of the rule to argue "if a triangle is equilateral, it is equiangular; but this triangle is equiangular and therefore it is equilateral."

6. Discuss the question whether the alternatives of a disjunctive syllogism are mutually exclusive. Show in what way the answer given to this question affects that form of the disjunctive syllogism known as *Modus ponendo tollens.*

7. Give a clear and precise explanation of the rule concerning the Middle Term of a syllogism.

8. When is it impossible to draw a valid conclusion from two given premises? Explain why the inference is impossible.

9. Prove that a particular premise necessitates a particular conclusion.

10. Show, from the general rules of syllogism, that in the third figure the minor premise must be affirmative and the conclusion particular.

11. Explain why the third figure of the syllogism is "suitable to the discovery or proof of instances and exceptions," and prove *by the syllogistic rules* that in the second figure the conclusion must be negative.

Construct an *example* (not a symbolical formula) in *Baroco* in the second figure and reduce it directly and indirectly.

12. Give the special rules for the quality and quantity of the premises in the second and third syllogistic figures and explain the reasons on which they are based. Reduce to the first figure the following syllogisms—

(1) Some logicians are not good reasoners; all logicians know how to reason well; therefore some who know how to reason well do not reason well.

(2) Some virtuous persons are bores; all bores show a lack of sympathetic imagination; therefore some persons who show a lack of sympathetic imagination are virtuous.

13. State the special canons of the four figures; construct examples of syllogisms in *Camenes*, *Darapti*, and *Baroco*, and reduce them to the first figure.

14. Why cannot O stand as a premise in the first, as a major in the second, as a minor in the third, or as a premise in the fourth figure?

15. Prove that in the first figure (1) the major premise must be universal, (2) the minor premise must be affirmative.

16. If the major term be universal in the premises and particular in the conclusion determine the mood and figure, it being understood that the conclusion is not a weakened one.

17. What is meant by Mood? Show from the rules of the syllogism what moods are alone admissible in the second figure.

18. Construct examples of syllogisms in *Camestres*, *Disamis*, *Ferison*, and reduce them to the first figure.

19. How far does the doctrine of Figure and Mood apply to

(1) Pure Disjunctive Syllogisms, (2) Pure Hypothetical Syllogisms?

20. Prove that in every figure, if the minor premise is negative, the major premise must be universal.

21. State the following argument in a syllogism of the third figure, and reduce it both directly and indirectly to the first—Some things worthy of being known are not directly useful, for every truth is worthy of being known, while not every truth is directly useful.

CHAPTERS XXII.-XXIII.

1. What are the valid moods in Mixed Syllogisms, and how are they determined?

2. Discuss, with examples, the reduction of Mixed Hypothetical Syllogisms to the categorical form.

3. Explain the *modus ponendo tollens*, and examine the assumptions upon which its validity depends. If I argue from the truth *of A to the falsity of B, what is the real relation between A and* B which is required to make the inference valid?

Regarding the following two arguments as examples of the *ponendo tollens*, point out analogies between them and introduce in each case the suppressed premise: (1) A man may be without a vote, though not a pauper; but, from the fact that Mr. A. has a vote, I am able to infer that he is not a pauper. (2) There would be nothing absurd in maintaining that the doctrines of Natural Selection and of Utilitarianism are both false; but from my conviction that the former is true, I derive the conclusion that the latter is not.

4. What is meant by Disjunctive Syllogisms, and what conclusion can be drawn from them? Give examples.

5. Is it possible to apply distinctions of figure either to Hypothetical or Disjunctive Syllogisms?

6. Consider the following argument—Those who agitate for the franchise do not deserve it and those who do not are indifferent. It should not therefore be conferred.

7. Construct in symbols a Trilemma corresponding to each of the four kinds of dilemma.

8. What are the essential characteristics of the Dilemma? How many different kinds of Dilemma are to be recognised? Give an example (in symbols) of each.

Indicate the most common sources of fallacy in dilemmatic arguments, with illustrations.

9. "The plan of meeting a dilemma by another dilemma is a purely rhetorical device and has no logical efficacy." Comment on this and give illustrations.

10. Exemplify, and analyse the different kinds of Sorites.

11. Classify the following and examine their validity—

(*a*) Those who have shall not receive; those who do not receive do not want.

(*b*) If we have a wet summer there is always a good clover crop. We shall therefore have a poor crop this year, for the summer has been very dry.

(*c*) If the train is late, I shall miss my appointment; if it is not late, I shall miss the train: but either it will be late or not late; therefore, in any case, I shall miss my appointment.

12. Reduce the following to logical form, and say if either contains any fallacy—

(*a*) Free trade is a great boon to the working man, for it increases trade, and thus cheapens articles of ordinary consumption; this gives a greater purchasing power to money, which is equivalent to a rise in real wages; and any rise in real wages is a boon to the working man.

(*b*) All thieves are dishonest; all dishonest persons are immoral; some immoral persons are not punished; therefore, some thieves are not punished.

CHAPTER XXIV.

1. "Certain individuals have a given attribute; an individual or individuals resemble the former in certain other attributes; therefore they resemble them also in the given attribute."—(Mill.)

Consider whether this adequately expresses the nature of syllogistic reasoning.

2. Upon what grounds has it been asserted that the conclusion of a syllogism is drawn, not from, but according to, the major premise? Are they valid?

3. (1) You do not argue from but only according to the major premise of a syllogism.

(2) If the major premise is not true, you cannot draw the conclusion.

Are these statements consistent, and, if so, in what sense?

4. (*a*) Explain carefully what you conceive to be the function in a syllogism of the major and minor premise respectively, and briefly examine the contention that when the major premise is admitted the conclusion is asserted.

(*b*) Prove that if the middle term of a syllogism is twice distributed, the conclusion must be particular.

5. If the conclusion of an inference is implied in the premises, can it give new knowledge? Discuss with special reference to Induction.

6. Determine the character and form of the following arguments—

(*a*) X lies to the south-east of Z, being due south of Y, which is due east of Z.

(*b*) A is taller than C, being taller than B.

7. Examine the following: All strictly formal reasonings can be reduced to the syllogistic form.

8. Examine Mill's view that the Syllogism, considered as an argument to prove its conclusion, is a *Petitio Principii.*

CHAPTERS XVIII.-XXIV.

[*The following exercises should be worked when the complete doctrine of the syllogism has been mastered.*

1. Put the following arguments as far as possible into logical form, examine their validity, and name any fallacy you may discover—

(i) He must be a Scotchman, for no Scotchman can see the force of a joke.

(ii) Partners in the same enterprise have the same interest. How then can there be antagonism between my workmen and me?

(iii) Only if an officer is cool-headed does he deserve the chief command, and as this officer has not been thought worthy of the appointment, we may conclude that he is not cool-headed.

(iv) The king, by the constitution, can do no wrong, and therefore he is not rightly subject to praise or blame.

2. Examine technically the following arguments—

(*a*) All men have equal rights; therefore, if A. has a right to £500 a year, B. has a right to £500 a year.

(*b*) The great majority of persons who have had small-pox have been vaccinated; therefore, vaccination can have no efficacy as a preventive of small-pox.

(*c*) Attention is a reflex act and is determined by interest. Now a reflex act may be defined as one that is determined by a physical stimulus. Therefore, interest is a physical stimulus.

3. State the following arguments in syllogistic form, and examine their validity—

(*a*) It is too cold here for this plant to grow.

(*b*) Some statesmen are also authors; for such are Beaconsfield, Gladstone, Balfour, and others.

(*c*) The literary style of these two books is similar, and therefore they are probably by the same author.

4. State the following arguments in syllogistic form where possible, and examine their validity—

(*a*) If a man be rightly entitled to the produce of his labour, then no one can be rightfully entitled to anything which is not the produce of his labour.

(*b*) Poetic genius and scientific ability are perfectly compatible, as the instance of Goethe proves.

(*c*) M is the only possible cause of P; if therefore we find P occurring, we may be sure of the presence of M.

(*d*) It is useless to give advice; for if a man is wise he will not need it, and if he is not wise he will go his own way.

5. Examine the following:—(1) Reason is identical in nature wherever it is found, and all men are rational, therefore all men should be treated as equal. (2) To make a good bargain is an advantage, therefore the more good bargains we make the better. (3) A person found guilty by a jury may be innocent; John Jones has been found guilty, therefore he may be innocent. (4) Do not interfere. For if he means to do wrong he will do it, and if he means to do right your interference will be impertinent.

6. Examine the following arguments—

(1) He is a very bad marksman; hence it is safest to stand in front of the object he is aiming at.

(2) The truly brave are never bullies: hence all bullies are cowards.

(3) Wisdom is inseparable from benevolence: hence all benevolent persons are wise.

(4) A man may smile and smile, and be a villain; my friend Smith is constantly smiling: therefore he may be a villain.

(5) This regiment behaves bravely in action: therefore it must be composed of brave men.

(6) B. condemns Napoleon and Caesar. But Napoleon and Caesar are great men; therefore he condemns great men.

7. Express in syllogistic form and examine the following arguments, naming the fallacies which you may detect—

(1) Idiots cannot be men, for man is a rational being.

(2) Everybody is benefited by a University training, therefore it would be a good thing if everybody had a University training.

(3) Trespassers will be prosecuted. But if there are none, none will be prosecuted. Therefore somebody will trespass.

(4) Apart from experience due to the state of the internal organs of the body, emotion can have no existence. Therefore emotional consciousness is wholly constituted by these experiences.

8. Reduce to syllogistic form (giving mood and figure) and examine the validity of the following arguments—

(*a*) Everyone must be either a socialist or a free-trader; but there are no free-traders who are pessimistic; hence it follows that only socialists are pessimistic.

(*b*) Few soldiers can be considered heroes; for anyone who is incapable of fear must be accounted a hero, and but few soldiers can be said to be incapable of fear.

(*c*) Englishmen admire all who are successful; they must, therefore, admire some persons who are politically dangerous, for assuredly there are some successful persons who are politically dangerous.

9. Examine the following, explaining what the character of the argument is, stating it in logical form, and describing it by its logical name, whether fallacious or not—

(*a*) It is difficult to understand why a statesman should lay himself open to the charge of little-mindedness by harping upon his consistency, which it is well known is the failing of little minds.

(*b*) I am accused of inciting to sedition by the address which I delivered to the meeting. But there is not one man present at the meeting who, if my remarks had been addressed to him privately, would have been moved to disloyalty.

(*c*) "To prove continuity and the progressive development of the intellectual faculties from animal to man is not the same

as proving that these qualities have been developed by natural selection. Because man's physical structure has developed from *an animal form* by *natural selection*, it does *not* necessarily follow that his mental nature, though developed *pari passu* with it, has been developed by the same cause only."

10. Test the following arguments and name any fallacies they may contain—

(i) Ill-managed business is unprofitable. Railways are never ill-managed. Therefore all railways are profitable.

(ii) A vacuum is impossible, for if there is nothing between bodies they must touch.

(iii) The governor of a country ought not to be blamed for using his influence to further his religious views, for every man has a right to inculcate his own opinions.

11. Set out the course of the following, bringing out the character of the argument and counter-argument, and stating that as far as possible in logical form—

"Schoolmasters speak of the disciplinary value of the classics (as giving them a superiority over the mother tongue). They are apt to forget that our teaching of Latin comes from a time when Latin was used as a real substitute for the mother tongue—when it was a living language. There is no need in any way to belittle the utilitarian or disciplinary advantages of teaching foreign languages, ancient and modern. They have at any rate one fundamental use in enabling children to distinguish between words and ideas, since the same idea is translated in different languages by different words. But to a large number of our children they must remain inaccessible; and in any case the teaching of a foreign language can as little replace the teaching of the mother tongue as a finger can replace the use of the hand."—(Hartog, *The Writing of English.*)

12. Examine the following arguments: express them where possible in syllogistic form: supply any premises or conclusions which are implied but not expressed: give the figure and mood of valid syllogisms, and, if fallacy occurs, determine its nature.

(*a*) "With regard to the high aristocratic spirit of Virginia and the southern colonies, it has been proposed, I know, to reduce it by declaring a general enfranchisement of their slaves. This project has had its advocates and panegyrists; yet I never could argue myself into any opinion of it. Slaves are often much attached to their masters. A general wild offer of liberty would not always be accepted. History furnishes few instances

of it. It is sometimes as hard to persuade slaves to be free as it is to compel freemen to be slaves; and in this auspicious scheme we should have both these pleasing tasks on our hands at once. But when we talk of enfranchisement, do we not perceive that the American master may enfranchise too; and arm servile hands in defence of freedom? A measure to which other people have had recourse more than once, and not without success, in a desperate situation of their affairs."—(Burke, *On Conciliation with the American Colonies.*)

(*b*) "THE DANGER OF TOUCHING THE PRESS IS THE DIFFICULTY OF MARKING ITS LIMITS. My learned friend, who has just gone out of Court, has drawn no line and unfolded no principle. He has not told us, if *this* book is condemned, *what* book may be written. If I may not write against the existence of a monarchy, and recommend a republic, may I write against any part of the Government? May I say that we should be better without a House of Lords, or a House of Commons, or a Court of Chancery, or any other given part of our establishment? Or if, as has been hinted, a work may be libellous for stating even *legal* matter with *sarcastic* phrase, the difficulty becomes the greater, and the liberty of the press more impossible to define."—(Henry, Lord Erskine, *Speech on behalf of Thomas Paine.*)

(c) "France has a representative government; and as the unjust privileges of the clergy and nobility are abolished; as she is blessed with a most wise, clear, and simple code of laws; as she is almost free from debt, and emancipated from odious prejudices, she is likely to prove an example and a light to the world."—(Daniel O'Connell, *Speech at a Meeting to Recover Catholic Rights.*)

(*d*) "Those are our present prospects of success. First, man is elevated from slavery almost everywhere, and human nature has become more dignified, and, I may say, more valuable. Secondly, England wants our cordial support, and knows that she has only to secede to us justice in order to obtain our affectionate assistance. Thirdly, this is the season of successful petition, and the very fashion of the times entitles our petition to succeed. Fourthly, the Catholic cause is disencumbered of hollow friends, and interested speculators. Add to all these the native and inherent strength of the principle of religious freedom and the inert and accumulating weight of our wealth,

our religion and our numbers, and where is the sluggard that shall dare to doubt our approaching success?"[1]—(O'Connell, *ibid.*)

CHAPTERS XXV.-XXVI.

1. Explain the general process of inductive reasoning, and describe its aim.

2. Distinguish induction from deduction, and contrast popular with scientific induction, so as to bring out the chief defects of the former.

3. Discuss the part played by deductive inference in inductive enquiry. Are we justified in supposing that every inductive enquiry must contain some deductive element?

4. What is meant by describing induction as the inverse process of deduction? Compare this with other accounts of inductive reasoning.

5. How would you express the Law of Causation? Examine the definition of Cause as the invariable and unconditional antecedent.

6. Define a Uniformity; and explain how Uniformities can be classified. Give illustrations.

What is meant by saying that "Uniformity pervades Nature"?

7. Inquire into the possibility of regarding the principle of the Uniformity of Nature as an induction from experience.

"If we ask what is involved in the conception of a cause *not* acting uniformly, we shall see that it is the same as denying the existence of causal connexions altogether." Discuss this.

8. What do you take to be the ultimate principle upon which inductive reasoning is based? What kind of justification would you offer for the principle in question?

Comment on the following:—"The contrary of every matter of fact is still possible; because it can never imply a contradiction. That the sun will not rise to-morrow is no less intelligible a proposition, and implies no more contradiction, than the affirmation, that it will rise."

9. What different meanings do the logician and the psychologist respectively attach to the question, "What is causal connexion?"

10. Consider carefully whether the cause of the fall of a stone

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[1] The extracts from speeches—(*a*) to (*d*)—are taken from *Political Orations*, ed. by William Clarke.

would be more correctly said to be (*a*) the earth, or (*b*) the force of gravitation, or (*c*) the previous process of raising the stone to the position from which it falls.

11. Is it necessary that a *cause*, as Science understands the term, should be (*a*) antecedent to its effect, (*b*) immediately antecedent? Discuss the view that one and the same effect may proceed from a number of alternative causes.

12. What different kinds of conditions have to be considered when we speak of the cause of an event as the sum-total of the conditions upon which the occurrence of the event depends?

13. Define precisely what Science understands by the Cause of an event, and illustrate your definition by a typical example of causation as determined by experiment.

What is the relation of the scientific conception of Cause to the conception of Cause as the sum-total of the conditions on the occurrence of which the event depends?

Can you account for the divergence between the scientific and popular views as to the Cause?

14. "The Uniformity of Nature is the ultimate Major Premise in all Induction."—(Mill.) Explain and discuss this statement.

CHAPTERS XXVII.-XXVIII.

1. To what extent do (*a*) unintentional inference and (*b*) selective interest enter into the process of scientific observation?

What precautions must be taken in observation and experiment to avoid error?

2. How does Experiment differ from Observation? Is the difference fundamental? Show the dependence of Observation on previous knowledge and in what sense it always involves inference.

3. "Experiment is always preferable to observation." Why is this? Explain also from the example of any sciences how observation and experiment supplement each other.

4. Examine the relation or distinction between (*a*) Experiment and (*b*) the use of Instruments of Observation. Give examples showing the utility of each.

5. What is the logical character of the appeal to "the testimony of the senses"? Does this testimony really bear witness to the motion of the sun relatively to the earth?

6. Indicate the special errors to which observation and experiment are liable. Give illustrations.

7. Examine how far Logic can assist us in avoiding the fallacies of Non-observation and of Mal-observation. Distinguish these fallacies from errors of observation, and explain how the latter can be eliminated by systematic experiment.

8. Discuss the place and importance of testimony in (*a*) history, (*b*) physical science.

9. If five witnesses swore to a prisoner's presence near the scene of crime, and only one swore to an *alibi*, would that be sufficient to prove the falsity of the latter assertion?

10. What are the general tests of the trustworthiness of testimony?

11. Discuss the statement that for the purposes of history a man writing thirty years after an event is not necessarily in any better position than a man writing three hundred years later—indeed is often less able to trace the course of events with accuracy.

12. What is the justification of textual criticism as a branch of historical enquiry?

13. "It doesn't matter who wrote the book, nor when it was written: read it for its elements of permanent value." How far can the historian accept this attitude?

14. Examine critically the following argument—

"What a barrister calls his 'case,' his theory or explanation, is a single and organic thing. It is a story. If the tale is true, it is coherent. If the tale is false then some other tale is true; one we do not know; one perhaps very different; one possibly that we cannot even imagine. But the clinching point is this. Until we know that other tale which is true we cannot tell *how much* of the first tale is false. That *any* of it is false is a reason for digging deeper. But *all* of it may be false. Whatever it was that tainted the admittedly tainted evidence may also taint the apparently untouched evidence. We cannot tell till we know what the taint was. Until we know why Jones, Brown, and Robinson lied we cannot be sure that Smith is telling the truth."

(G. K. Chesterton, in *Daily News*, April 12, 1911.)

CHAPTERS XXIX.-XXX.

1. It is said that all induction depends on hypothesis. How far is this true?

When may a hypothesis be regarded as established?

2. Examine and illustrate the characteristics of hypothesis as a

conscious process of scientific investigation. Show that a rejected hypothesis need not necessarily have been fruitless.

3. Exemplify various ways in which hypotheses are suggested, and explain why no rules can be given for their formation.

4. Indicate clearly the nature and the place of hypothesis in scientific investigation. State and illustrate the conditions to which a legitimate hypothesis must conform.

5. Distinguish between a working hypothesis and an established hypothesis, so as to bring out the conditions upon which the latter depends.

6. A Hypothesis is sometimes defined as "a generalisation adopted on avowedly insufficient grounds." Comment on this definition, and inquire whether any other meaning can be attached to the term Hypothesis.

Why have Hypotheses (as above defined) any utility?

7. Describe *inductio per enumerationem simplicem*, indicating its mode of origin and the process of thought that seems to be implied in it.

8. What is the nature of the argument from Analogy? How would it be expressed in syllogistic form? Can it ever be regarded as conclusive?

Show, with illustrations, the place of analogical reasoning in the process of scientific discovery.

9. Define the true place of analogical reasoning in scientific explanation. How would you differentiate a sound from an unsound analogy? Give illustrations.

10. What is the connexion between the argument from analogy and the process of enumerative induction?

11. What is the logical force of the following?—

(*a*) "During the reigns of the kings of Spain of the Austrian family, whenever they were at a loss in the Spanish councils, it was common for their statesmen to say that they ought to consult the genius of Philip the Second. The genius of Philip the Second might mislead them; and the issue of their affairs showed that they had not chosen the most perfect standard. But, Sir, I am sure that I shall not be misled, when in a case of constitutional difficulty, I consult the genius of the English constitution."—(Burke, *On Conciliation with the American Colonies.*)

(*b*) England has a democratic franchise, therefore India should have a democratic franchise too.

(c) "I have somewhere read, I think in Sir Walter Raleigh's *History of the World*, of a most bloody and fatal battle which was fought by two opposite armies, in which almost all the combatants on both sides were killed, 'because,' says the historian, 'though they had offensive weapons on both sides, they had none for defence.' So, in this war of words, if we are to use only offensive weapons, if we are to indulge only in invective and abuse, the contest must be eternal."—(Charles James Fox, *On the French Overtures for Peace, Feb. 3rd, 1800.*)

(d) All the great empires that have ever existed have lost their position of eminence, hence no great empire in the future will maintain its supremacy.

CHAPTERS XXXI.-XXXII.

1. Describe the main preliminary processes or assumptions that have to be made before practically applying such methods of Induction as have been formulated by Mill.

2. What are the so-called Inductive Methods, and what is their use? Explain, with an illustration, the Method of Difference, and contrast the principle upon which it proceeds with that of the Method of Agreement. What is the connexion between it and the Method of Concomitant Variations?

3. Examine and compare Mill's "Method of Agreement," "Method of Difference," and "Joint Method," so as to show how far each possesses complete logical cogency.

4. Show exactly how Mill's Method of Agreement differs from Simple Enumeration.

On what presuppositions is the former Method applicable? What are its defects, and its utility, in scientific discovery?

5. Explain precisely (with an illustration) the principle of the Method of Difference, contrasting it with that of the Method of Agreement. What defects are there in the former and how can they be overcome?

6. Bring out the importance of the negative instance in the experimental methods, and point out under what circumstances it ceases to be available.

7. Illustrate the maxim: In experiment one circumstance only should be varied at a time. Explain the reason of this maxim, and point out the practical difficulties in carrying it out.

8. How does inductive enquiry deal with the difficulties due (*a*) to counteracting influences, (*b*) to exceptional phenomena?

What does Science understand by an "exception to a law of nature"?

9. "A perfect experiment establishes a law." On what grounds is this certainty of inference from experiment rested? Point out the difficulties in the way of securing data of the kind required.

10. What is meant by the Plurality of Causes? What difficulty in applying the methods of induction arises from the Plurality of Causes, and how far and in what way can the difficulty be met?

11. Explain, with examples, the Method of Concomitant Variations, showing the presuppositions under which it can be applied, its logical character, and its relation to the Method of Difference.

12. Explain the Method of Concomitant Variations. Give examples of cases where its application is especially profitable, and point out the limitations attaching to its use.

13. What special kinds of reasoning are involved in the establishment of quantitative uniformities? (such as the formula which connects the volume with the temperature of a body, or that of gravitation according to the inverse square of the distance.)

14. What do you consider to be the main function of the Method of Residues?

15. Examine the relation between the 'direct' and 'indirect' methods of inductive enquiry.

16. How far are the direct methods of inductive enquiry correctly described as methods of elimination?

17. Why are social phenomena so difficult to treat scientifically? What methods may be employed in treating them?

18. Describe the so-called "Deductive Method," and explain for what kinds of subjects it needs to be used in contrast with the so-called "experimental methods."

19. "In worms the sense of smell apparently is confined to the perception of certain odours, and is feeble. They were quite indifferent to my breath, as long as I breathed on them very gently. This was tried, because it appeared possible that they might thus be warned of the approach of an enemy. They exhibited the same indifference to my breath whilst I chewed some tobacco, and while a pellet of cotton-wool with a few drops of millefleurs perfume or of acetic acid was kept in my mouth. Pellets of cotton-wool soaked in tobacco juice, in millefleurs perfume, and in paraffin, were held with pincers and were waved about within two or three inches of

several worms, but they took no notice. On one or two occasions, however, when acetic acid had been placed on pellets, the worms appeared a little uneasy, and this was probably due to the irritation of their skins. The perception of such unnatural odours would be of no service to worms; and as such timid creatures would almost certainly exhibit some signs of any new impression, we may conclude that they did not perceive these odours."—(Darwin, *Earthworms*.)

Analyse the above passage so as to bring out the inductive character of the process by which the conclusion is reached.

20. "Wealth is greatly increased by the change from production on the small to production on a large scale, by the introduction of machinery and the division of labour. This holds equally if we compare a railway with a stage-coach, or a coach with a pack horse; a cotton mill with a spinning-wheel, or a spinning-wheel with a distaff and spindle. Under every form, at every stage, and in every period, wealth has been increased by improved and extended co-operation between human beings. This complex co-operation of many-sided individual effort, then, appears as the main-spring of industrial progress. Where it is not we have stagnation—primitive barbarism; where it is found, in whatever form or degree, there by one means and another industry is improved, and the material side of life made perfect."

Analyse the above example of inductive reasoning, point out what inductive method is employed, and consider whether the conclusion is or is not completely established.

21. Analyse logically the following passages, indicating the methods of reasoning employed, and estimating the logical validity of the conclusions drawn—

(*a*) "The general rule among vertebrates, as regards colour, is for the two sexes to be alike. This prevails with only a few exceptions, in fishes, reptiles, and mammalia; but in birds diversity of sexual colouring is exceedingly frequent, and is not improbably present in a greater or less degree in more than half of the known species. . . . The most fundamental characteristic of birds from our present point of view is a *greater intensity of colour in the male. This is the case in* hawks and falcons; in many thrushes, warblers, and finches; in pigeons, partridges, rails, plovers, and many others. . . . It is in tropical regions, where from a variety of causes colour has been developed to its fullest extent, that we find the most remarkable examples of sexual divergence of colour. The most

gorgeously coloured birds known are the birds of paradise, the chatterers, the tanagers, the humming birds, and the pheasant-tribe, including the peacocks. In all these the females are much less brilliant, and, in the great majority of cases, exceptionally plain and dull coloured birds. . . . The reason of this phenomenon [the dull colours of female birds] is not difficult to find, if we consider the essential conditions of a bird's existence, and the most important function it has to fulfil. In order that the species may be continued, young birds must be produced, and the female birds have to sit assiduously on their eggs. While doing this they are exposed to observation and attack by the numerous devourers of eggs and birds, and it is of vital importance that they should be protectively coloured in all those parts of the body which are exposed during incubation. To secure this end all the bright colours and showy ornaments which decorate the male have not been acquired by the female, who often remains clothed in the sober hues which were probably once common to the whole order to which she belongs. . . . This principle is strikingly illustrated by the existence of considerable numbers of birds in which both sexes are similarly and brilliantly coloured,—in some cases as brilliantly as the males of many of the groups above referred to. . . . When searching for some cause for this singular apparent exception to the rule of female protective colouring, I came upon a fact which beautifully explains it; for in all these cases, without exception, the species either nests in holes in the ground or in trees, or builds a domed or covered nest, so as completely to conceal the sitting-bird. . . . It has been objected that the domed nests of many birds are as conspicuous as the birds themselves would be, and would, therefore, be of no use as a protection to the birds and young. But, as a matter of fact, they do protect from attack, for hawks or crows do not pluck such nests to pieces, as in doing so they would be exposed to the attack of the whole colony. . . . Such birds as jays, crows, magpies, hawks, and other birds of prey, have also been urged as an exception; but these are all aggressive birds, able to protect themselves, and thus do not need any special protection for their females during nidification."—(Wallace, *Darwinism*, pp. 275-281.)

(*b*) "Lavoisier, reasoning with himself on the cause of the increase of weight in metals after calcination, argued as follows:

'if the increase in the weight of metals calcined in closed vessels is due, as Boyle had thought, to the addition of the matter of the flame and the fire which penetrate the pores of the glass and combine with the metals, then it follows that on introducing a known weight of metal into a glass vessel and sealing this hermetically, determining the weight exactly and then proceeding to calcination by a charcoal fire—just as Boyle had done—and then finally after calcination, before opening it, again weighing the same vessel, this weight must be found augmented by that of the whole quantity of fire matter which had been introduced during calcination. But if, said I to myself, the increase in the weight of the metal calx is not due to the addition of fire matter nor of any other extraneous matter, but to a fixation of a portion of the air contained in the vessel, the whole vessel after calcination must be no heavier than before and must merely be partially void of air, and the increase in the weight of the vessel will not occur until after the air required has entered.'" Two experiments were made with tin. In the first a small retort was used: in the second a large retort. The same quantity of tin—8 oz.—was used in each case. In both retorts a residue of tin remained after heating. The change of weight in the retort and its contents due to heating alone was (1) ·27 grains, (2) 1 grain. The increase in weight of the tin on calcination was (1) 3·12 grains, (2) 10 grains. The weight of air which took the place of that absorbed in calcination came out at (1) 3·13 grains, (2) 10·06 grains. Lavoisier concluded that (1) 'in a given volume of air only a fixed quantity of tin can be calcined.' (2) This quantity is greater in a large retort than in a small one. (3) The hermetically sealed retorts, weighed before and after the calcination of the tin contained in them, showed no difference of weight, which evidently proves that the increase in weight of the metal arises neither from the fire matter nor from any other matter extraneous to the vessel.—(Freund, *op. cit.*, pp. 47-49.)

(c) Bentley rejected the *Epistles of Phalaris* as forgeries He proved that "if the Epistles so belauded were true, Phalaris. had borrowed money from men who lived 300 years after his death, had destroyed towns that were not founded, and conquered nations that had no names: that he had overturned the chronology of Thucydides and Herodotus and proved their

histories mere fables: that he had transposed the events of his own day with those that happened long after he was dead: and finally that he had written in an Attic dialect which no man used during his lifetime, and which bore no resemblance to that which he, a Dorian, could have used."—(Craik, *Life of Jonathan Swift*, Vol. I., p. 88.)

(*d*) "Wages are necessarily determined by the minimum that is absolutely necessary for the support of the labourer and his family. . . . If we take this theory literally as meaning that the workman's wages can never rise above what he absolutely requires to live on, it is much too pessimistic and is manifestly contrary to facts. The purely material wants of life are, on the whole, of relatively little consequence. Irish and French peasants find it possible to live on next to nothing. If, then, the indispensable minimum for the bare support of life constituted an 'iron law of wages,' innumerable common-place facts would be inexplicable. Why is the rate of wages not the same in all trades? Must an engraver or a skilled mechanic consume more food-stuffs, more nitrogen and carbon, than a stone-breaker or a street-cleaner? Why, moreover, are wages higher in the United States than in France, Germany, or England? Is there any physiological reason why an American should eat more than an Englishman,—despite the fact both of them belong to the same race? Why are wages higher to-day than a century ago,—a fact which is beyond all question? Have we greater appetites than our forefathers? Again, why are the wages of farm labourers lower in winter, when they are obliged to spend more for heating and clothing, than in summer, when food is so cheap and life in the country is so easy that Victor Hugo calls this the 'poor man's season'?"—(Gide, *Principles of Political Economy*, Eng. Trans., pp. 502-503.)

(*e*) All acids contain oxygen. On this assumption the combination of ammonia, which does not contain oxygen, with hydrochloric acid would give as one of its products water which does contain oxygen. Hence the oxygen would have been present previously in the hydrochloric acid. But when the experiment of combining the two substances was made in 1812 it was found that no more than a slight dew was formed which was sufficiently accounted for by unavoidable imperfections in carrying out the experiment. Therefore hydrochloric acid does not contain oxygen, and the initial assumption

is disproved.—(Cf. Whewell, *History of the Inductive Sciences*, Vol. III., p. 124.)

(*f*) Faraday invented an instrument to measure the amount of electrolytical action when substances are decomposed by means of electricity. "In this instrument, the amount of action was measured by the quantity of water decomposed: and it was necessary in order to give validity to the mensuration, to show (as Faraday did show) that neither the size of the electrodes, nor the intensity of the current, nor the strength of the acid solution which acted on the plates of the pile, disturbed the accuracy of this measure. He proved, by experiments on a great variety of substances, of the most different kinds, that the electro-chemical action is definite in amount according to the measurement of the new instrument."—(Whewell, *op. cit.*, Vol. III., pp. 144-145.)

(*g*) It was at one time "a current opinion in science that the smallest bubble of oxygen or of air which might come in contact with a preserve would be sufficient to start its decomposition." To this it was objected: "'How can the germs of microscopic organisms be so numerous that even the smallest bubble of air contains germs capable of developing themselves in every organic infusion? If such were the case the air would be encumbered with organic germs.' M. Pouchet said and wrote that they would form a thick fog, as dense as iron." Was the assumption true? Pasteur prepared a series of bulbs half-filled in each case with a liquid which would very readily show the appearance of microscopic organisms. The liquid was boiled, the air expelled, the bulbs sealed. When the sealed end of a bulb was broken off, air was immediately admitted, and the bulb could then be sealed up again. Under these circumstances a change in the liquid could be readily observed. Pasteur showed that in every place where the experiment was carried out a certain number of bulbs would escape alteration. If, however, a room were dusted or swept the liquid in all the bulbs to which the air was then admitted became altered owing to the great quantity of germs raised and remaining suspended in the air. In towns most of the bulbs showed alteration. Of twenty opened in the country far from all habitations and then sealed again, eight contained organised productions: of twenty opened on the heights of the Jura, five only were altered: of twenty opened at a much greater altitude

in the vicinity of a glacier from which a strong wind was blowing, one alone was altered. Pasteur's conclusions were as follows: "More thickly spread in towns than in the country, the germs become fewer in proportion as they recede from human habitations. Mountains have fewer than plains, and at a certain height they are very rare."—(From *Louis Pasteur*, by his Son-in-Law, Eng. Trans., pp. 105-107.)

(*h*) "When those who were in the secret had decided on the time and place of the poisoning, Sejanus, with the most consummate daring, reversed his plan, and, whispering an accusation against Drusus of intending to poison his father, warned Tiberius to avoid the first draught offered him as he was dining at his son's house. Thus deceived, the old emperor, on sitting down to the banquet, took the cup and handed it to Drusus. His suspicions were increased when Drusus, in perfect unconsciousness, drank it off with youthful eagerness, apparently, out of fear and shame, bringing on himself the death which he had plotted against his father.—These popular rumours, over and above the fact that they are not vouched for by any good writer, may be instantly refuted For who, with moderate prudence, far less Tiberius with his great experience, would have thrust destruction on a son, without even hearing him, with his own hand too, and with an impossibility of returning to better thoughts. Surely he would rather have had the slave who handed the poison, tortured, have sought to discover the traitor, in short would have been as hesitating and tardy in the case of an only son hitherto unconvicted of any crime, as he was naturally even with strangers. But as Sejanus had the credit of contriving every sort of wickedness, the fact that he was the emperor's special favourite, and that both were hated by the rest of the world, procured belief for any monstrous fiction, and rumour too always has a dreadful side in regard to the deaths of men in power. Besides the whole process of the crime was betrayed by Apicata, Sejanus's wife, and fully divulged, under torture, by Eudemus and Lygdus. No writer has been found sufficiently malignant to fix the guilt on Tiberius, though every circumstance was scrutinised and exaggerated." —(Tacitus, *Annales*, Bk. iv., 10, 11. Trans. by Church and Brodribb, p. 118.)

CHAPTERS XXXIII.-XXXIV.

1. If a logician holds that no result of induction is absolutely certain, can he consistently still distinguish between inductive and probable reasoning?

2. "Calculations of probability in general do not express what will actually occur in the future, but only the degree of subjective confidence which we repose in their occurrence." Discuss this.

3. A bag contains three balls, each of which is known to be red, white, or blue. What is the chance of all three being white? Give reasons for your answer.

4. Discuss: "Probability after the event is based on exactly the same principles, and is formally the same process of reasoning as probability before the event."

5. Suppose that two witnesses, the probability of whose accuracy is $\frac{3}{4}$ and $\frac{2}{3}$ respectively, agree in affirming the occurrence of an event which, in itself, is as likely to have happened as not to have happened—*i.e.* whose antecedent probability is $\frac{1}{2}$—what is the probability that the event really did happen?

6. In what ways can errors in measurement be best allowed for?

7. Explain and exemplify the Methods of Means and of Least Squares.

8. What is meant by a scientific explanation? In what way can we explain (1) facts, (2) laws of nature?

9. How far is it true to say that explanation is the ideal of science?

10. Discuss the connexion between the scientific explanation of a phenomenon and the assignment of its cause.

11. "The object of science is explanation."

"Science never explains; she only reduces complex events to simple ones of the same kind, as when she deals with certain phenomena of magnetism by supposing every ultimate atom of the substance to act as if it were a magnet."

Carefully consider these statements.

12. Bring out the logical peculiarities of mathematical reasoning, and enquire whether it is radically distinct from other forms of scientific reasoning.

13. What is the function of statistics in scientific investigation?

14. What is a law of nature, and what is its difference from an empirical law? Explain the use for science of the discovery of empirical laws.

15. In what different ways may a uniformity, based upon a more or less satisfactory induction, be confirmed by deductive explanation? Under what circumstances has a wider uniformity a greater certainty than a narrower uniformity?

16. Indicate carefully the meaning of the term 'law' as employed in natural science. Distinguish between empirical laws, laws of nature, and ultimate laws.

"A truly universal law is not demonstrable truth." Discuss this.

17. Define the terms Fact, Theory, Hypothesis, Law, Cause, as used in Science.

In the light of your definitions, comment on the following statement: "What is called the scientific explanation of a fact is nothing more than showing it to be a case of a more general fact, which though more general, is still a fact merely."

MISCELLANEOUS QUESTIONS

1. Define Judgment, Proposition, Term, Concept; and illustrate from a logical point of view, the process of forming a concept.

Is every general term the name of a class? Give your reasons.

2. Write a brief logical comment, with illustration, on each of the following—"Circular Definition," "Proving too much," "Seeing is believing," "The exception proves the rule."

3. Define the following logical terms:—Undistributed Middle; Hypothetical Syllogism; Simple Destructive Dilemma; Enthymeme. Give an example of each.

4. Explain, with illustrations, the meaning of the following—relative terms, cross-division, a redundant definition, a prosyllogistic train of reasoning, a genetic definition, an *argumentum ad hominem*.

5. Indicate, with examples, the meaning of the following terms in Logic:—(i) Fundamentum divisionis; (ii) Strengthened syllogisms and Weakened syllogisms; (iii) Reduction *per impossible*; (iv) Universe of Discourse; (v) Quantification of the Predicate.

6. Define the following technical expressions, and illustrate in each case by an example—Inseparable Accident, Argument *à fortiori*, Sorites, *Ignoratio elenchi*.

7. Explain and illustrate the following Fallacies—*Petitio Principii*, *A dicto secundum quid ad dictum simpliciter* (and the converse), Composition, Division.

8. Write brief notes on, and illustrate, each of the following—'Proving too much'; 'Fallacy of Many Questions'; 'Definition by type'; 'Crucial Experiment.'

9. To what kinds of Fallacy is Inductive Reasoning most liable?

10. Comment on the following: "All so-called material fallacies are in reality either formal, or not fallacies at all, but mere mistakes in regard to matters of fact."

11. Explain, with examples, the following—A gratuitous hypothesis, colligation of facts, immaterial conditions of a phenomenon, predisposing and exciting causes, residual phenomenon.

12. Explain and discuss briefly the following—(i) mathematical induction, (ii) 'perfect' induction, (iii) mechanical and chemical composition of causes, (iv) empirical laws.

13. Write a short explanatory comment on (i) Intermixture of Effects, (ii) *Vera causa*, (iii) Coincidence as distinguished from a real connection of facts, (iv) *Non sequitur*.

14. Explain, with examples, the following—A crucial experiment, an adequate hypothesis, an empirical law, the composition of causes, the fallacy of mal-observation.

15. Make a logical analysis of the following—

"The catastrophe in Ireland fitted in with the governing moods of the hour, and we know only too well how simple and summary are the syllogisms of a rooted distrust. Ireland was papist, and this was a papist rising. The queen was a papist, surrounded at Somerset House by the same black brood as those priests of Baal who on the other side of St. George's Channel were described as standing by while their barbarous flock slew old men and women wholesale and in cold blood, dashed out the brains of infants against the walls in sight of their wretched parents, ran their skeans like Red Indians into the flesh of little children, and flung helpless protestants by scores at a time over the bridge at Portadown. Such was the reasoning, and the damning conclusion was clear. This was the queen's rebellion, and the king must be her accomplice. Sir Phelim O'Neil, the first leader of the Ulster rebellion, declared that he held a commission from the king himself, and the story took quick root. It is now manifest that Charles was at least as much dismayed as any of his subjects, yet for the rest of his life he could never wipe out the fatal theory of his guilt."—(John Morley, *Oliver Cromwell*, p. 108.)

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